

# CONTROLLING ARRIVALS FOR A MARKOVIAN QUEUEING SYSTEM WITH A SECOND OPTIONAL SERVICE

Jau-Chuan Ke<sup>1</sup>, Chia-Jung Chang<sup>2</sup> and Fu-Min Chang<sup>3</sup>

<sup>1</sup>Department of Applied Statistics,  
National Taichung Institute of Technology,  
Taichung 404, Taiwan  
Email: [jauchuan@ntit.edu.tw](mailto:jauchuan@ntit.edu.tw)

<sup>2</sup>Department of Accounting,  
National Taichung Institute of Technology,  
Taichung 404, Taiwan

<sup>3</sup>Department of Finance,  
Chaoyang University of Technology,  
Taichung County 413, Taiwan

This paper considers the optimal management problem of a finite capacity M/M/1 queueing system with  $F$ -policy in which some customers may demand a second service in addition to the first essential service. The  $F$ -policy investigates the frequent issue of controlling arrival to a queueing system and its required startup time before allowing customers to enter the system. This system has potential applications in the wireless communication networks, the transport service and production system. By applying the birth and death process, some important performance measures are derived. A cost model, developed to determine the optimal control  $F$ -policy at a minimum cost, and sensitivity analysis are also studied.

**Keywords:** Control; F-Policy; Server startup; Optional service

**Significance:** We study the controlling arrivals of a Markovian queueing system through simple mathematical tools, where some customers may demand a second service in addition to the first essential service. Such a system has potential applications in the wireless communication networks, the transport service and production system, etc.

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## 1. INTRODUCTION

In this paper, we deal with the issue of controlling arrivals for an M/M/1/K queueing system with an  $F$ -policy, which some customers may demand a second service in addition to the first essential service. In the proposed model, when the number of customers in the system reach its capacity  $K$  ( $K < \infty$ ), no further arriving customers are allowed to enter into the system until a certain number of customers, who are already in the system, have been served in order to make sure the number of customers in the system decreases to a predetermined threshold  $F$  ( $0 \leq F \leq K - 1$ ). At that time, the server needs to take an exponential startup time and allow customers to enter into the system. Moreover, the server provides the first essential service as well as a second optional service. Upon completion of the essential service, the customer may opt for an optional service with probability  $q$ .

Controllable queueing models aim to find the optimal operating policy, that is, rules for turning the server on and off that result in the lowest long-run cost. Past works regarding such queueing problems could be divided into two parts according to whether the system is considered to control the service or the arrival. The issue of controlling the service includes: the N-policy introduced by Yadin and Naor (1963), the T policy proposed by Heyman (1977) and the D-policy introduced by Balachandran (1973). Among said operating policies, the studies of N-policy queueing models gained a lot of attention due to analytical being easily tractable. The variations and extensions of these vacation models with N policy can be referred to Lee et al. (1994, 1995), Ke (2003), Arumuganathan and Jeyakumar (2005), Moreno (2008), and others. The developments and applications on service control of queueing systems are rich and diverse (see Tadj and Choudhury (2005)). In the other hand,  $F$ -policy, the pioneering work in the issue of controlling the arrivals, was first investigated by Gupta (1995). The intention of the  $F$ -policy is to control the arrival process when service control is not possible through N-policy.

As for optional service, Madan (2000) first investigated an M/G/1 queueing system with a second optional service, in which some of arrivals may require a second optional service immediately after completion of the first essential service. In Madan's work (2000), the service times of the first essential service are assumed to be a general distribution

and those of the second optional service are exponential. He also cited some important applications in day-to-day life situations. Madan's work (2000) is extended to 'optional service' with general distribution case by Medhi (2002). Later, Choudhury and Paul (2006) studied the queue size distribution at a random epoch as well as at a departure epoch for an  $M^x/G/1$  queueing system with second optional channel under N-policy. They also derived a simple procedure to obtain optimal stationary policy under a suitable linear cost structure. The reliability measures were examined by Wang (2004) for the ordinary  $M/G/1$  queue with channel breakdowns and second optional service. Recently, Tadj and Ke (2008) examined the optimal control policy for a two-phase bulk service queueing system under N policy with multiple vacation and setup, where the group of customers has the option to choose the service type in either phase of service.

