

A MULTI-VARIATE/MULTI-ATTRIBUTE APPROACH FOR PLANT LAYOUT DESIGN

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This paper presents an integrated multivariate and multi attribute analysis approach for solving plant layout design problems. The integrated approach discussed in this paper is based Principal Component Analysis (PCA) and Analytic Hierarchy Process (AHP). The validity of the model is verified and validated by Numerical Taxonomy (NT) approach. Furthermore, a non-parametric correlation method, namely, Spearman correlation experiment is used to show the correlation between the findings of PCA and NT. The integrated PCA AHP approach of this study presents exact whereas previous DEA AHP studies presents incomplete and non exact plant layout alternatives. Furthermore, the superiority and effectiveness of PCA AHP approach is compared with previous DEA AHP study through a case study. The integrated PCA AHP would help policy makers and top managers to have precise understanding and improve existing systems with respect to facility layout performance. Furthermore, it provides complete and exact rankings of the plant layout alternatives. Moreover, Numerical Taxonomy is used to verify and validate the findings of PCA whereas previous DEA AHP does not have verification and validation feature.

Significance: The paper demonstrates an integrated method to solve the plant layout design problems. Moreover, the prescribed method (PCA AHP) can be used to obtain complete and exact ranking of various alternatives with respect to qualitative as well as quantitative criteria, whereas, with the previous method (DEA AHP) doesn't provide exact and complete rankings of plant layout alternatives. Furthermore, the PCA AHP model is verified and validated by Numerical Taxonomy and spearman correlation technique.

Keywords: Plant layout, PCA, AHP, Numerical taxonomy, DEA

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1. INTRODUCTION

Plant Layout involves the spatial arrangement of equipment within the steel structure or building of a plant and considers the inter-connections through pipes and ducts as well as walks and vehicle transportation. An optimal layout has to ensure operability, adequate safety and an economic design. Therefore, it is influenced by a whole range of factors such a process needs, maintenance, operational requirements, safety considerations and the available site. Plant layout is one of the main fields in industrial engineering where much research effort was spent and numerous approaches were developed before. However, most of them emphasized on the design stage and very few results were accomplished in the evaluation stage.

Algorithmic approach usually simplifies both design constraints and objectives to reach a total objective function, the solution of which can then be obtained. The majority of the existing literature reports and studies on algorithmic approach (Heragu, 1997). Algorithmic approach can generate layout alternatives efficiently, particularly, when commercial software is available such as Spiral (Goetschalckx, 1992) and LayOPT (Bozer et al., 1994). However, the resulting quantitative results of these software often do not capture all of the design objectives.

It should be noted that procedural approaches can incorporate both qualitative and quantitative objectives in the design process (Muther, 1973). For such approaches, the design process is divided into several steps that are solved sequentially. The success of a procedural approach implementation is mainly dependent on the generation of quality design alternatives, often provided by an experienced designer.

Layout generation and evaluation is often challenging and time consuming task due to its inherent multiple objective natures and its cumbersome data collection process (Lin and Sharp, 1999 b). Past and emerging research has been aimed at developing solution methodologies to meet these requirements. However, algorithmic approaches have focused mainly on minimizing flow distance in order to minimize material handling costs. Moreover, procedural approaches have relied heavily on experts' experience and judgment. One of the studies in this area reveals that neither an algorithmic nor a procedural layout design methodology is necessarily effective in solving practical design problems (Yang et al., 2000).

Foulds and Partovi (1998) applied AHP to evaluate a closeness relationship among planning departments for a layout problem. Cambron and Evans (1991) used different computer-aided layout design methods to generate a set of design alternatives that were then evaluated by AHP against a set of design criteria. Yang et al. (2000) used AHP to evaluate multiple-objective layout design alternatives generated from Muther's systematic layout planning (SLP) procedure (Muther, 1973). AHP is capable of providing objective weights against a set of qualitative layout evaluation criteria but is neither efficient in evaluating large number of alternatives nor in choosing the performance frontiers. Shang and Sueyoshi [9] proposed a unified framework to facilitate decision-making in the design and planning stage for the problem of selecting the most appropriate flexible manufacturing system (FMS). Yang et al (2003) used DEA-BCC model without inputs which is proposed by Knox et al (1999) and AHP for a similar study. They used a computer-aided layout planning tool to facilitate the layout alternative generation process as well as to collect quantitative performance data such as flow distance (or material handling cost), adjacency score, and shape ratio.

