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Determining the Periodic Maintenance Interval for Guaranteeing the Availability of a System with a Linearly Increasing Hazard Rate

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This paper discusses the determination of the optimal frequency for performing preventive maintenance (PM) actions on a system that exhibits a linearly increasing hazard rate and a constant repair rate. Under a periodic maintenance policy, where a predetermined availability is guaranteed, the time interval between PM actions incurring the minimal maintenance cost is selected. Considering both cost and system availability, guidelines for determining the maintenance time interval are presented. A simple example is also provided to illustrate these ideas.

Keywords: Availability; Hazard rate; Corrective maintenance; Preventive maintenance

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Nomenclature:

λ : failure rate
μ : repair rate
$h(t)$: hazard function
$H(t)$: cumulative hazard function
A^* : a predetermined availability
A_1 : availability when Policy 1 is performed
A_2 : availability when Policy 2 is performed
$\tilde{A}_2$: availability incurring the minimal maintenance cost when Policy 2 is performed
c_p : cost of a preventive maintenance (PM) action replacing a system by a new one
c_r : cost of a corrective maintenance (CM) action through minimal repair
c_f : cost of a corrective maintenance (CM) action replacing a system by a new one
c_o : opportunity cost per unit time due to the inability of a system while replacing it
MC_1 : maintenance cost per unit time when Policy 1 is performed
$MC_2(T)$: maintenance cost per unit time over $(0, T)$ when Policy 2 is performed
T : time interval between PM actions
t^* : time interval between PM actions guaranteeing the predetermined availability A^*
T^* : time interval incurring the minimal maintenance cost per unit time

1. INTRODUCTION

Engineering systems such as aircrafts or manufacturing equipment are becoming more complex. At the same time, the users demand a very high availability which can be achieved in minimizing of system downtime. Yet, most engineering systems undergo degradation due to abrasion and aging. Such degradation causes the system to increase failures. These failures make the system lifetime shorten and total cost, including operation cost and opportunity cost during the failure time, increase. In order to reduce such losses, the system needs maintenance actions.

Maintenance actions are divided into corrective maintenance (CM) and preventive maintenance (PM). CM actions are unscheduled and are intended to restore a system from a failed state to a working state either through repair or replacement of failed components. On the other hand, PM is scheduled active and can be carried out either to reduce the likelihood of a failure or to improve the availability of the system [1]. As the PM effort increases, the PM cost increases and the CM cost decreases. Since the total cost of PM and CM actions together first decreases and then increases with increasing PM effort, there is an optimal level of PM effort that will minimize the total maintenance cost. Minimization of the total cost is one approach to determining the optimal maintenance actions.

Many authors have proposed several PM models, either theoretical or practical, and obtained the best PM policies by optimizing several criteria regarding the operating cost. References to these can be found in papers as follows.

Most of the PM studies of systems consider a cost based approach. As Wang [16] has pointed out, all maintenance actions aim to improve the reliability performance of the system, whereas PM modeling traditionally focuses on the cost optimization as the main criterion, ignoring the system reliability requirements. While Kardon and Fredendall [11] did take machine reliability into consideration when developing PM plans, their study did not consider the cost of performing unnecessary maintenance actions on some of the machines, as called for in block replacement policies. More comprehensive discussion in PM from both theoretical and application point of view can be found in Juang and Anderson [10]. They studied a Bayesian approach on the random parameters of a Weibull distribution to solve the optimal replacement problem for both block replacement protocol with minimal repair and the age replacement protocols by minimizing the expected long-run average cost. Also Wu and Derek [18] consider the scenarios when the maintenance quality is a random variable. It assumes that both the adjustment factor and the restoration interval are random variables with certain probability distributions. Prior research assumes that the quality of PM is a fixed constant, which is usually not the case in many real situations. Their paper studied the optimization problem of PM policies for the situations where the quality of PM is a random variable with a certain probability distribution, which would be more practical than the situation when the quality of maintenance is assumed to be a fixed constant.

A viewpoint of an availability of PM has recently being receiving more attention. Cui and Xie [4] have discussed the instantaneous availability function under two models, when the perfect repair or replacement time is a constant or random. When the repair or replacement time is a constant, or when the random repair or replacement time has a discrete distribution, they provided some recursive equations for the instantaneous availabilities for two models. Also Tsai et al. [15] presented availability-centered PM in simultaneously considering three actions, mechanical service, repair and replacement for a multi-components system based on availability.