Existing research, however, seldom addresses the optimization issue in such queueing systems with optional service. Besides the lack of research works on such problem, our study is also motivated by some practical systems. For example, in the wireless communication system, arriving (new and handoff) calls are granted, or denied, access to the network by the call admission scheme (CAC) based on predefined criteria, taking the network loading conditions into consideration. When the system capacity has reached the predefined level, the arriving calls will be restricted entrance until the number of calls drops to a threshold level. When system capacity reduces to the threshold level, the calls are allowed to enter the system. This will help to prevent the system from becoming over-loaded ( $F$ -policy). In addition, multimedia services such as voice, video, data, and audio are to be offered with various QoS profiles in the third generation and beyond wireless systems. Hence, except for handling the normal accesses from wireless mobile devices (essential service), the access gateway may need additional work (optional service) to negotiate with other gateway for QoS control depending on the request of wireless mobile devices.

The other applications in our model are the transport service and production system. Considering the transport service based on Ke (2006), all ships arriving at a port may need unloading service on arrival. Due to the limited working place and labor power, the control policy should be executed to maintain the quality of unloading service. When the capacity of working place is full, the arriving ships will be restricted entrance until the capacity of working place drops to a threshold level. When the capacity of working place reduces to the threshold level, the ships are admitted to enter the port ( $F$ -policy). Yet, some of ships may require a re-loading service (which is optional) soon after the unloading (essential service). In the production system, items are allowed to be produced by the machine based on predefined criteria, taking the machine loading conditions into consideration. When the machine capacity is reached the predefined level, the arriving items will be restricted entrance until the number of items drops to a threshold level. When machine capacity reduces to the threshold level, the items are allowed to enter the system. This will help to maintain the production performance of the machine ( $F$ -policy). The machine producing items, however, may require two of services such as periodic checking (essential service) and repairing (optional service) during the processing of raw materials. At times, some of units may either left the system after periodic checking or rest of units may send to workshop for the reprocessing service.

The remainder of this paper is organized as follows: In Section 2, the queueing model is briefly described. In Section 3, the mathematical model is developed and its analytical steady-state solutions are derived. Various system performance measures are presented in Section 4. In Section 5, we develop the total expected cost function per unit time for the  $F$ -policy  $M/M/1/K$  queueing system with a second optional service, and then provide the numerical illustration and sensitivity analysis in Section 6. Finally, some concluding remarks are drawn.

## 2. MODEL DESCRIPTIONS

We consider the issue of controlling arrivals for the  $F$ -policy  $M/M/1/K$  queueing system with a second optional service. It is assumed that arriving customers follow a Poisson process with parameter  $\lambda$ . A general case for a customer service is considered. All arriving customers require the essential service and some customers may further demand a second optional service. On completion of the essential service, the customer may leave the system with probability  $p$  or may opt for the second optional service with probability  $q$  ( $1-p$ ), the times of the essential service and the optional service are assumed to be exponential distribution with parameter  $\mu_1$  and  $\mu_2$ , respectively. Arriving customers form a single waiting line based on the order of their arrivals. In other words, they are queued according to the first-come, first-served (FCFS) discipline. The server can only provide either the essential or the optional service for one customer at a time and that the service is independent of the arrival of customers. A customer who arrives and finds the server busy must wait in the queue until he is available. In addition, the server operates  $F$ -policy when the number of arrivals reaches its capacity  $K$  (*i.e.*, the system becomes full). As soon as the number of the customers drops to a predetermined threshold value  $F$  ( $0 \leq F \leq K - 1$ ) since the system becomes full, the server immediately requires an exponential startup time with parameter  $\gamma$  to start allowing customers in the system. The system operates normally until the number of customers in the system reaches its capacity at which time the above process is repeated all over again.