This study proposes an Analytic Hierarchy Process (AHP) and Principal Component Analysis (PCA) methodology to solve plant layout design problems. This paper highlights the fact that using proposed methodology is more appropriate; it gives a complete efficiency account and hence ranks the decision making units (DMUs) in a better fashion. PCA is a popular ranking method in multidimensional analysis, but no practical application by PCA to the plant design problem has been reported in the literature. AHP is applied to collect qualitative performance data and PCA is used to solve the layout design problem by simultaneously considering both the quantitative and qualitative performance data. Moreover, Numerical Taxonomy is used to verify the findings of the proposed algorithm. The case study illustrated by Yang and Kuo (2003) is used and applied to show the superiority of the integrated AHP DEA model over previous DEA AHP model. Moreover, the prescribed method (PCA AHP) can be used to obtain complete and exact ranking of various alternatives with respect to qualitative as well as quantitative criteria, whereas, with the previous method (DEA AHP) doesn't provide exact and complete rankings of plant layout alternatives. Furthermore, the PCA AHP model is verified and validated by Numerical Taxonomy and spearman correlation technique.

1.1 Principle Component Analysis

Principal Component Analysis (PCA) is widely used in multivariate statistics such as factor analysis. It is used to reduce the number of variables under study and consequently ranking and analysis of decision-making units (DMUs), such as industries, universities, hospitals, cities, etc (Azadeh et al 2004, Azadeh et al 2001, Azadeh et al 2002, Azadeh et al 2003). These DMUs utilize a variety of sources as inputs to produce several outputs. The purpose of another paper was to describe and demonstrate the applicability of two multivariate statistical techniques, namely principal component analysis and corresponding analysis, as analysis tools for quality professional. Using a principal component factor analysis with a rotation technique identifies seven hotels out of 33 hotel attributes and determines the level of satisfaction and service quality of Hong Kong hotels among Asian and Western travelers (Tat et al 2000). A multivariate analysis proposed a framework for measuring the efficiency of investment in IT that addresses some shortcomings include measurement errors, lags between investment and benefits, redistribution of profits, and mismanagement of IT resources. A paper suggests the utilization of multivariate technique as an objective optimization approach for the comparative efficiency evaluation of the maintenance department (Subhi, 2000). Furthermore, PCA captured the measurement correlations and reconstructed each variable to define associated residuals and sensor validity index. The beverage data was analyzed using PCA and cluster analysis (Rossi, 2001). A multivariate analysis was used to test whether there is any relationship between airline flight delays and the financial situation of an airline (Bhat, 1995). The PCA methodology has the advantage to be fairly simple to exploit, since it has been available in "off-the-shelf" computer packages for decades (Petroni et al, 2000).

In other words, the objective of PCA is to identify a new set of variables such that each new variable, called a principal component, is a linear combination of original variables. Second, the first new variable y_1 accounts for the maximum variance in the sample data and so on. Third, the new variables (principal components) are uncorrelated. PCA is performed by identifying Eigen structure of the covariance or singular value decomposition of the original data.

1.2 Analytic Hierarchy Process

Analytic hierarchy process (AHP) is a simple decision-making tool to deal with complex, unstructured and multi-attribute problems, Figure 1 Flow chart of the inference mechanism process and was first developed by Saaty (1980). Applications of AHP have been reported in numerous fields such as conflict resolution, project selection, budget allocation, transportation, health care, and manufacturing. The strength of AHP lies in its ability to mimic the management judgment about the importance that would be attached to different influential factors and to structure a complex and multi-attribute system matrix. The AHP consists of three basic steps: design of hierarchy, the prioritization procedure, and calculation of results. AHP initially breaks down a complex multi-criteria decision-making problem into a hierarchical structure. The decomposition forms the structure of the decision problem according to its main components. The top level of hierarchy, referred to as focus, consists of a single element or goal, which is the overall objective. The elements that affect the decision are called attributes or criteria, and are included in the subsequent levels, each of which may have several

elements. Attributes are mutually exclusive and their priorities are independent of the elements positioned below them in the hierarchy. The lowest level of hierarchy is referred to as alternatives, which are decision options (Saaty, 1980).