In this study, two maintenance policies are considered:

- Policy 1: As a CM action, a system is replaced by a new one whenever it fails. The time taken to replace a failed system is a random variable with a constant mean time. There is no PM action.

- Policy 2: As a PM action, a system is periodically replaced by a new one at set times $t = kT$, $k = 1, 2, \ldots$. The time taken to replace a system is the same random variable as in Policy 1. Since the replacement cost of Policy 2 (c_p) might include the opportunity cost caused by replacing a nonfailed system by a new one and the replacement cost of Policy 1 (c_f) might include the penalty for failure, which replacement cost is larger depends on the results of cost analysis. As a CM action, any failure between replacement times is repaired minimally. When a failed system is subjected to minimal repair, the hazard rate of the system after a repair is the same as the hazard rate of the system immediately prior to failure. The time required to repair a failed system minimally is negligible. It is common that the minimal repair cost (c_r) would be smaller than the above two kinds of replacement cost.

In this paper, we analyze the determination of the optimal frequency for performing PM actions on a system when a combination involving both availability and cost issues is used as a measure of the system performance. It is in particular assumed that the decision for selecting a maintenance policy between Policy 1 and Policy 2 is based on the following criteria: Primarily, the system should be maintained to guarantee a predetermined desired availability, and, secondly, as long as the predetermined availability can be guaranteed, the maintenance policy which incurs a smaller cost is preferred. The system is assumed to exhibit a linearly increasing hazard rate. It is also assumed to have a constant repair rate as in [5].

2. SYSTEM AVAILABILITY AND COST UNDER THE MAINTENANCE POLICY WITHOUT PM ACTIONS

Engineering systems typically degrade over time and thus more likely to exhibit a failure distribution characterized by a strictly increasing hazard function. The Weibull distribution is considered to be the most widely used distribution for modeling failure in reliability applications because of its substantial flexibility. The hazard function of the Weibull distribution has the form of

$$h(t) = \alpha\beta^{\alpha} t^{\alpha-1} \tag{1}$$

for all $t \geq 0$, where $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters, respectively. The mean of the Weibull distribution is

$$\frac{1}{\beta}\Gamma\left(1+\frac{1}{\alpha}\right) = \frac{1}{\alpha\beta}\Gamma\left(\frac{1}{\alpha}\right), \tag{2}$$

where $\Gamma(\bullet)$ is the gamma function. The hazard function in equation (1) is appropriate for modeling lifetimes with constant ($\alpha = 1$), strictly increasing ($\alpha > 1$), and strictly decreasing ($\alpha < 1$) hazard functions. One special case occurs when $\alpha = 2$. The Weibull distribution in this case is commonly known as the Rayleigh distribution [12] whose hazard function is a line with slope $2\beta^2$, i.e.,

$$h(t) = 2\beta^2 t\ . \tag{3}$$

Therefore, a system that exhibits a linearly increasing hazard function is characterized by the Rayleigh distribution. Substituting 2 for α in equation (2), the mean of the Rayleigh distribution becomes

$$\frac{1}{2\beta}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2\beta}\ . \tag{4}$$

The system's availability, where $\bar{U}$ and $\bar{D}$ denote the average length of the up and down periods of a system respectively, is $A = \frac{\bar{U}}{\bar{U}+\bar{D}}$. If the operating and repair time distributions for a system are arbitrary continuous distributions with respective means $1/\lambda$ and $1/\mu$, then it follows from the theory of alternating renewal processes [13] that the availability A is

$$A = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{\mu}{\mu+\lambda}\ . \tag{5}$$

