QUALITY DESIGN OF TAGUCHI'S DIGITAL DYNAMIC SYSTEMS

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Taguchi's method is a popular methodology utilized within many different industries in order to improve product quality and process performance. Digital dynamic systems denote the problems where both the input and output are digital values in Taguchi's method. When the signal factor levels are classified into two classes and the output is classified into two or more classes, two or more errors will occur in experiments. The digital dynamic systems are generally applied in the fields of telecommunication, computer operations, chemistry and tests of detection of medicine or environmental pollution. The SN ratio recommended by Taguchi is based on the errors with the same loss coefficient to optimize the problems. However, the losses due to the errors are not equal in practice. This paper proposes a general model for optimizing parameter design and selecting threshold value for the digital dynamic systems where the output is classified into four classes. The implementation and the effectiveness of the proposed approach are illustrated through two case studies.

Significance: The digital dynamic systems are usually applied in the fields of telecommunication, computer operations, chemistry and tests of detection of medicine or environmental pollution. This article presents an effective method to optimize such problem.

Keywords: Taguchi's method, Digital dynamic system, Error, SN ratio, Threshold value.

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1. INTRODUCTION

Stringent market competitiveness has driven manufacturers to enhance product quality. Taguchi's method (robust design) is a widely used methodology in many different industries for improving product quality and process performance of both static and dynamic systems. In dynamic systems, if a product or process to be optimized has a signal input that directly decides the output, the optimization involves determining the best control factor levels so that the input/output ratio is closest to the ideal function. In the digital dynamic system, the ideal function is that whenever an input signal is "Input 1", the output should be "Input 1", and whenever the input signal is "Input 2", the output should be "Input 2".

Several researchers have studied robust design problems concerning dynamic systems. Kapur and Chen (1988) developed the SN ratio for four cases of dynamic characteristic problems. They also presented the method to compute SN ratios both equispaced and non-equispaced intervals for levels of signal factors. Phadke and Dehnad (1988) derived a two-step procedure for optimizing the designs of products and processes. Taguchi (1992) proposed the SN ratio based on the errors with the same loss coefficient to optimize the problems. Miller and Wu (1996[a]) classified signal-response systems into two broad types: measurement systems and multiple target systems to analyze the dynamic systems. Miller and Wu (1996[b]) pointed out the deficiencies in Taguchi's dynamic SN ratio approach and then provided two strategies for modeling and analyzing data of dynamic systems. Wasserman (1996) illustrated the parameter design of dynamic system with the regression perspective. Lunani, Nair and Wasserman (1997) demonstrated the limitations of data analysis methods recommended by Taguchi and then proposed two graphical methods for identifying suitable measures of dispersion and for data analysis. McCaskey and Tsui (1997) developed an appropriate two-step procedure for dynamic systems under an additive model. This procedure reduces the dimension of the optimization problem and allows for future changes of the target slope without re-optimization. Su and Hsieh (1998) presented an approach based on neural network technique to achieve optimization of dynamic systems. Tsui (1999) investigated the response model analysis under an additive model and a linear response-to-signal relationship. Su, Chiu and Chang (2000) employed the neural network and genetic algorithm to search for the optimal parameter combination. Li (2001) presented three models for the establishment of the optimal parameter conditions by selection of the optimal threshold value for digital dynamic system. Miller (2002) covered three approaches, including the use of Taguchi's dynamic signal-to-noise ratio, a graphical technique and joint effects plot for analyzing the dynamic system. Nair, Taam and Ye (2002) analyzed the location and dispersion effects for optimizing the performance over a range of input-output values. Tong and Wang (2002) presented an alternative approach based on principal component analysis and grey relational analysis to determine an overall quality performance index for multiple responses. Tong, Wang, Houng and Chen (2002) used the dual response surface method to optimize the dynamic multi-response problems. Chen (2003) provided a stochastic optimization modeling procedure to accommodate dynamic characteristics. The advantage of this method is that it does not require any performance measure as the SN ratio.

