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A Mathematical Model for a Multi-Commodity, Two-Stage Transportation and Inventory Problem

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This paper presents a mathematical model for two-stage planning of transportation and inventory for many sorts of products (multi-commodity). The situation considered in this paper, which happens in a local furniture manufacturing firm, is that the total supply in origins exceeds the current-stage's total demand from all destinations (markets). Therefore, the problem is how to arrange the current-stage's shipping in consideration of next-stage's (that is, future's) inventory in both origins and destinations. A mathematical model is proposed for the problem with the objective of minimizing the total cost of both shipping and inventory for all products within two stages. Meanwhile, since the next-stage's shipping costs usually are unknown, this paper presents a new concept of rational unit shipping cost: a forecasted average cost with weight of next-stage's shipping amount. Finally, a numerical example extracted from the furniture manufacturing company with 4 origins, 4 destinations and 4 commodities is illustrated in the paper.

Significance: This paper presents a mathematical model for multi-commodity, two-stage transportation and inventory problem. The developed model is flexible for adjusting of the next-stage's shipping plan in accordance with the practical conditions at that time. It is also proved that many variables can be reduced by use of the rational unit shipping cost.

Key words: Transportation, inventory, network programming, multi-commodity, rational unit shipping cost.

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1. INTRODUCTION

With the rapid development of global marketing and manufacturing, the competition among manufacturing firms becomes more intense, so a well-established logistic system plays an important role in a manufacturing firm. A complete logistic system involves four steps: moving raw materials from suppliers, converting raw materials into products, shipping products to various warehouses, and finally delivering products to customers. Obviously, transportation and inventory are two key elements in the entire logistic system, and have attracted many researchers (Lari and Nag, 1993; Castro and Nabona, 1996; Lee et al., 2003; Ahn and Seo, 2005). However, due to complexities and difficulties, transportation and inventory are often studied separately although they are closely related, especially for those multi-product (multi-commodity) problems. While shipping plan for current-stage's demands (without inventory) is made, how to hold other surplus commodities over current-stage's demands has to be considered. This paper will present a mathematical model for this problem.

Mathematically, the multi-commodity flow problem, as a complicated network, has a variety of practical applications, such as, fleet management problem (Vemuganti et al., 1989), facility location problem (Lee, 1993), flight scheduling problem (Gu et al., 1994), commodity allocation problem (Bassok and Ernst, 1995), and lot sizing problem (Millar and Yang, 1993; Stadtler, 1996). For arborescent supply chain with discrete-period variable demand, Chiu et al. (2003) proposed a mixed integer programming model to minimize the total supply chain cost including transportation cost and inventory holding cost over the periods (stages). In 2005, Ahn and Seo presented a multi-commodity ordering model by considering transportation constraint on the inventory management during the planning horizon. Huang et al. (2005) investigated a one-warehouse multi-retailer system in which each retailer faces a constant and deterministic demand over an infinite horizon, and then formulated the transportation and inventory problem as a 0-1 integer linear program. However, all the above papers assumed unit shipping cost to be fixed throughout the periods, which is seldom the case in the actual situation. Compared with the general multi-period (multi-stage) multi-commodity network flow problems, this paper studies a special case, that is, the two-period network flow problem. The reason for this is obvious: the longer the time period to be considered, the more uncertain in future's activities. The problem considered in this paper will be addressed in the next section. Then, a mathematical model will be formulated and the complexity of the model will be analyzed to demonstrate the advantage of the model. Finally, a numerical example with sensitivity analysis is illustrated in the paper.

2. PROBLEM DESCRIPTION

The problem originates from a local furniture manufacturing company. The company locates in the Pearl River Delta, while its markets are mainly in USA. The USA buyers require that furniture must be finally assembled, so it requires a large space to store. The furniture manufacturing company has a two-stage transportation and inventory problem, as shown in Figure 1. When the company makes a transportation plan for its commodities, the company also considers the inventory for the next stage, that is, it has to consider where to store its commodities, at origins (the current locations) or destinations (warehouses in USA)? Obviously, the commodities should be shipped to the destination at the current stage and stored there for future's consumption. However, the commodities may not be stored at destinations because (1) the holding cost at destinations is higher than at origins, (2) the inventory space at destinations is limited. When the inventory space at destinations is not enough to hold those commodities for the next-stage's (future's) demand, certainly, these commodities have to be stored at origins. However, in the case of higher inventory cost at a destination, the commodities may or may not be stored at origins because the shipping costs in both the current stage and the next stage must be compared. While the shipping cost at the current stage can be known from transportation business companies, like airlines or vessel companies, however, the shipping cost for the next stage is uncertain. This makes the problem more complicated.