When Policy 1 is applied, the availability of the system which follows the Rayleigh lifetime distribution and has a constant repair rate μ is

$$A_1 = \frac{\frac{\sqrt{\pi}}{2\beta}}{\frac{\sqrt{\pi}}{2\beta}+\frac{1}{\mu}} = \frac{\mu\sqrt{\pi}}{\mu\sqrt{\pi}+2\beta}, \tag{6}$$

and the maintenance cost per unit time becomes

$$MC_1 = \lambda c_f + \frac{\lambda}{\mu} c_o. \tag{7}$$

Since the reciprocal of the mean of the Rayleigh distribution can replace λ, the maintenance cost per unit time (equation (7)) can be rewritten as

$$MC_1 = \frac{2\beta}{\sqrt{\pi}}\left(c_f + \frac{c_o}{\mu}\right). \tag{8}$$

When corrective maintenance without PM actions (Policy 1) does not guarantee a predetermined availability A^* of the system, that is, when $A_1 < A^*$, performing PM actions (Policy 2) achieves this purpose. It should be noted, however, that even when corrective maintenance with no PM actions (Policy 1) guarantees the predetermined availability A^* of the system, that is, when $A_1 > A^*$, it is necessary to choose between performing no PM action (Policy 1) and performing PM actions (Policy 2). This is because the system has to achieve the minimal maintenance cost per unit time.

3. SYSTEM AVAILABILITY AND COST UNDER THE MAINTENANCE POLICY WITH PM ACTIONS

When corrective maintenance with Policy 1 does not guarantee a predetermined availability A^* of the system, that is, when $A_1 < A^*$, implementing Policy 2 should be considered. Under a periodic maintenance policy (Policy 2), the times at which preventive maintenance occurs are multiples of a cycle length T, i.e., $t = kT$, $k = 1, 2, \ldots$, and under this policy the system availability becomes

$$A_2 = \frac{T}{T + \frac{1}{\mu}}. \tag{9}$$

Since the cycle cost is the sum of the preventive replacement cost at the end of the cycle and the cost of corrective repairs during the cycle, the maintenance cost per unit time is

$$\begin{aligned} MC_2(T + \frac{1}{\mu}) &= \frac{c_p + c_r \int_0^T 2\beta^2 t\,dt + \frac{c_o}{\mu}}{T + \frac{1}{\mu}} \\ &= \frac{c_p + c_r\,\beta^2 T + \frac{c_o}{\mu}}{T + \frac{1}{\mu}}. \end{aligned} \tag{10}$$

Setting $\frac{dMC_2(T)}{dT} = 0$ and solving yields the time interval resulting in the minimal maintenance cost per unit time:

$$T^* = \sqrt{\frac{1}{\mu^2} + \frac{\mu c_p + c_o}{\mu c_r\,\beta^2}} - \frac{1}{\mu}, \tag{11}$$

the corresponding cost of which is

$$MC_2(T^* + \frac{1}{\mu}) = \frac{c_p + c_r\,\beta^2 T^* + \frac{c_o}{\mu}}{T^* + \frac{1}{\mu}}. \tag{12}$$

From equation (9), the corresponding availability of the system $\tilde{A}_2$ now would be

$$\tilde{A}_2 = \frac{T^*}{T^* + \frac{1}{\mu}} = \frac{\sqrt{\frac{1}{\mu^2} + \frac{\mu c_p + c_o}{\mu c_r \beta^2}} - \frac{1}{\mu}}{\sqrt{\frac{1}{\mu^2} + \frac{\mu c_p + c_o}{\mu c_r \beta^2}}}. \tag{13}$$

By equation (9), the time interval between PM actions t^* guaranteeing the predetermined availability A^* is

$$A^* = \frac{t^*}{t^* + \frac{1}{\mu}} \quad \Rightarrow \quad t^* = \frac{A^*}{\mu(1-A^*)}, \tag{14}$$

Equation (10) shows that the maintenance cost per unit time corresponding to t^* guaranteeing the predetermined availability A^* is

$$MC_2(t^* + \frac{1}{\mu}) = \frac{c_p + c_r \beta^2 \frac{A^*}{\mu(1-A^*)} + \frac{c_o}{\mu}}{\frac{A^*}{\mu(1-A^*)} + \frac{1}{\mu}} \tag{15}$$

Equation (9) also shows that under the periodic maintenance policy (Policy 2), to maintain the availability A^*, the time interval between two consecutive PM actions must be equal to or larger than t^*.