## 3. STEADY-STATE SOLUTIONS

The following pair describes the states of the  $F$ -policy  $M/M/1/K$  system with second optional service channel  $(i, n)$ , where  $i$  represents server's state and  $n$  represents the number of customers in the system. Let  $i=0$  denote the customer is not allowed to enter into the system when the server is providing the essential service,  $i=1$  denote the

customer is not allowed to enter into the system when the server is providing the optional service,  $i=2$  denote the customer is allowed to enter into the system when the server is providing the essential service,  $i=3$  denote the customer is allowed to enter into the system when the server is providing in the optional service. Thus  $(i, n) \in \{(i, n) : i = 0; n = 0, 1, 2, \dots, K\} \cup \{(i, n) : i = 1; n = 1, 2, \dots, K\} \cup \{(i, n) : i = 2; n = 0, 1, 2, \dots, K - 1\} \cup \{(i, n) : i = 3; n = 1, 2, \dots, K - 1\}$ .

In steady-state, the following notation are used,

$P_0(n)$  : Probability that there are  $n$  customers in the system when the arrivals are not allowed to enter into the system, and the server is providing the essential service, where  $n = 0, 1, 2, \dots, K$ .

$P_1(n)$  : Probability that there are  $n$  customers in the system when the arrivals are not allowed to enter into the system, and the server is providing the optional service, where  $n = 1, 2, \dots, K$ .

$P_2(n)$  : Probability that there are  $n$  customers in the system when the arrivals are allowed to enter into the system, and the server is providing the essential service, where  $n = 0, 1, 2, \dots, K - 1$ .

$P_3(n)$  : Probability that there are  $n$  customers in the system when the arrivals are allowed to enter into the system, and the server is providing optional service, where  $n = 1, 2, \dots, K - 1$ .

Referring to the state-transition diagram depicted in Figure 1, the steady-state equations for  $P_i(n)$  ( $i = 0, 1, 2, 3$ ) are given by:

$$\mu_2 P_1(n) = q\mu_1 P_0(n), \quad 1 \leq n \leq K \quad \dots \quad (1)$$

$$\gamma P_0(0) = \mu_2 P_1(1) + p\mu_1 P_0(1) \quad \dots \quad (2)$$

$$(\gamma + \mu_1) P_0(n) = \mu_2 P_1(n+1) + p\mu_1 P_0(n+1), \quad 1 \leq n \leq F \quad \dots \quad (3)$$

$$\mu_1 P_0(n) = \mu_2 P_1(n+1) + p\mu_1 P_0(n+1), \quad F+1 \leq n \leq K-1 \quad \dots \quad (4)$$

$$\mu_1 P_0(K) = \lambda P_2(K-1), \quad \dots \quad (5)$$

$$\lambda P_2(0) = p\mu_1 P_2(1) + \mu_2 P_3(1) + \gamma P_0(0), \quad \dots \quad (6)$$

$$(\mu_1 + \lambda) P_2(n) = p\mu_1 P_2(n+1) + \mu_2 P_3(n+1) + \gamma P_0(n) + \lambda P_2(n-1), \quad 1 \leq n \leq F \quad \dots \quad (7)$$

$$(\mu_1 + \lambda) P_2(n) = p\mu_1 P_2(n+1) + \mu_2 P_3(n+1) + \lambda P_2(n-1), \quad F+1 \leq n \leq K-2 \quad \dots \quad (8)$$

$$(\mu_1 + \lambda) P_2(K-1) = \lambda P_2(K-2), \quad n = K-1 \geq F \quad \dots \quad (9)$$

$$(\mu_2 + \lambda) P_3(1) = q\mu_1 P_2(1), \quad \dots \quad (10)$$

$$(\mu_2 + \lambda) P_3(n) = \lambda P_3(n-1) + q\mu_1 P_2(n), \quad 2 \leq n \leq K-2 \quad \dots \quad (11)$$

$$\mu_2 P_3(K-1) = \lambda P_3(K-2) + q\mu_1 P_2(K-1), \quad n = K-1 \geq F \quad \dots \quad (12)$$