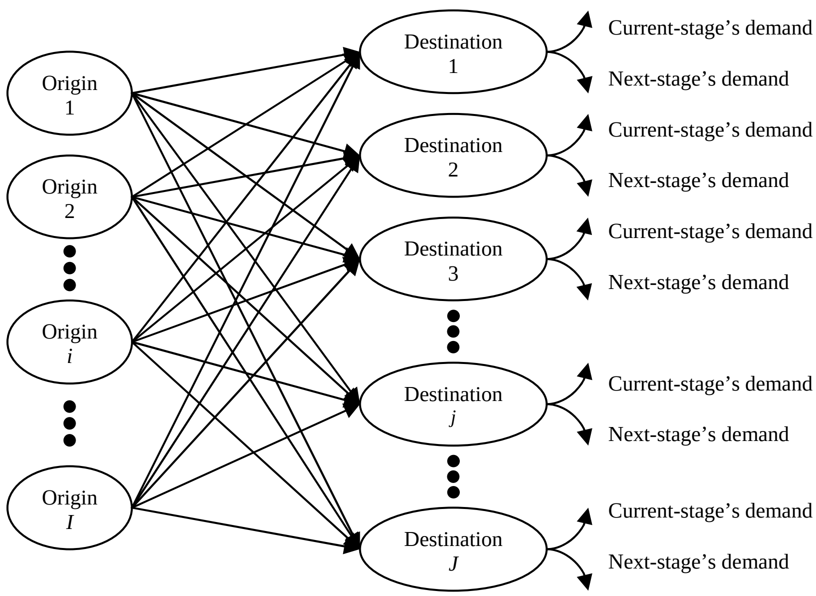

Figure 1. The two-stage transportation and inventory problem

As the unit shipping cost in the next stage (actually, in future) is not clear, it is unreasonable to determine a concrete shipping plan for the next stage using the current-stage's unit shipping cost. In order to get a reasonable unit shipping cost for the next stage, the mathematical model developed in this paper will use a new concept, the rational unit shipping cost. The rational unit shipping cost here corresponds to the most probable unit shipping cost in the next stage for the total commodities held from one origin to all destinations. Actually, the rational unit shipping cost is a weighted average unit shipping cost with the weight of next-stage's shipping amount. This shipping amount can be calculated by the Gravity Model for the Trip Distribution Problem (Adib, 1983; Bruton, 1985). Therefore, the total shipping cost from the origin (which holds commodities) to all destinations is the rational unit shipping cost times the holding amount in this origin. By using the rational unit shipping cost, the number of variables in the mathematical model will be reduced so the model of the problem becomes simpler. The detailed derivation of the rational unit shipping cost will be addressed later.

In summary, the conditions considered in the paper are: (1) demands within two stages in destinations, (2) current-stage's unit shipping costs from origins (factories) to destinations, (3) holding capacity in destinations, (4) current-stage's unit holding costs in both origins and destinations, and (5) occupying space for holding one unit of different products. By knowing these conditions, the objective of the model is to make a plan for transportation and inventory in order to minimize the total cost of transportation and inventory. The mathematical model developed later in the paper can decide: 1) reasonable shipping amounts at the current-stage from different origins to different destinations, which cover current-stage's demands and inventory for partial or entire demands in destinations for the next-stage; and 2) reasonable amounts of current-stage's inventory in origins, which will be shipped in future to different destinations. Mathematically, the model developed here is a special case of the general multi-commodity, multi-period network problems.