Once the problem has been decomposed and the hierarchy constructed, the prioritization procedure starts in order to determine the relative importance of the elements within each level. The pair-wise judgment starts from the second level (first level of attributes) and finishes in the lowest level alternatives. In each level, the elements are compared pair-wise with each other, according to their levels of influence upon an element position in the immediate higher level. The decision maker must express his/her preference between each pair of elements. Each pair-wise comparison is scored as equally important (1), weakly more important (3), strongly more important (5), very strongly more important (7), and absolutely more important (9) (Saaty, 1980; 1982). An even preferential number scoring system can also be used to represent comparisons among a pair of attributes. This method of ranking enables the decision maker to incorporate his/her experience and knowledge in an intuitive and natural manner. After forming the preference matrices, the mathematical process commences in order to normalize and find the priority weights for each matrix. The AHP process then determines the consistent nature of the pair wise comparisons (i.e. consistency ratio (CR)) for all matrices. If the CR value is larger than 0.10 (which is the acceptable upper limit for CR (Saaty, 1982)), it implies that there is a 10 per cent chance that the elements are not compared well. In this case, the decision maker must review the comparisons again. The mathematical process then starts to integrate the assigned weights in order to develop an overall evaluation process (i.e. the mathematical process to determine the CR values and the corresponding weights for each alternative). Although the mathematical process of AHP is tedious, the use of expert system makes it simple and accurate to apply (Turban, 1993).

1.3 Numerical Taxonomy

Numerical Taxonomy approach is capable of identifying homogeneous from non-homogeneous cases. Furthermore, a group of DMUs by given indexes is divided to homogeneous sub-groups (Agarwala, 1999; Azadeh & Ebrahimipour 2002; Azadeh & Ebrahimipour & Ataei, 2003). It also ranks the DMUs in a particular group. After defining new measures, for following Taxonomy process, the distance of every two DMUs for each indicator is computed and by notice of the distance matrix, we are able to rank the DMUs (Thomas, 1987; Cohen & Farach, 1997; Ricketts & Taylor, 1999; Gaibraith, & Haines, 2000; Harbottle, 2000).

1.3.1 Cluster Analysis

Cluster analysis is a mathematical approach used for combining observations into homogenous clusters or groups. Each cluster is homogenous with respect to particular characteristics. Also, each cluster is different from other clusters with respect to the same characteristics. There are two main types of analytical clustering techniques, namely: hierarchical and non-hierarchical (He, 1999;).

This study utilizes the non-hierarchical clustering approach. As mentioned, in the case of numerical taxonomy classification if all dis are within L1 and L2, homogeneity is achieved and cluster analysis is not required. Otherwise, cluster analysis is performed until all DMUs are homogenous. The cluster analysis is performed by the following algorithm:

1. The objective is to form n homogenous clusters. The first cluster is identified by checking the minimum distance between all the paired DMUs. The selected minimum distance is defined as centroid or seed.
2. The third DMU is assigned to the first cluster closest to the centroid of the first two selected DMUs.
3. The L1 and L2 of the three DMUs are evaluated. If the third DMU is not within the preceding range, the first cluster is formed and the same process as stated in step 1 is repeated by considering only the reminders of all DMUs for the formation of the new cluster. However, if the third DMU is within the preceding range, then it is selected as a member of the first cluster. This means that the next member of the first cluster must be identified. This is done through evaluation of the new centroid by considering the three DMUs. This process is repeated until the members of the cluster are identified. In general, steps 1, 2 and 3 are repeated until all new clusters are identified.

2. MULTIVARIATE/MULTI-ATTRIBUTE MODEL

This study concentrates on integrated layout generation and evaluation in global basis. To achieve the objectives of this study, a comprehensive study was conducted to locate layout design variables, which influence plant layout performance of industrial sectors. The steps for solving the layout design problem are illustrated in this section and shown in Figure 1. We used a practical case study to illustrate the efficiency and effectiveness of the proposed methodology. This case study is in regards to IC packaging process; it was used by Yang and Kuo (2003). The manager of plant would like to assure their future plant layout is efficient in supporting production activities. If the current layout is not efficient, company would like to know what layout alternatives are efficient. The experience learned from this study will provide the guidelines for future layout designs optimization and planning.