Therefore, in case of $A_1 < A^*$, Policy 1 is neglected and the time interval between the PM actions of Policy 2 is determined as follows: If $T^* \geq t^*$, the time interval is optimal, since the system availability is $\tilde{A}_2$, and the maintenance cost per unit time is minimized. On the other hand, if $T^* < t^*$, selecting t^* as the time interval between PM actions can guarantee the availability A^*, and the system availability is A^*. It incurs the maintenance cost per unit time of equation (15). Consider the case of $A_1 \geq A^*$. Here, the minimal maintenance cost per unit time over the time interval $(0, T^*)$ of equation (12) should be compared with the maintenance cost per unit time of equation (8). If the former cost is greater than the latter cost, that is, when the following inequality (16) holds

$$MC_2(T^*) > MC_1 \\ \Rightarrow 2\beta\sqrt{\frac{\mu c_p c_r + c_p c_o}{\mu}} > \frac{2\beta\mu c_f + \sqrt{\pi}\, c_o}{\mu\sqrt{\pi}}, \tag{16}$$

then Policy 1 should be applied, resulting in system availability A_1. In the opposite case of inequality (16), when the minimal maintenance cost incurred from Policy 2 is less than the maintenance cost per unit time incurred by Policy 1, the optimal maintenance policy should be determined as follows: (i) If $T^* \geq t^*$, the time interval between PM actions is T^*, and the system availability in turn becomes $\tilde{A}_2$, which is greater than A^*. (ii) If $T^* < t^*$, the maintenance costs per unit time of the two policies should now be compared. If $MC_2(t^*) < MC_1$, the system should be maintained by Policy 2, and in particular, the time interval between PM actions and the system availability are t^* and A^*, respectively. On the other hand, if $MC_2(t^*) > MC_1$, the system should be maintained by Policy 1, where the system availability becomes A_1.

4. NUMERICAL EXAMPLE

This section present numerical example about the decision-making process determining the optimum maintenance policy and interval guarantee the predetermined availability. As we have mentioned earlier, the failure system assumed linearly increasing hazard rate.

The following numerical cases show the selection of a maintenance policy (Policy 1 or 2) while considering system availability and its maintenance cost:

[Case 1] Consider the case that Policy 1 does not guarantee the predetermined availability or the time interval between PM actions incurring the minimal maintenance cost guarantees the predetermined availability: A system is found to exhibit an linearly increasing hazard rate, $h(t) = 8\times10^{-6}t$ (t is in hours) and a constant repair rate $\mu = 2\times10^{-2}$. The desired availability of the system is predetermined to be $A^* = 0.96$.

By equations (3) and (6), β^2 is 4×10^{-6} , and the corresponding availability A_1 is 0.898. Since A_1 is less than A^*, PM actions are applied. By equation (14), the time interval between two consecutive PM actions must be equal to or larger than $t^* = 1617$ hours in order to keep the availability $A^* = 0.97$.

Furthermore, it is assumed that the cost of a PM action replacing a system by a new one (c_p) is \$4000 and the cost of a CM action through minimal repair (c_r) is \$100. Also, opportunity cost per unit time (c_o) is \$20. Using equations (11) and (10), the time interval incurring the minimal maintenance cost per unit time T^* is 3162 hours, which is larger than $t^* = 1617$ hours, and the minimal maintenance cost per unit time is \$4.25. Figure 1 shows the above result. In this case, the resulting system availability $\tilde{A}_2$ becomes 0.984.

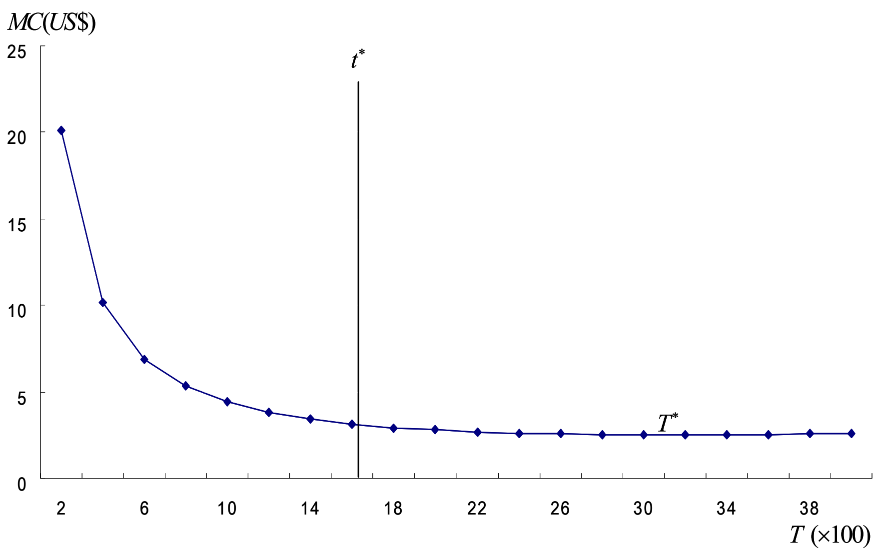

Figure 1. Policy 1 does not guarantee the predetermined availability: The minimal maintenance cost per unit time incurs when Policy 2 is performed with $T^* = 3162$.