3. NOTATION

3.1. Parameters

I	number of origins (i =1, 2,…, I)
J	number of destinations (j =1, 2, …, J)
L	number of commodities (l =1, 2, …, L)
A^l_i	supply amount of commodity l at origin i
B^l_j	current-stage's demand of commodity l in destination j
$\bar{B}^l_j$	next-stage's demand of commodity l in destination j
C^l_{ij}	current-stage's unit shipping cost of commodity l from origin i to destination j
o^l_i	unit inventory cost of commodity l at origin i
d^l_j	unit inventory cost of commodity l in destination j
D^l_{ij}	Sum of unit inventory cost of commodity l in destination j and current-stage's unit shipping cost of commodity l from origin i to destination j, that is,

$$D^l_{ij} = d^l_j + C^l_{ij} \quad \ldots \quad (1)$$

P_i	inventory capacity for standard commodity in origin i
Q_j	inventory capacity for standard commodity in destination j
M	incidence vector representing a relationship of the occupying inventory space of various commodities with the standard one.

Here, $M = (M^1, M^2,\ldots, M^l,\ldots, M^L)$, an element M^l denotes the proportion of occupying space for a unit commodity l to a unit standard commodity. Assuming S^d and S^l denotes occupying inventory space for the standard commodity and commodity l, respectively, then $M^l= S^l / S^d$.

3.2. Variables

X^l_{ij}	current-stage's shipping amount of commodity l from origin i to destination j ($j \neq J + 1$). This amount will consumed at the current-stage in the destination j, which means this amount will not be held at either origin i or destination j.
X^l_{iJ+1}	inventory amount of commodity l at origin i.
V^l_{ij}	inventory amount of commodity l in destination j shipped at the current stage from origin i but held at destination j for the next-stage's consumption.

3.3. Prerequisite

In addition, a condition has to be met that the total supply amount at all origins must exceed the total current-stage's demand in all destinations. Mathematically, this can be expressed as follows:

$$\sum_{i=1}^{I} A^l_i > \sum_{j=1}^{J} B^l_j \qquad \forall l \quad \ldots \quad (2)$$

Otherwise, if this condition can not be met, the problem will degenerate into a general transportation model, which has been studied by many researchers (Gass, 1990).

4. MATHEMATICAL MODEL

4.1. The Model

Based on the previous discussion, the mathematical model of the problem can be detailed as follows:

minimize

$$\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J+1} C^l_{ij} X^l_{ij} + \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} D^l_{ij} V^l_{ij} \quad \ldots \quad (3)$$

subject to

$$\sum_{j=1}^{J+1} X^l_{ij} + \sum_{j=1}^{J} V^l_{ij} = A^l_i \qquad \forall i,l \quad \ldots \quad (4)$$

$$\sum_{i=1}^{I} X^l_{ij} = B^l_j \qquad \forall j,l \quad \ldots \quad (5)$$

$$\sum_{i=1}^{I} V_{ij}^{l} \le \overline{B}_j^l \qquad \forall\, j, l \qquad \ldots \qquad (6)$$

$$\sum_{l=1}^{L} X_{iJ+1}^{l} M^{l} \le P_i \qquad \forall\, i \qquad \ldots \qquad (7)$$

$$\sum_{l=1}^{L}\left(\sum_{i=1}^{I} V_{ij}^{l}\right) M^{l} \le Q_j \qquad \forall\, j \qquad \ldots \qquad (8)$$

$$X_{ij}^{l}, V_{ij}^{l} \ge 0 \text{ and integer } \ \forall\, i,\, l, \text{ and } j = 1, 2, \ldots, J+1 \qquad \ldots \qquad (9)$$

Obviously, the objective (3) of the model is to minimize the total cost, including shipping and holding. Constraint (4) represents that total amount of a commodity *l* at origin *i* can be shipped to some destinations for consumption at current stage, or shipped to some destinations but held for the next-stage's usage, or kept at origin *i*, that is, the item $X^l_{i,J+1}$. Constraint (5) addresses that the total amount of a commodity *l*, received at a destination *j* from all origins, should meet the current-stage's demand at the destination. Constraint (6) states that in a destination, the total amount of a commodity held for the next stage should not exceed the next stage's demand of that commodity. Constraints (7) and (8) are from the inventory space requirements at origins and destinations, respectively.