Since the closed-form solutions of equations (1) - (12) are too complicated to obtain by using a recursive method, a computer software (MATLAB) is used to compute the  $P_i(n)$  ( $i = 0, 1, 2, 3$ ) by using the following normalizing conditions:

$$\sum_{n=0}^K P_0(n) + \sum_{n=1}^K P_1(n) + \sum_{n=0}^{K-1} P_2(n) + \sum_{n=1}^{K-1} P_3(n) = 1. \quad \dots \quad (13)$$

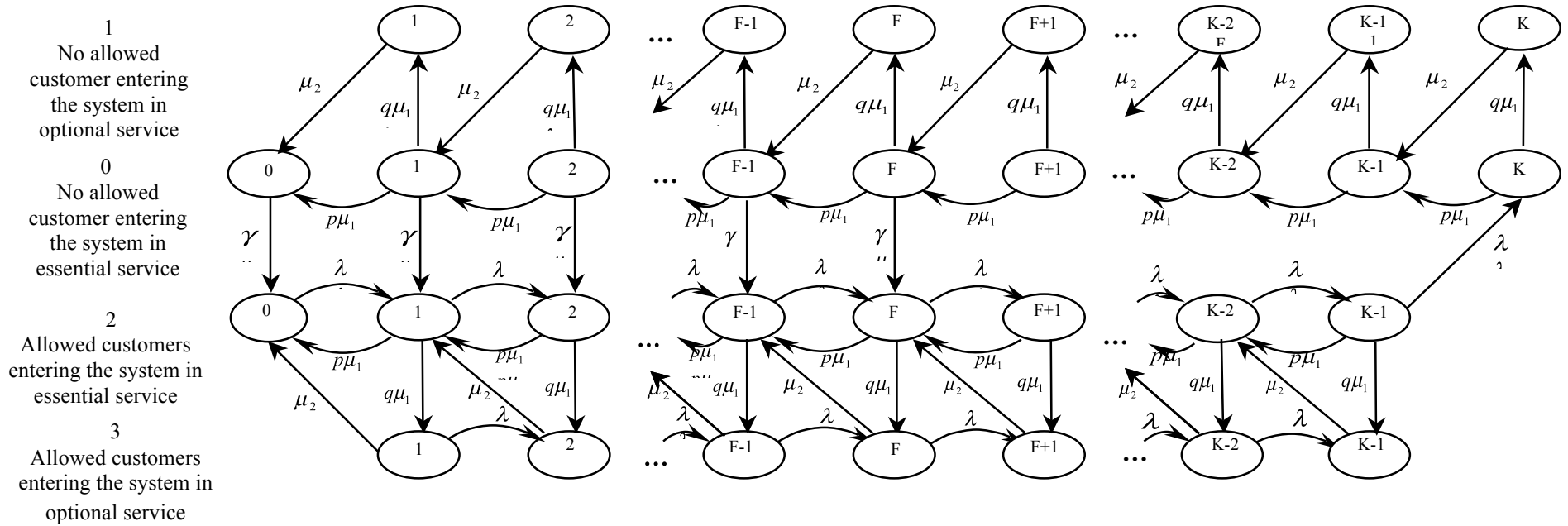


Figure 1. State-transition rate diagram for the F-policy M/M/1/K queueing system with optional service

#### 4. SYSTEM PERFORMANCES

Some important system performance measures of the  $F$ -policy M/M/1/K queueing system with a second optional service are defined as follows:

$L$   $\equiv$  the expected number of customers in the system;

$P_e$   $\equiv$  the probability that the server is providing the essential service;

$P_{op}$   $\equiv$  the probability that the server is providing the optional service;

$P_s$   $\equiv$  the probability that the server requires a startup time before starting the service;

$P_b$   $\equiv$  the probability that the system is blocked

$W$   $\equiv$  the expected waiting time in the system

$E(S)$   $\equiv$  the expected number of customers when the server starts to allow customers entering the system;

$E(B)$   $\equiv$  the expected number of customers when the system is blocked;