[Case 2] Consider the case that both policies can guarantee the predetermined availability, and Policy 1 incurs a maintenance cost less than the minimal maintenance cost incurred by Policy 2: Consider the case of $h(t) = 2\times10^{-6}t$, $\mu = 2\times10^{-2}$, $A^* = 0.92$, $c_p = \$4000$, $c_r = \$300$, and $c_f = \$1500$ $c_o = \$30$.

From equation (6), the availability is $A_1 = 0.947$. Since A_1 is greater than A^*, both policies are options. This case needs the policy that incurs the minimal maintenance cost. Equations (8) and (12) show the maintenance cost per unit time of Policy 1 (\$1.69) is less than the minimal maintenance cost per unit time of Policy 2 (\$4.36). Figure 3 shows the result. Therefore, Policy 1 is the optimal maintenance policy, and the system availability A_1 is 0.947.

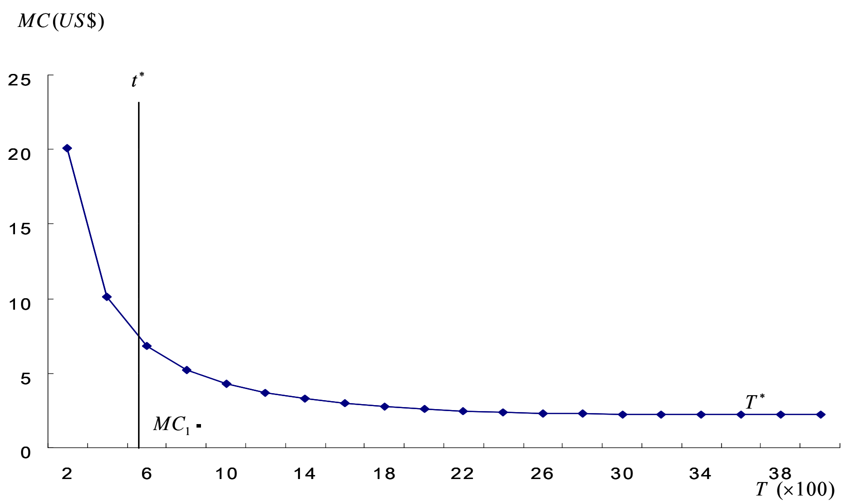

Figure 2. Both Policy 1 and Policy 2 can guarantee the predetermined availability: The maintenance cost per unit time is minimal when Policy 1 is applied.

Table 1 summarizes the data used and the experimental results explained in the above two cases.

Table 1. Experimental results

Case	A^*	c_p	c_r	c_f	c_o	$h(t)$	μ	t^*	$MC_2(t^*)$	T^*	$MC_2(T^*)$	$\tilde{A}_2$	A_1	MC_1
1	0.97	4000	100	2000	20	0.000008	0.02	1617	3.12	3162	4.25	0.984	0.899	4.51
2	0.92	4000	300	1500	30	0.000002	0.02	575	7.13	3651	4.36	0.986	0.947	1.69

5. MAINTENANCE POLICY SELECTION

A particular maintenance policy is usually pre-specified under practical considerations such as the cost of equipment and maintenance difficulty. Maintenance policies of repairable systems have been widely investigated in the literature [3, 7, 16, 18] and can be summarized as follows [3, 15].

1. Corrective maintenance policy: No inspection is performed. The system is replaced only when it is in the failed-state.
2. Block-based maintenance policy: The equipment is subject to PM at the end of each replacement to eliminate the wear-out failures during the replacement. Regardless of any CM operations between the two scheduled PMs, the PM operations are always carried out as scheduled at the end of the replacements without affecting the production schedule.
3. Age-based maintenance policy: PM is scheduled at the end of a replacement, but the PM time changes as the equipment undergoes CM. If the equipment fails and a CM is carried out before the next PM, then the next PM is rescheduled when the CM is completed.
4. Opportunity-triggered maintenance policy: PM operations are carried out only when they are triggered by failure. Thus, CM as well as PM is applied to the system at the time of a failure.
5. Conditional opportunity-triggered maintenance policy: This is a special case of opportunity-triggered maintenance policy. In this policy, PM is performed on each machine at either scheduled times or when a specified condition based on the occurrence of a CM arises. The maintenance operation can define the specified condition.