Lesperance and Park (2003) proposed a joint generalized linear model based on standard regression modelling techniques. The model uses all of the data rather than relying on summary statistics, and the approach is applicable to a wide range of dynamic systems. Tong, Wang, Chen and Chen (2004) utilized the principal component analysis to simplify the dynamic multi-response problems and determine the optimization direction by using the variation mode chart. The proposed procedure transforms the correlated multiple responses into uncorrelated components through PCA, thereby simplifying the optimization process. Hsieh, Tong, Chiu and Yeh (2005) proposed a procedure utilizing the statistic regression analysis and desirability function to optimize the multiresponse problem. The approach can effectively departure those control factors which significant affect the response's variability or system's sensitivity and the requirement of minimizing variability and adjusting system's sensitivity can be achieved. Su, Chen and Chan (2005) used the neural networks to simulate the relationship between the control factor values and corresponding responses and then employed the scatter search to obtain the optimal parameter settings. Wang and Tong (2005) proposed a procedure including the technique for order preference by similarity to ideal solution, the multiple attribute decision-making method and the grey relational analysis to assess the respective response performance. The proposed procedure simultaneously considers the ideal and negative ideal solutions of each response so that it can explicitly depict the multiresponse performances and accurately determine an optimal factor/level combination. Wu and Yeh (2005) presented an approach to optimize multiple dynamic problems based on quality loss. The objective of proposed method is to minimize the total average quality loss for multiple dynamic quality characteristics experiment.

Bae and Tsui (2006) developed a two-step optimization procedure to substantially reduce the process variance by dampening the effect of both explicit and hidden noise variables based on a generalized linear model. The proposed method provides more reliable results through iterative modeling of the residuals from the fitted response model. Chang (2006) proposed an approach based on backpropagation neural networks and desirability functions to optimizing parameter design of the dynamic multi-response. The response model can predict all possible multi-responses of the system by presenting full parameter combinations. Through evaluating the performance measurement of the predicted dynamic multi-response, the best parameter setting can be obtained by maximizing the single index. Lee and Kim (2007) proposed a multiple response optimization model which minimizes quality loss function by extending the concept of a dual response surface approach to the multiple response systems. The overall performance of a multiple response problem is evaluated by obtaining mean and variance responses for each quality characteristic, and covariance responses among quality characteristics Wang (2007) developed a procedure using principal component analysis (PCA) and multiple criteria evaluation of the grey relation model to optimize dynamic multi-responses. PCA can consider the correlations among multiple quality characteristics to obtain uncorrelated components. These components are then substituted into multiple criteria evaluation of the grey relation model to determine the optimal factor level combination. Wu (2007) deduced the quality loss function of digital-digital dynamic system based on the two error rates with unequal loss coefficient. The model corrects the inappropriate formula suggested by Taguchi for optimizing the control factor settings. Chang (2008) employed the artificial neural networks to build a system's response function model and then used the desirability functions to evaluate the performance measures of multiple responses. Finally, he applied the simulated annealing algorithm to obtain the best factor settings through the response function model. The obtained best factor settings can be any values within their upper and lower bounds so that the system's multiple responses have the least sensitivity to noise factors along the magnitude of the signal factor.

Tong, Wang and Tsai (2008) presented a procedure for optimizing a dynamic system based on data envelopment analysis. The relative efficiencies of location effects and dispersion effects resulting from DEA are used as quality performance measures for the product/process mean and variance, respectively.

This paper gives a general model to optimize the digital dynamic system where the signal is classified into two classes and the output is classified into four classes. Two examples demonstrate the computation of error rates, threshold and standardized SN ratio, and the optimization of the problem.

2. DIGITAL DYNAMIC SYSTEM

In digital dynamic systems, if signal (Input 1 or Input 2) and response (output: y) are classified into two classes, the output is a continuous random variable affected by control factors and noise factors, the criterion for judging the output is the threshold value R. Since the transmission process is affected by noise factors, we do not have the ability to observe the distributions of the continuous variable received at the output terminal when "Input 1" or "Input 2" is transmitted. The optimal parameter design is performed to minimize the quality loss that occurred by two error rates p and q. This can be accomplished by a leveling operation, such as changing the threshold. Figure 1 shows the relationship between error rates and threshold.