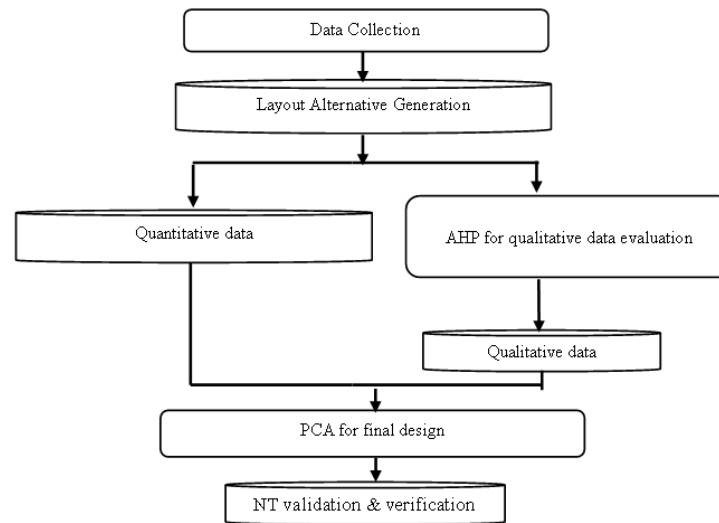


Figure 1: Steps to solve the design layout problem

2.1 Data Collection

Data collection should include characteristics of products, quantities, routing, support, and time considerations, in order to assure the validity of the input data at the design stage. The design objectives usually include both quantitative and qualitative criteria. The outputs of this stage become the inputs for the proposed hierarchical analysis procedure.

The IC packaging process is divided into 10 departments. The material flow is from departments 1 to 10 sequentially, regardless of product type. The departmental sizes are shown in Table 1. Also available floor space is 99.25 m (width) \times 27 m (length). The performance measures are flow distance, adjacency score, shape ratio, flexibility, accessibility, and maintenance.

Table 1: Determined sizes of the case study

No.	Department name	Size(m^2)
1	Wafer sawing	89.21
2	Die bond	181.51
3	Wire bond	577.38
4	Molding	599.57
5	Dejunk/trimming and curing	183.71
6	Electro deflash/solder plating	500.13
7	Marking	199.94
8	Forming and singulation	186.4
9	Lead scanning/inspection	110.78
10	Packaging	51.09

The flow distance is equal to the sum of the products of flow volume and rectilinear distance between the centroids of two departments. The adjacency score is the sum of all positive relationships between adjacent departments. There is a positive relationship between each two consecutive departments along the process routing. Shape ratio is defined as the maximum of the depth-to-width and width-to-depth ratio of the smallest rectangle that completely encloses the department. For a layout design problem, we would like to minimize both the shape ratio and flow distance, while maximizing adjacency score. Flexibility involves two aspects. The first is the capability to perform a variety of tasks under a variety of operating conditions. The second is the flexibility of future expansion. Accessibility involves material handling and operator paths. Finally, the maintenance issue involves the required space for maintenance engineers and tool movement.

2.2 Layout Alternative Generation

This step adopts a computer-aided layout planning tool, e.g., (Spiral, 1999), ALDEP (See Hof and Evans, 1967), BLOCPLAN (Donaghey and Pire, 1990), etc., to efficiently investigate a large number of design alternatives. When only

few alternatives are evaluated, the final solution may be trapped in a local optimum and may be inferior. A computer-aided tool can quickly provide quantitative performance measures that become parts of the DMU outputs.

Spiral was used for layout generation. Flow routing was converted to a from-to chart, which together with departmental sizes then became the input to Spiral. Spiral generated a layout alternative based on its embedded algorithm. We then choose its improvement algorithm, three-way pair-wise interchange, to generate a large number of alternatives ranking by flow distance in ascending order. Note that the strength of the proposed methodology is to efficiently evaluate a large number of layout alternatives to assure solution quality.

A preliminary study selected 17 alternatives for further evaluation. The quantitative measures for those design alternatives are shown in Table 2. Due to the larger-the-better criterion for DMU outputs, we take the reciprocal values of distance and shape ratio as the DMU outputs.

Table 2: Quantitative measures for plant layout alternatives

<i>Plant layout alternative</i>	<i>Distance (m)</i>	<i>Adjacency</i>	<i>Shape ratio</i>
1	185.95	8	8.28
2	207.37	9	3.75
3	206.38	8	7.85
4	189.66	8	8.28
5	211.46	8	7.71
6	264.07	5	2.07
7	228	8	14
8	185.59	9	6.25
9	185.85	9	7.85
10	236.15	8	7.85
11	183.18	8	2
12	204.18	8	13.3
13	225.26	8	8.14
14	202.82	8	8
15	170.14	9	8.28
16	216.38	9	7.71
17	179.8	8	10.3
18	185.75	10	10.16

2.3 AHP for Qualitative Data Evaluation

The purpose of the AHP is to provide a vector of weights expressing the relative importance of those layout alternatives for each criterion. AHP requires four steps: (1) structuring the hierarchy of criteria and alternatives for evaluation; (2) assessing the decision-makers evaluations by pair wise comparisons; (3) using the eigenvector method to yield priorities for criteria and for alternatives by criteria; and (4) synthesizing the priorities of the alternatives by criteria into composite measures to arrive at a set of ratings for the alternatives. The scale of importance is defined in Table 3 according to Saatty 1–9 scale for pair wise comparison.