It should be noticed that the item $C^l_{i,\,J+1}$ in the objective (3) has not been defined yet. In fact,

$$C_{iJ+1}^{l} = o_i^l + \overline{C}_i^l \qquad \forall\, i, l \qquad \ldots \qquad (10)$$

Where, $\overline{C}_i^l$ denotes the rational unit shipping cost from origin *i* to all destinations. So, $C^l_{i,\,J+1}$ represents the total cost of the unit inventory cost of commodity *l* in origin *i* and the rational unit shipping cost of commodity *l* from origin *i* to all destinations.

The rational unit shipping cost $\overline{C}_i^l$ can be derived as follows: Assuming that T^l_{ij} denotes next-stage's distribution amount (i.e., shipping amount) of holding amount $X^l_{i,J+1}$ of commodity *l* from origin *i* to destination *j*, that is,

$$\sum_{j=1}^{J} T_{ij}^{l} = X_{iJ+1}^{l} \qquad \forall\, i, l \qquad \ldots \qquad (11)$$

In order to get T^l_{ij}, the Gravity Model Method for the Trip Distribution Problem (Adib, 1983; Bruton, 1985) can be used. The basic idea of the gravity model method is that the shipping amount from origin *i* to destination *j* is direct proportional to the inventory held at origin *i* (that is, $X^l_{i,J+1}$) and the next-stage's demand at destination *j* (that is, $\overline{B}_j^l$), but reverse proportional to the current shipping cost C^l_{ij}, so,

$$T_{ij}^{l} = \alpha X_{iJ+1}^{l} \overline{B}_j^l \big/ C_{ij}^{l} \qquad \forall\, i, j, l \qquad \ldots \qquad (12)$$

where α is a coefficient which can be obtained by the calibration method for the Trip Distribution Problem (Adib, 1983; Bruton, 1985). Now the rational unit shipping cost can be represented as an average shipping cost weighted by the next-stage's shipping amount, that is,

$$\overline{C}_i^l = \sum_{j=1}^{J} T_{ij}^{l} C_{ij}^{l} \Big/ \sum_{j=1}^{J} T_{ij}^{l} \qquad \forall\, i, l \qquad \ldots \qquad (13)$$

By combining (12) and (13), then

$$\overline{C}_i^l = \sum_{j=1}^{J} \overline{B}_j^l \Big/ \sum_{j=1}^{J} \overline{B}_j^l \big/ C_{ij}^{l} \qquad \forall\, i, l \qquad \ldots \qquad (14)$$

Current-stage's shipping cost and next-stage's demand are two key factors that impact next-stage's shipping cost. On the one hand, the current-stage's shipping cost has a positive influence on the next-stage's shipping cost, that is, if the current-stage's shipping cost is high, the next-stage's shipping cost is usually also high. On the other hand, when next-stage's demand of commodity *l* in destination *j* (that is, $\overline{B}_j^l$) increases, the next-stage's shipping cost from origin *i* to all destinations (that is, $\overline{C}_i^l$) normally will be affected more by current-stage's shipping cost of commodity *l* from origin *i* to destination *j* (that is, C^l_{ij}). All these have been considered in the above formulation. Therefore, it is reasonable to determine next-stage's shipping plan by using the rational unit shipping cost developed in the paper.

In order to ensure that a feasible solution exists, the following condition is essential:

$$\sum_{l=1}^{L}\left(\sum_{i=1}^{I} A_i^l - \sum_{j=1}^{J} B_j^l\right) M^{l} \le \sum_{i=1}^{I} P_i + \sum_{j=1}^{J} Q_j \qquad \ldots \qquad (15)$$

which states the inventory space requirement for all commodities can not exceed the available space at both origins and destinations.

4.2. Complexity Analysis

This model has $I \times (2J+1) \times L$ variables including $I \times (J+1) \times L$ $X^l_{i,J}$'s and $I \times J \times L$ $V^l_{i,J}$'s, and $I \times L$ constraints from (4), $J \times L$ constraints from (5), $J \times L$ constraints from (6), I constraints from (7), and J constraints from (8).

If the next-stage's unit shipping cost is considered as the same with the current-stage's unit shipping cost, the objective function of the model will be:

minimize

$$\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} C^l_{ij}X^l_{ij} + \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} D^l_{ij}V^l_{ij} + \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} (o^l_i + C^l_{ij})T^l_{ij} \quad \ldots \tag{16}$$

Compared with the original objective (3), the first summation item in (3) is divided into two parts in (16).