The expression for  $L$ ,  $P_e$ ,  $P_{op}$ ,  $P_s$ ,  $P_b$  and  $W$  are given by:

$$L = \sum_{n=0}^K nP_0(n) + \sum_{n=1}^K nP_1(n) + \sum_{n=0}^{K-1} nP_2(n) + \sum_{n=1}^{K-1} nP_3(n), \quad \dots \quad (14a)$$

$$P_e = \sum_{n=0}^K P_0(n) + \sum_{n=0}^{K-1} P_2(n), \quad \dots \quad (14b)$$

$$P_{op} = \sum_{n=1}^K P_1(n) + \sum_{n=1}^{K-1} P_3(n), \quad \dots \quad (14c)$$

$$P_s = \sum_{n=0}^F P_0(n), \quad \dots \quad (14d)$$

$$P_b = \sum_{n=0}^K P_0(n) + \sum_{n=1}^K P_1(n). \quad \dots \quad (14e)$$

The effective arrival rate is  $\lambda_{eff} = \lambda \left[ \sum_{n=0}^{K-1} P_2(n) + \sum_{n=1}^{K-1} P_3(n) \right]$ ,

$$W = \frac{L}{\lambda_{eff}}, \quad \dots \quad (15a)$$

$$E(S) = \sum_{n=0}^F nP_0(n), \quad \dots \quad (15b)$$

$$E(B) = \sum_{n=0}^K nP_0(n) + \sum_{n=1}^K nP_1(n). \quad \dots \quad (15c)$$

#### 5. COST SENSITIVITY ANALYSIS

We construct a total expected cost function per unit time for the  $F$ -policy M/M/1/K queueing system with a second optimal service, in which  $F$  and  $K$  are a decision variable. Our objective is to determine the optimum threshold  $F$ , say  $F^*$ , and the optimum system capacity  $K$ , say  $K^*$ , simultaneously at minimum cost. The joint optimal values  $(F^*, K^*)$  and various system performance measures are obtained based on assumed numerical values given to the system parameters. Let us define the following cost elements:

$C_h$ : the holding cost per unit time for each customer present in the system;

$C_e$ : the cost per unit time for the busy server with essential service;

$C_{op}$ : the cost per unit time for the busy server with optional service;

$C_s$ : the setup cost per unit time for the preparatory work of the server before starting the service;

$C_b$ : the fixed cost for each lost customer when the system is blocked;

$C_w$ : the waiting cost per unit time when one customer is waiting for service;  
 $C_7$ : the fixed cost per customer's space;

The total expected cost function per unit time is determined by

$$TC(F,K)= C_hL+C_eP_e+C_{op}P_{op}+C_sP_s+C_b\lambda P_b+C_wW+C_7K, \quad \dots \quad (16)$$

where  $L$ ,  $P_e$ ,  $P_{op}$ ,  $P_s$ ,  $P_b$  and  $W$  are defined above.

The cost parameters in Eq. (16) are assumed to be linear in the expected number of the indicated quantity. Substitution of Eqs. (14) and (15a) into Eq. (16), the cost function is too ample to be shown here. In many practical optimization problems, we must focus our attention on finding the absolute minimum (or absolute maximum) of a function of two variables. We should mention that the first and second partial derivatives are powerful tools for locating and categorizing the relative extrema (relative minimum or relative maximum) of a function of two variables. Following the Calculus textbook, the second derivative test is used to find and classify the relative extrema of a function of two variables. Unfortunately, the analytic study of the behavior of the cost function  $TC(F, K)$  would have been an arduous task to undertake, or at least extremely difficult to develop the optimal solution  $(F^*, K^*)$  symbolically due to the highly non-linear and complex nature of the optimization problem. To the best of the authors' knowledge, no new and efficient methods to solve this optimization problem currently exist. This is due to the fact that there are two decision variables,  $F$  and  $K$ , involved in our model. Here, we should explicitly point out that the solution really gives the minimum value and the second partial derivative of  $TC(F, K)$  at the  $(F^*, K^*)$  is greater than 0 when the values of system parameters satisfy suitable conditions. Yet, it is quite boring to provide the explicitly expression. Thus, we will present extensive numerical experiments to show that the function is truly convex and that the solution actually gives a minimum. An efficient and direct procedure is used to obtain  $(F^*, K^*)$ . Following Hilliard (1976), we carry out the following steps for achieving the joint optimal values  $(F^*, K^*)$ .