Table 3: Scale of importance

Intensity of importance	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong or demonstrated importance
9	Extreme importance
2,4,6,8	For compromise between the above values

In order to avoid potential comparative inconsistency between pairs of categories, a consistency ratio (CR), an index for consistency, was calculated to assure the appropriateness of the comparisons. The resulting CR values for this case

study are 0.097, 0.088 and 0.098 for flexibility, accessibility and maintenance respectively. Since CR is smaller than the commonly critical value of 0.1, there is no evidence of inconsistency. The resulting weights are shown in Table 4.

Table 4: Qualitative measures for layout alternatives

<i>Plant layout alternative</i>	<i>Flexibility</i>	<i>Accessibility</i>	<i>Maintenance</i>
1	0.0494	0.0294	0.013
2	0.0494	0.0147	0.0519
3	0.037	0.0147	0.0519
4	0.037	0.0147	0.0519
5	0.0617	0.0147	0.039
6	0.0494	0.0147	0.0519
7	0.0247	0.0735	0.0649
8	0.037	0.0441	0.039
9	0.0741	0.0441	0.0519
10	0.0741	0.0588	0.0649
11	0.0864	0.1029	0.0909
12	0.037	0.0588	0.026
13	0.0247	0.0735	0.0519
14	0.0247	0.0588	0.0519
15	0.0864	0.1176	0.1169
16	0.0741	0.0735	0.0519
17	0.0988	0.1324	0.0909
18	0.0741	0.0588	0.039

2.4 PCA for Final Design

In this section, the PCA approach will be discussed. After identifying major variables that influenced plant layout design, principle component analysis is performed as follows:

There are 6 variables (indicators) and 18 plant layout alternatives. The six indicators must be normalized and have the same order to be used in PCA. Distance and shape ratio variables have opposite order compared to the rest of the variables. To solve this problem, these variables are inverted. Thus $X = (x_1 \dots x_6)_{18 \times 6}$ is a matrix composed by x_{ij} 's defined as the value of j^{th} variable (Distance, Adjacency, Shape ratio, Flexibility, Accessibility and Maintenance) for i^{th} layout. For obtaining PCA scores the following steps are performed:

Step 1: Standardize the variables (indicators) x_1 to x_6 . For this, calculate the sample mean vector \bar{x} and covariance matrix S:

$$\bar{x} = (\bar{x}_1 \dots \bar{x}_6)_{1 \times 6}$$

In which,

$$\bar{x}_j = \frac{1}{18} \sum_{i=1}^{18} x_{ij} \text{ for } j = 1 \dots 6$$

$$S = (s_{jq})_{6 \times 6} = \frac{1}{17} (X - \bar{x})^T (X - \bar{x}) \text{ for } q = 1 \dots 6$$

The indicators are standardized and shown in Table 5.

Step 2: Evaluate the correlation matrix. This matrix shows the values of linear correlation between variables (indicators) \hat{x}_1 to \hat{x}_6 .

Table 5: Standardized matrix for the 18 layout alternatives with respect to the 6 indicators

	\hat{X}_1	\hat{X}_2	\hat{X}_3	\hat{X}_4	\hat{X}_5	\hat{X}_6
1	0.760772	-0.2215	-0.368812	-0.259938	-1.121826	-1.749563
2	-0.266366	0.775248	0.798498	-0.259938	-1.076012	-0.149662
3	-0.223592	-0.2215	-0.31588	-0.783569	-1.076012	-0.149662
4	0.566257	-0.2215	-0.368812	-0.783569	-1.076012	-0.149662
5	-0.438831	-0.2215	-0.297372	0.259469	-1.076012	-0.680221
6	-2.180928	-3.211743	2.530136	-0.259938	-1.076012	-0.149662
7	-1.073173	-0.2215	-0.763621	-1.302976	0.508587	0.385009
8	0.780061	0.775248	-0.054952	-0.783569	-0.283713	-0.680221
9	0.766122	0.775248	-0.31588	0.7831	-0.283713	-0.149662
10	-1.353062	-0.2215	-0.31588	0.7831	0.112437	0.385009
11	0.91114	-0.2215	2.66542	1.302507	1.300887	1.454352
12	-0.127056	-0.2215	-0.733541	-0.783569	0.112437	-1.214892
13	-0.974526	-0.2215	-0.352192	-1.302976	0.508587	-0.149662
14	-0.066331	-0.2215	-0.33499	-1.302976	0.112437	-0.149662
15	1.68479	0.775248	-0.368812	1.302507	1.697036	2.523694
16	-0.637656	0.775248	-0.297372	0.7831	0.508587	-0.149662
17	1.100899	-0.2215	-0.558322	1.826138	2.095881	1.454352
18	0.771479	1.771996	-0.547618	0.7831	0.112437	-0.680221