The variable number in the above model (16) is $3 \times I \times J \times L$, but the model presented in this paper is $I \times (2J+1) \times L$, so about $(J-1)/(2J+1)100\%$ variables can be decreased for the objective function. In other words, if there is an example with 10 origins, 10 destinations and 10 commodities, then about $(10-1)/(2\times10+1)= 43\%$ variables will be reduced. Apart from this, the model in section 4 can not only reduce the variables in the objective, but also reduce variables in the constraints of (4) and (7). Constraint (4) can be reduced $(J-1)/(2J+1)100\%$; constraint (7) is $(J-1)100\%$, respectively. So, the conclusion is when the larger the size of the problem, the more variables can be decreased. Furthermore, the number of variables to be reduced is depending on the destination number J only, regardless of the number of origins, I, and the number of commodities, L. In other words, with the world economy globalization, more markets are to be expanded for a company, then the model will become more complicated. In the case of the future unit shipping cost is not clear, the problem will be simplified by use of the model presented in this paper.

5. NUMERICAL EXAMPLE

5.1. Example Data

Table 1. Basic data of l=1

C^l_{ij} D / O	1	2	3	4	o^l_i	A^l_i
1	3	6	3	4	5	900
2	4	3	5	5	2	800
3	6	3	7	3	5	1000
4	7	5	3	2	5	1100
d^l_j	6	4	7	5		
B^l_j	420	380	280	300		
$\overline{B}^l_i$	530	440	480	505		

Table 2. Basic data of l=2

C^l_{ij} D / O	1	2	3	4	o^l_i	A^l_i
1	2	3	5	1	3	680
2	3	7	4	5	7	890
3	4	2	3	5	2	550
4	6	8	3	5	4	800
d^l_j	2	1	3	4		
B^l_j	620	680	550	430		
$\overline{B}^l_i$	706	808	645	558		

Table 3. Basic data of l=3

C^l_{ij} D / O	1	2	3	4	o^l_i	A^l_i
1	11	15	16	27	10	950
2	22	30	19	20	13	860
3	19	22	24	28	15	780
4	30	27	23	18	26	980
d^l_j	17	16	15	14		
B^l_j	800	760	550	610		
$\overline{B}^l_i$	797	1023	907	886		

Table 4. Basic data of l=4

C^l_{ij} D / O	1	2	3	4	o^l_i	A^l_i
1	10	8	9	11	13	1000
2	9	8	10	13	11	1010
3	12	14	15	12	10	800
4	10	9	8	7	11	950
d^l_j	9	8	6	7		
B^l_j	600	590	320	480		
$\overline{B}^l_i$	830	680	654	589		

To apply the above model, an example extracted from the furniture manufacturing company is solved by CPLEX, a commercial software package. Due to business confidentiality, the real data have been modified. The parameters of the example with 4 origins, 4 destinations, and 4 commodities are shown in Table 1, 2, 3, and 4, which are the data corresponding to commodity 1, 2, 3, and 4, respectively. Other data are as follows: M= (1, 2, 3, 2), P_1=900, P_2=840, P_3=1080, P_4=1500, and Q_1=1400, Q_2=900, Q_3= 1500, Q_4=2500.

5.2. Numerical Results

The rational unit shipping costs calculated with formulation (14) are shown at Table 5.

Table 5. The rational unit shipping costs $\overline{C}_i^l$

C^l_{ij} (i \ l)	1	2	3	4
1	3.646	2.075	15.698	9.357
2	4.106	4.357	22.252	9.561
3	4.143	2.995	22.884	13.083
4	3.393	4.983	23.597	8.485

The example results solved by CPLEX are given in Table 6 and 7. The final objective is 152109.679. The numbers at upper left, upper right, lower left, and lower right in a cell in Table 6 and 7 are the results of commodity 1, 2, 3, and 4, respectively.