Step 1: Find the optimal system capacity  $K^*$ , for threshold value  $F$ , i.e.,

$$\min_K TC(F, K) = TC(F, K^*)$$

Step 2: Find the set of all minimum cost solutions for  $F=1, 2, \dots, K-1$ ,

$$\Theta = \{TC(F, K^*): F=1, 2, \dots, K-1\}$$

Step 3: Find the minimum cost solution for  $F=1, 2, \dots, K-1$ ,

$$\min_F \Theta = TC(F^*, K^*)$$

An example (such as the transport service system mentioned in Section 1) is provided to illustrate the direct search procedure. For example:

- The ships arrive follows a Poisson process with rate  $\lambda=1.5$ .
- The preparing interval for admitting ship entrance is an exponential random variable with rate  $\gamma=0.1$ .
- The unloading and re-loading time are according to an exponential distribution with rate  $\mu_1=1.0$  and  $\mu_2=1.2$ , respectively.
- Upon the completion of unloading, the ships require re-loading with probability  $p=0.6$ .
- Holding cost  $C_h=\$10/\text{unit}$ , unloading cost  $C_e =100/\text{day}$ , re-loading cost  $C_{op}=60/\text{day}$ , Setup cost  $C_s =300/\text{day}$ , blocked cost  $C_b=400/\text{unit}$ , waiting cost  $C_w=8/\text{day}$ , and fixed cost  $C_7=5/\text{unit}$ .

Step 1 Find  $K^*$  for threshold value  $F$ , where  $F=1, 2, 3, \dots, 29$  (see Table 1)

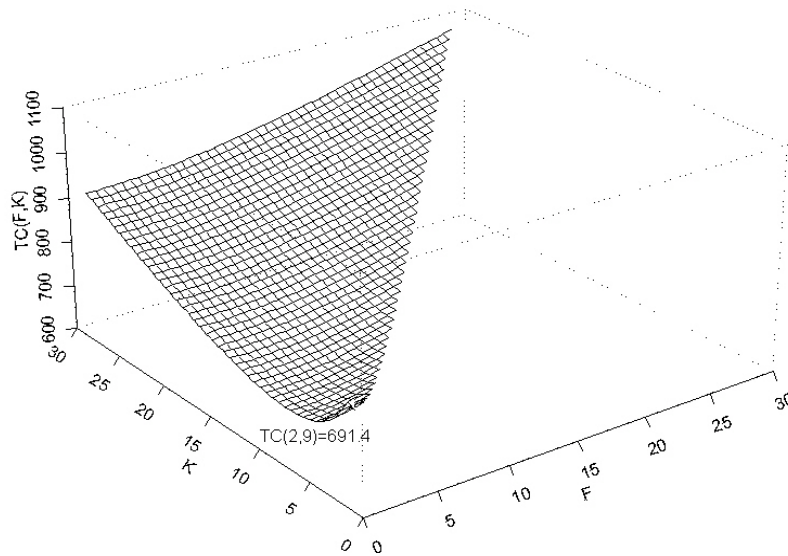
Step 2 From Table 1 the set of all minimum cost solutions  $\Theta=\{\$692.09, \$691.42, \$692.22, \dots, \$1063.63\}$

Step 3 From Step 2, the optimal solution  $TC(F^*, K^*)= \$691.42$  is achieved at  $F^*=2$  and  $K^*=9$ .

Alternatively, we also sketch this example to demonstrate that the expected cost function is convex and the solution achieves a global minimum. From Figure 2, we can find the expected cost function attains a minimum \$ 691.42 at  $F^*=2$  and  $K^*=9$ .