$R = C_1 / \sqrt{s_{jj}} \cdot S \cdot C_1 / \sqrt{s_{jj}}$ where $C_1 / \sqrt{s_{jj}}$ is a $p \times p$ diagonal matrix whose j th diagonal element is $1 / \sqrt{s_{jj}}$ for $j = 1 \dots 6$. Then solve the following equation:
 $|R - \lambda I_6| = 0$ where I_6 is a 6×6 identity matrix. We obtain the ordered 6 characteristic roots (Eigen values) $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_6$ with $\sum_{j=1}^6 \lambda_j = 6$ and the related 6 characteristic vectors (eigenvectors) (l_{m1}, l_{m2}, l_{m6}).

Those characteristic vectors compose the principal components Y_i . The components in eigenvectors are respectively the coefficients in each corresponding Y_i :

$$Y_m = \sum_{j=1}^6 l_{mj} \hat{x}_{ij} \quad \text{for } m = 1 \dots 6 \text{ and } i = 1 \dots 18$$

The Eigen analysis for the data set is presented in Table 6.

Table 6: Eigenanalysis for layout problem

Variable	Eigenvalues	Weight	Eigenvectors					
1	2.67	0.44	0.45	-0.26	0.45	-0.60	-0.4	0.12
2	1.70	0.28	0.33	-0.55	0.20	0.14	0.70	-
3	0.71	0.12	-0.08	0.64	0.54	-0.23	0.34	-
4	0.41	-0.07	0.47	0.22	0.36	0.72	-0.26	0.11
5	0.29	0.05	0.51	0.15	-0.49	-0.08	-0.15	-
6	0.22	-0.04	0.45	0.39	-0.32	-0.20	0.38	0.60

Step 3: Finally, calculate the weights (w_j) of the principal components and PCA scores (z_i) of each layout ($i = 1, \dots, 18$). Furthermore, the z vector ($z_1 \dots z_{18}$) where z_j shows the score of j^{th} plant layout alternative is given by:

$$w_j = \lambda_j / \sum_{j=1}^6 \lambda_j = \lambda_j / 6 \quad j = 1 \dots 6$$

$$z_i = \sum_{j=1}^6 w_j Y_j \quad i = 1 \dots 18$$

The values of principal components, their aggregated weights and consequently layout rankings are shown in Table 7. Next section utilizes Numerical Taxonomy to verify and validate the findings of PCA. It should be mentioned that previous DEA AHP does not have verification and validation feature in comparison with the integrated PCA AHP model of this study.

Table 7: The values and scores of principal components for layout alternatives

Plant layout alternative	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	z _i (scores)	Rank
1	-0.96	-1.16	0.93	-0.18	-1.08	-0.32	-0.778	18
2	-0.70	-0.12	0.96	0.04	1.11	0.17	-0.179	8
3	-1.19	-0.40	-0.02	-0.22	0.18	0.70	-0.630	16
4	-0.78	-0.67	0.35	-0.74	-0.20	0.83	-0.494	12
5	-1.02	-0.32	0.45	0.75	-0.22	0.47	-0.549	15
6	-2.97	3.65	0.31	0.16	-0.39	0.08	-0.254	9
7	-0.67	-0.16	-1.74	-0.31	0.40	-0.06	-0.510	13
8	-0.22	-1.15	0.57	-0.75	0.21	-0.31	-0.290	10
9	0.78	-0.76	0.81	0.34	-0.08	0.27	0.183	4
10	-0.09	0.61	-0.72	1.34	0.21	0.24	-0.015	7
11	2.08	2.62	1.17	-0.68	0.38	-0.66	1.894	1
12	-0.94	-0.94	-0.42	-0.12	-0.63	-0.58	-0.729	17
13	-0.94	-0.11	-1.34	-0.30	0.34	-0.53	-0.533	14
14	-0.74	-0.39	-0.74	-0.80	0.05	-0.15	-0.451	11
15	3.65	0.42	-0.50	-0.52	0.12	0.70	1.690	2
16	0.54	-0.26	-0.22	1.12	0.37	-0.46	0.111	6
17	3.07	0.75	-0.70	0.27	-1.03	0.03	1.423	3
18	1.10	-1.61	0.85	0.60	0.27	-0.42	0.112	5