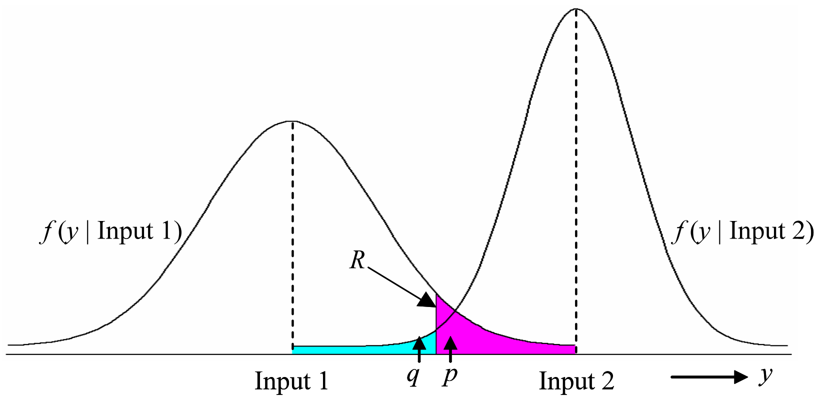

Figure 1. Error rates *p* and *q* are not equal when the threshold is at *R*

The digital dynamic system is widely applied in the fields of telecommunication, computer operations, chemistry and tests of detection of medicine or environmental pollution. For example, the chemical processes of metal refinement, element extraction and element separation can be viewed as a digital dynamic system that has two kinds of errors. In the health sciences field, a widely used application of experiments is found in the evaluation of screening tests and diagnostic criteria. An enhanced ability to correctly predict the presence or absence of a particular disease from a knowledge of test results (positive or negative) and/or the status of presenting symptoms (present or absent) is of interest to clinicians. The evaluation of screening tests and diagnostic criteria is based on the sensitivity and specificity. The disease status and screening test result can be viewed as a digital dynamic system that has two kinds of errors (false-positive and false-negative).

Suppose the signal is classified into two classes and the output is classified into four classes. The criterions for judging the output have two threshold values for signals "Input 1" and "Input 2", respectively. When signal is "Input 1", if output y is larger than R_1, the output y is judged as "Good 1"; if output y is between R_1 and R, the output y is judged as "Bad 1" (error rate p_1); if output y is smaller than R, the output y is judged as "Bad 2" (error rate p_2). When signal is "Input 2", if output y is smaller than R_2, the output y is judged as "Good 2"; if output y is between R_2 and R, the output y is judged as "Bad 2" (error rate q_2); if output y is larger than R, the output y is judged as "Bad 1" (error rate q_1). An input/output table can be developed, as shown in Table 1

Table 1. Input/output table in terms of error rate

Input \ Output	Good 1	Bad 1	Bad 2	Good 2	Total
Input 1	$1-p_1-p_2$	p_1	p_2	0	1
Input 2	0	q_1	q_2	$1-q_1-q_2$	1
Total	$1-p_1-p_2$	p_1+q_1	p_2+q_2	$1-q_1-q_2$	2

To obtain one accurate output "Good 1", on average, we have to obtain $p_1/(1-p_1-p_2)$ pieces wrong output "Bad 1" and $p_2/(1-p_1-p_2)$ pieces wrong output "Bad 2" when signal is "Input 1". Similarly, on average, we have to obtain $q_1/(1-q_1-q_2)$ pieces wrong output "Bad 1" and $q_2/(1-q_1-q_2)$ pieces wrong output "Bad 2" for obtaining one accurate output "Good 2" when signal is "Input 2". Hence, the data in Table 1 can be converted into those in Table 2 for processing one accurate output.

Table 2. Input/output table in terms of processing one accurate output

Input \ Output	Good 1	Bad 1	Bad 2	Good 2
Input 1	1	$p_1/(1-p_1-p_2)$	$p_2/(1-p_1-p_2)$	0
Input 2	0	$q_1/(1-q_1-q_2)$	$q_2/(1-q_1-q_2)$	1

Since the distributions of the continuous variable received at the output terminal are unknown, when signals "Input 1" and "Input 2" are transmitted, the thresholds values, R_1 and R_2, are fixed in the original system design. The optimal parameter design is performed to minimize the quality loss occurred by error rates p_1, p_2, q_1 and q_2. This can be accomplished by a leveling operation to change the threshold R to R'. Figure 2 shows the relationship among error rates and thresholds.