Table 6. Transportation amount X^l_{ij} of commodity l from origin i to destination j

X^l_{ij} (O \ D)	1		2		3		4	
1	420			250	280		1	430
	707			26		320		
2		620						
	83	216		564	550			
3			380	430				
	10	384	760					
4						550	299	
							610	480

Table 7. Inventory amount of commodity l: $X^l_{i,J+1}$ in origin i and V^l_{ij} in destination j from origin i

V^l_{ij} (O \ D)	1		2		3		4		$X^l_{i,J+1}$	
1	28								171	
						654			243	
2		270							800	
				230	64		150		13	
3			440						180	120
		416							10	
4									801	250
							370	470		

In Table 3, take origin 2 and commodity 3 for example: the supply is 860; meanwhile, the current-stage's demands and the next-stage's demands for destination 1, 2, 3, and 4 are 800, 797; 760, 1023; 550, 907; and 610, 886; respectively. The results in Table 6 show that current-stage's shipping amounts of commodity 3 from origin 2 to destination 1 and 3 are X^3_{21}=83 and X^3_{23} =550, respectively. Variable X^3_{25} has a value of 13, which indicates that 13 units of commodity 3 are held at origin 2. In Table 7, X^3_{23} =64 and X^3_{24} =150 mean that inventory amounts of commodity 3 in destination 3 and 4 shipped at the current stage from origin 2 but held at destination 3 and 4 for the next-stage's consumption are 64 and 150, respectively. From the relevant parameters, the above results are reasonable. The same conclusion can be obtained for other origins and commodities.

Meantime, for this particular problem, if the traditional approach is used, that is, two stages are considered separately, the minimal total transportation and inventory cost is 171466.86. With the model developed in the paper, the optimal cost is 152109.679. The percentage of cost reduction is 11.29% ((171466.86-152109.679)/171466.86 *100%).

5.3. Sensitivity Analysis

Based on the previous numerical example, sensitivity analyses are carried out by changing (increasing and decreasing) the various parameters by 1%, taking one at a time and keeping the remaining parameters at their original values. We use OBJ'/OBJ as the measure of sensitivity, where OBJ is the objective cost under the original parameters and OBJ' is the objective cost with the changed parameters. The results of the sensitivity analysis are shown in Table 8. It should be noticed that the changed parameters in the constraints are rounded to integers to be consistent with the realities.

Table 8 (a). Sensitivity analysis of the model

Parameters	% changed	Objective	OBJ'/OBJ
$C^l_{ij}(j\neq J+1)$	+1%	153287.715	1.007745
$C^l_{ij}(j\neq J+1)$	-1%	150945.972	0.992350
o^l_i	+1%	152222.674	1.000743
o^l_i	-1%	151995.487	0.999249
d^l_j	+1%	152349.129	1.001574
d^l_j	-1%	151888.062	0.998543
A^l_i	+1%	154792.972	1.017641
A^l_i	-1%	149522.038	0.982988

The main conclusions drawn from the sensitivity analysis are as follows:

1) The range of OBJ'/OBJ is from 0.982988 to 1.017641. The average value of OBJ'/OBJ is 1.000073.
2) The value of OBJ'/OBJ is more sensitive to the parameters of A^l_i, $C^l_{ij}(j\neq J+1)$, and B^l_j, and less sensitive to the parameters of $\overline{B}^l_j$, o^l_i, and Q_j.
3) When next-stage's demands change, the objective almost remains the same.
4) When current-stage's unit shipping cost, unit inventory cost at both origin and destination, and supply amount increase, the objective costs also increase. The reverse is true for both current-stage's and next-stage's demand, and inventory capacity at both origin and destination.

Table 8 (b). Sensitivity analysis of the model

Parameters	% changed	Objective	OBJ'/OBJ
B^l_j	+1%	151188.641	0.993945
B^l_j	-1%	153044.229	1.006144
$\overline{B}^l_i$	+1%	152108.762	0.999994
$\overline{B}^l_i$	-1%	152126.407	1.000110
P_i	+1%	152085.514	0.999841
P_i	-1%	152141.104	1.000207
Q_j	+1%	151991.442	0.999223
Q_j	-1%	152242.057	1.000870

6. CONCLUSIONS

The problem studied here is a special case of the multi-commodity, multi-period, transportation and inventory problem. In this paper, a mathematical model was presented. A new concept, the rational unit shipping cost, was also introduced. The purpose of the model presented in the paper is to make the plan for both transportation and inventory in the current stage and the next stage. The developed model has two advantages: 1) it can reduce many variables by use of the rational unit shipping cost; 2) it is flexible for adjusting of the next-stage's shipping plan in accordance with the practical conditions at that time.

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