2.5 Validation and Verification

The integrated PCA AHP model is verified and validated by Numerical Taxonomy (NT) approach. Note that, cluster analysis is not required because none of the plant layout alternatives are out of limits. The Numerical Taxonomy approach is achieved as follows:

Step 1: Same as PCA, standard matrix is calculated (similar to step 1 in section 2.4).

Step 2: The distance of every two plant layout alternatives for each variable is computed. This is done to homogenize the plant layout alternatives. Therefore, the distance matrix $D = [d_{ij}]_{6 \times 6}$ and vector $d = [d_i]_{6 \times 1}$ where d_i is the minimum of i^{th} row of matrix D are identified (Table 8). To identify homogenous scenarios, the upper (L1) and lower (L2) limits of vector d is computed as $L1 = \bar{d} + 2s_d$ and $L2 = \bar{d} - 2s_d$ where \bar{d} and s_d are the mean and standard deviation of vector d respectively. If all d_{ij} are within L_1 and L_2 , homogeneity is achieved and we go to next step. Otherwise, cluster analysis is performed until all layouts are homogenous.

Step 3: The distance of each plant layout alternative from the ideal plant layout alternative for each index is computed as:

$$c_i = \sqrt{\sum_{j=1}^6 (\hat{x}_{ij} - \hat{x}_{j_{max}})^2}$$

Where, $\hat{x}_{j_{\max}}$ is the maximum of the j^{th} variable. The growth level for each layout is:

$$f_i = \frac{c_i}{c^*} \text{ where } c^* = \bar{c} + 2s_c$$

And \bar{c} and s_c are mean and standard deviation of c_i 's respectively. f_i 's are between 0 and 1 with 1 as the worst and 0 as the best scores. Table 9 shows f_i 's and layout rankings.

Table 8: Distance matrix for the layout problem

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		2.50	2.04	1.74	1.78	5.32	3.24	1.64	2.19	3.31	5.11	1.47	2.89	2.26	5.33	2.86	4.78	2.63
2	2.50		1.58	1.85	1.67	4.70	2.82	1.76	2.04	2.50	4.12	2.50	2.56	2.21	4.79	2.25	4.73	2.61
3	2.04	1.58		0.90	1.18	4.53	1.98	1.78	2.29	2.32	4.83	1.68	1.85	1.34	4.95	2.49	4.70	3.07
4	1.74	1.85	0.90		1.59	4.99	2.44	1.44	2.04	2.85	4.71	1.81	2.35	1.51	4.66	2.77	4.50	2.87
5	1.78	1.67	1.18	1.59		4.48	2.59	2.09	1.95	1.92	4.73	1.78	2.38	2.08	5.01	2.05	4.50	2.72
6	5.32	4.70	4.53	4.99	4.48		5.00	5.66	5.83	4.52	5.40	5.14	4.70	4.86	7.49	5.45	6.81	6.74
7	3.24	2.82	1.98	2.44	2.59	5.00		2.60	3.12	2.20	4.92	1.95	0.68	1.20	4.61	2.44	4.27	3.59
8	1.64	1.76	1.78	1.44	2.09	5.66	2.60		1.67	3.07	4.47	1.65	2.35	1.61	4.43	2.35	4.32	1.96
9	2.19	2.04	2.29	2.04	1.95	5.83	3.12	1.67		2.47	3.92	2.39	3.04	2.51	3.51	1.65	3.28	1.22
10	3.31	2.50	2.32	2.85	1.92	4.52	2.20	3.07	2.47		4.15	2.60	2.22	2.49	4.22	1.40	3.57	3.13
11	5.11	4.12	4.83	4.71	4.73	5.40	4.92	4.47	3.92	4.15		5.07	4.80	4.58	3.47	3.99	3.37	4.54
12	1.47	2.50	1.68	1.81	1.78	5.14	1.95	1.65	2.39	2.60	5.07		1.59	1.25	5.03	2.29	4.44	2.75
13	2.89	2.56	1.85	2.35	2.38	4.70	0.68	2.35	3.04	2.22	4.80	1.59		0.98	4.86	2.34	4.44	3.47
14	2.26	2.21	1.34	1.51	2.08	4.86	1.20	1.61	2.51	2.49	4.58	1.25	0.98		4.56	2.41	4.26	3.08
15	5.33	4.79	4.95	4.66	5.01	7.49	4.61	4.43	3.51	4.22	3.47	5.03	4.86	4.56		3.79	1.70	3.87
16	2.86	2.25	2.49	2.77	2.05	5.45	2.44	2.35	1.65	1.40	3.99	2.29	2.34	2.41	3.79		3.25	1.89
17	4.78	4.73	4.70	4.50	4.50	6.81	4.27	4.32	3.28	3.57	3.37	4.44	4.44	4.26	1.70	3.25		3.73
18	2.63	2.61	3.07	2.87	2.72	6.74	3.59	1.96	1.22	3.13	4.54	2.75	3.47	3.08	3.87	1.89	3.73	