Among the five policies stated above, the corrective maintenance and the block-based policies are focused in this research. Under these policies, decision processes for selecting a maintenance policy (Policy 1 or 2) are developed while considering system availability and its maintenance cost. General situations with the conditions on system availability and its maintenance cost are as follows.

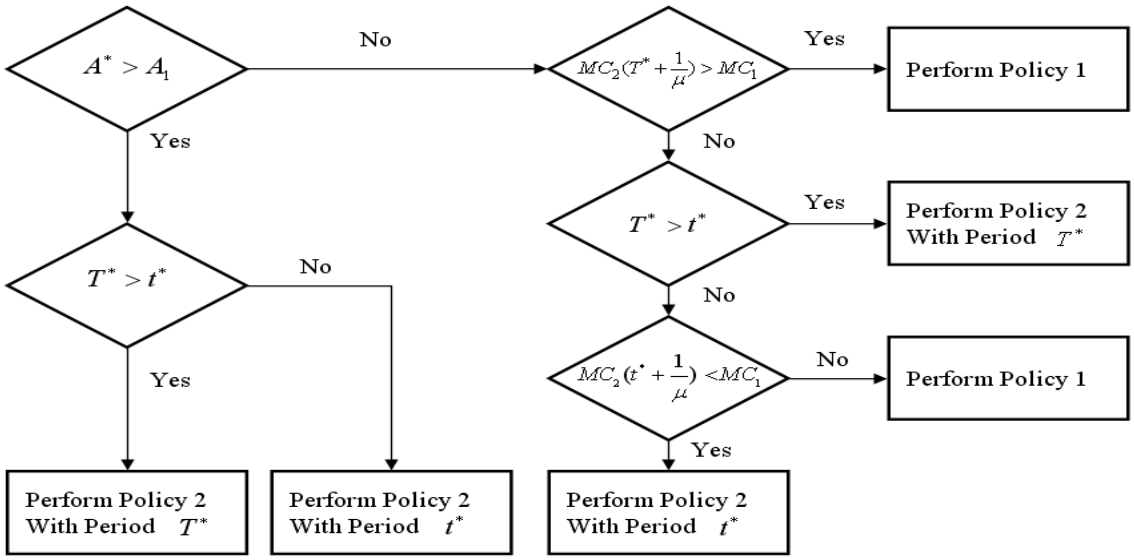

Figure 3. Decision process for selecting the optimal maintenance policy

6. CONCLUSIONS

This research proposed the optimal policy of preventive maintenance for a system exhibiting a linearly increasing hazard rate. A system showing a linearly increasing hazard rate exhibits a Rayleigh lifetime distribution. Hence, when availability is considered as an operational measure of the system performance, the system's availability can be calculated.

When corrective maintenance with no PM actions (Policy 1) does not guarantee a desired and predetermined availability of the system, applying PM actions (Policy 2) is a way to achieve this purpose. However, it should be noted that even when applying corrective maintenance with no PM actions (Policy 1) guarantees the predetermined availability of the system, it is necessary to choose between whether to apply PM actions (that is, to choose between policies) to minimize maintenance cost per unit time.

The problem then becomes that determination of the optimal maintenance policy or the time interval between PM actions. When a combination involving both availability and cost issues is used as a measure of the system performance, three cases can occur: (1) Corrective maintenance with no PM actions (Policy 1) does not guarantee the predetermined availability, but performing PM actions (Policy 2) with the PM time interval incurring the minimum maintenance cost does. (2) Both policies can guarantee the predetermined availability, but Policy 1 incurs a maintenance cost less than that incurred by Policy 2. (3) Policy 1 guarantees the predetermined availability, but Policy 2 (with the time interval incurring the minimum maintenance cost) does not guarantee the predetermined availability. These guidelines thus exhaust all cases.

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BIOGRAPHICAL SKETCH

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