**Table 1. The expected cost  $TC(F,K^*)$  for given**

$F$	1	2	3	4	5	6	7	8	9	10
$(F,K^*)$	(1,9)	(2,9)	(3,9)	(4,9)	(5,9)	(6,10)	(7,10)	(8,11)	(9,11)	(10,12)
$TC(F,K^*)$	692.09	691.42	692.22	694.54	698.40	703.03	708.96	716.34	724.09	733.52
$F$	11	12	13	14	15	16	17	18	19	20
$(F,K^*)$	(11,12)	(12,13)	(13,14)	(14,15)	(15,16)	(16,17)	(17,18)	(18,19)	(19,20)	(20,21)
$TC(F,K^*)$	743.15	754.12	766.47	780.01	794.60	810.11	826.43	843.49	861.20	879.50
$F$	21	22	23	24	25	26	27	28	29	
$(F,K^*)$	(21,22)	(22,23)	(23,24)	(24,25)	(25,26)	(26,27)	(27,28)	(28,29)	(29,30)	
$TC(F,K^*)$	898.34	917.67	937.45	957.63	978.20	999.10	1020.33	1041.84	1063.63	

**Figure 2. The expected cost  $TC(F,K)$  for different values of different  $F$  and  $K$ .**

## 6. NUMERICAL ILLUSTRATION

The numerical model is analyzed to illustrate how the management of the transport service system can use the above results to make the decision regarding the system capacity and the threshold level to minimize the total expected cost (such as Table 1 and Figure 2). The managers can find the optimal system capacity (working place) and the optimal threshold level (for admitting ship entrance) are 9 and 2, respectively, which attain the minimum cost. Based on above cost setting listed above, we also perform a sensitivity analysis for changes in the optimal value ( $F^*$ ,  $K^*$ ) along with changes in specific values of the system parameters. Under optimal operating condition, numerical results are presented in which various system performance measures are calculated. Various values of ( $\lambda$ ,  $\mu_1$ ,  $\mu_2$ ) are considered when  $\gamma=0.1$  and  $p=0.6$ . The optimal value ( $F^*$ ,  $K^*$ ), the minimum expected cost  $TC(F^*, K^*)$ , and various system performance measures are displayed in Table 2.

From Table 2, one can see that (i) the optimal value ( $F^*$ ,  $K^*$ ) and the minimum expected cost  $TC(F^*, K^*)$  increase as  $\lambda$  increases; and (ii) the optimal value ( $F^*$ ,  $K^*$ ) and the minimum expected cost  $TC(F^*, K^*)$  non-increase as  $\mu_1$  increases. Moreover, ( $F^*$ ,  $K^*$ ) do not change as  $\mu_2$  increases and  $TC(F^*, K^*)$  slightly decreases with  $\mu_2$  increases. It is reasonable that the cost of maintaining the transport service system increases as the number of arriving ships increases. On the other hand, the optimal system capacity (working place) and the optimal threshold level don't almost change when re-loading service rate varies. The cost of maintaining the transport service system slightly decreases as the rate of re-loading service increases. We believe that the numerical investigation is useful and significant for the manager to make intelligent decision (such as the determination of working place and threshold level for admitting ship entrance, etc.).

**Table 2. System performance measures for different values of  $\lambda$ ,  $\mu_1$  and  $\mu_2$  under optimal operating conditions**

$(\lambda, \mu_1, \mu_2)$	(0.1,1.0,1.5)	(0.4,1.0,1.5)	(0.4,2.0,1.5)	(0.4,3.0,1.5)	(0.4,1.0,3.0)	(0.4,1.0,4.0)	(0.9,1.0,1.5)	(0.4,2.0,3.0)	(0.4,2.0,4.0)
$(F^*, K^*)$	(2,3)	(4,6)	(4,5)	(3,4)	(4,6)	(4,6)	(4,9)	(4,5)	(4,5)
$TC(F^*, K^*)$	129.4397	163.3724	136.4057	129.0396	159.3366	158.5444	370.5292	134.9840	134.7584
$L$	0.1397	0.8794	0.4231	0.3037	0.7458	0.7159	3.2937	0.3265	0.3051
$E(B)$	0.0058	0.0505	0.0064	0.0049	0.0379	0.0353	0.9419	0.0042	0.0039
$E(S)$	0.0021	0.0172	0.0027	0.0016	0.0144	0.0138	0.1426	0.0022	0.0021