Table 9: The plant layout rankings by Numerical Taxonomy

Plant layout alternative	f_i	Rank
1	0.82	15
2	0.67	8
3	0.78	14
4	0.76	12
5	0.78	13
6	0.96	18
7	0.78	16
8	0.69	9
9	0.61	4
10	0.69	7
11	0.40	2
12	0.81	17
13	0.77	11
14	0.74	10
15	0.32	1
16	0.62	5
17	0.49	3
18	0.63	6

PCA scores with Taxonomy rankings of layouts are shown in Table 10. Table 10 also reports the results of nonparametric statistical tests of the relationship between PCA and Taxonomy rankings by Spearman correlation technique, which result in the rejection of H_0 at 0.01 significant levels. In addition, the Kendall's Tau test of correlation verifies this finding at the same level of significance. There is a direct relationship between PCA and Taxonomy in terms of data set presented for layouts. Hence, the results of PCA are verified by Numerical Taxonomy. Yang et al (2003) in their conclusion shows that the plant layouts 11, 15 and 18 are equally efficient, but the approach adopted in this paper shows that the above plant layout alternatives have exact and distinct ranking with respect to their efficiency as shown in Table 10.

Table 10: Test of correlation between PCA, DEA and Taxonomy rankings

Plant layout alternative	PCA rank	Taxonomy rank	DEA Efficiency scores by Yang et al (2000)	Equivalent DEA ranks
1	16	15	91.69	8
2	8	8	98.45	2
3	14	14	86.39	13
4	13	12	89.94	9
5	11	13	86.44	12
6	15	18	96.62	4
7	17	16	80.77	16
8	9	9	96.51	5
9	4	4	95.69	6
10	6	7	87.44	10
11	2	2	100	1
12	18	17	85.72	15
13	12	11	86.29	14
14	10	10	86.66	11
15	1	1	100	1
16	5	5	96.74	3
17	3	3	94.63	7
18	7	6	100	1
Result of tests of correlation between PCA and Taxonomy	$r_s = 0.922$ & $\tau = 0.979$ Reject at $\alpha = 0.01$			

3. CONCLUSION

This paper presents an integrated multivariate and multi-attribute approach, based on the use of principle component analysis and analytical hierarchical process respectively, aimed at helping managers to solve plant layout design problem. An application of methodology using a case study which was illustrated by Yang and Kuo (2003) is described. The prescribed method (PCA AHP) can be used to obtain complete and exact ranking of various alternatives with respect to qualitative as well as quantitative criteria, whereas, with the previous method (DEA AHP) doesn't provide exact and complete rankings of plant layout alternatives. Furthermore, the PCA AHP model is verified and validated by Numerical Taxonomy and spearman correlation technique.

The strength of the proposed method lies in using of PCA which its property is in simultaneously considering multiple inputs and outputs without any need for a prior assignment of weights. PCA also has a distinct advantage over the traditional methods of evaluating layouts performance in that it is not necessary to state the performance measures in the same unit. Moreover the integrated PCA AHP methodology enables evaluation of large number of design alternatives with less process than the traditional methods. In summary, the integrated PCA AHP approach of this study presents exact rankings whereas previous DEA AHP studies presents incomplete and non exact plant layout alternatives. Furthermore, the superiority and effectiveness of PCA AHP approach is compared with previous DEA AHP study through a case study. The integrated PCA AHP would help policy makers and top managers to have precise understanding and improve existing systems with respect to facility layout performance. Furthermore, it provides complete and exact rankings of the plant layout alternatives. Moreover, Numerical Taxonomy is used to verify and validate the findings of PCA whereas previous DEA AHP does not have verification and validation feature.

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