**Table 3. System performance measures for different values of  $\gamma$  and  $p$  under optimal operating conditions**

$(\gamma, p)$	(0.01, 0.7)	(0.5, 0.7)	(0.1, 0.2)	(1, 0.2)	(10, 0.2)	(1, 0.5)	(1, 0.8)	(0.1, 0.5)	(0.1, 0.8)
$(F^*, K^*)$	(1,8)	(2,6)	(1,7)	(2,4)	(2,3)	(2,4)	(2,5)	(2,8)	(3,10)
$TC(F^*, K^*)$	981.1184	524.8738	714.7991	502.1438	426.5778	492.0476	475.9678	699.7748	667.1461
$L$	0.6893	3.0147	2.6301	2.4426	2.1537	2.3212	2.6669	3.0246	3.8226
$E(B)$	0.3750	1.6911	1.5687	1.4840	1.3365	1.4038	1.5246	1.7077	1.9437
$E(S)$	0.0083	0.1216	0.0307	0.1349	0.0478	0.1501	0.1233	0.0894	0.1579

Alternatively, Table 3 shows the optimal value  $(F^*, K^*)$ , the minimum expected cost  $TC(F^*, K^*)$ , and various system performance measures for different values of  $\gamma$  and  $p$ , in which  $\lambda=1.5$ ,  $\mu_1=1.0$  and  $\mu_2=1.2$ . Table 3 reveals that (i) the optimal value  $K^*$  and the minimum expected cost  $TC(F^*, K^*)$  increase as  $\gamma$  decreases, but  $F^*$  increases as  $\gamma$  increases; and (ii) the optimal value  $(F^*, K^*)$  increases as  $p$  increases, but the minimum expected cost  $TC(F^*, K^*)$  increases with  $p$  decreases. It is logical that the cost of maintaining the transport service system decreases as the startup rate (preparing for admitting ships entrance) increases or the occurrence of ships re-loading  $(1-p)$  decreases. Moreover, the optimal system capacity (working place) increases as the startup rate decreases and the optimal threshold level increases as the startup rate increases.

## 7. CONCLUSIONS

In this paper, we studied an  $F$ -policy  $M/M/1/K$  queueing system with a second optional service, and obtain the steady-state analytic solutions. This model generalizes (i) a generally  $M/M/1/K$  queueing system with a second optional service; and (ii) an  $F$ -policy  $M/M/1/K$  queueing system (see Gupta (1995)). We have provided a method to determine the optimal threshold and system capacity simultaneously, to minimize the expected cost function, and calculated various system performance measures under optimal operating conditions. This research presents an extension of the Markovian model theory and the analysis of the model will provide a useful performance evaluation tool for more general situations arising in practical applications, such as the wireless communication networks, the transport service and production system, and many other related systems.

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**BIOGRAPHICAL SKETCH**

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Jau-Chuan Ke is a professor of Applied Statistics at National Taichung Institute of Technology, Taiwan. He received his M.S. in Applied Mathematics at National Cheng-Kung University, Taiwan, and his Ph.D. in Applied Mathematics at National Chung-Hsing University, Taiwan. His research interests include reliability, statistics, and queueing theory.

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Chia-Jung Chang is an instructor of statistics in the Department of Accounting Information, National Taichung Institute of Technology, Taiwan. She received her Master. Degrees in Business Administration from Tamkung University, Taiwan, R.O.C., in 1987. Her current research interests include the area of applied statistics.

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Fu-Min Chang is an Assistant Professor in the Department of Finance, ChaoYang University of Technology, Taiwan. He received his Master and Ph.D. degrees from National Chung-Hsing University, Taiwan, R.O.C., in 1992 and 2005, respectively. His current research interests include the area of queueing network, network performance evaluation, and system and network management.

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