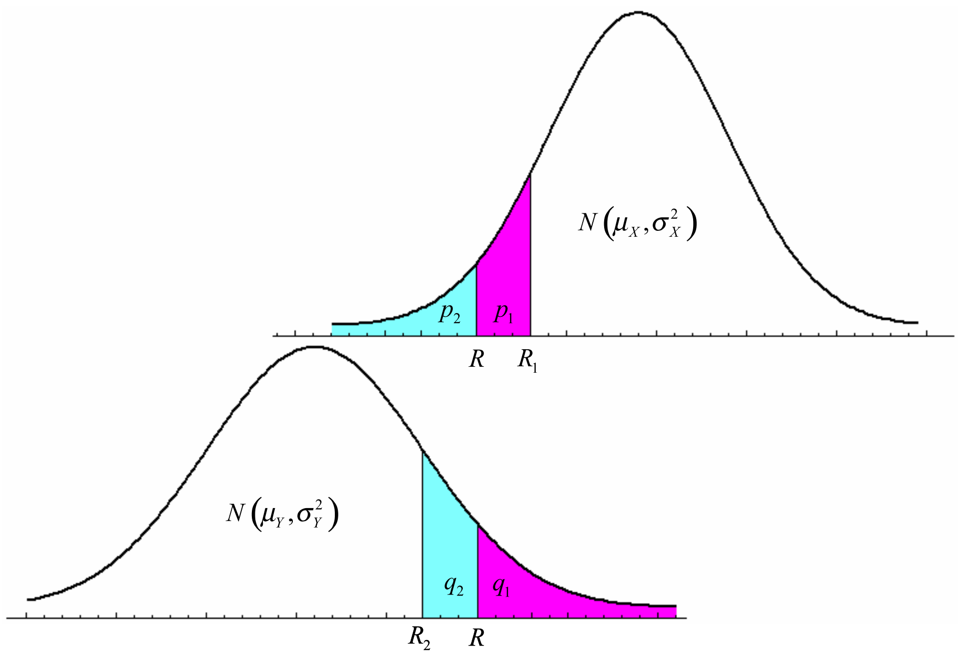

Figure 2. Relationship among error rates and thresholds

3. OPTIMIZATION OF ROBUST DESIGN FOR DIGITAL DYNAMIC SYSTEMS

Let K_{11} be the loss coefficient when signal is "Input 1" and output is "Bad 1"; K_{12} be the loss coefficient when signal is "Input 1" and output is "Bad 2"; K_{21} be the loss coefficient when signal is "Input 2" and output is "Bad 1"; K_{22} be the loss coefficient when signal is "Input 2" and output is "Bad 2". The total quality loss (L) can be expressed as

$$L = \left(K_{11} \cdot \frac{p_1}{1-p_1-p_2} + K_{12} \cdot \frac{p_2}{1-p_1-p_2} \right) + \left(K_{21} \cdot \frac{q_1}{1-q_1-q_2} + K_{22} \cdot \frac{q_2}{1-q_1-q_2} \right) \quad \cdots \quad (1)$$

Suppose that random variable X of signal "Input1" is normal distribution $N(\mu_X, \sigma_X^2)$ and random variable Y of signal "Input2" is normal distribution $N(\mu_Y, \sigma_Y^2)$. The following equalities hold.

$$P(R < X < R_1) = p_1 \quad \cdots \quad (2)$$

$$P(X < R) = p_2 \quad \cdots \quad (3)$$

$$P(Y > R) = q_1 \quad \cdots \quad (4)$$

$$P(R_2 < Y < R) = q_2 \quad \cdots \quad (5)$$

Since the original thresholds R_1, R and R_2 are known and error rates p_1, p_2, q_1 and q_2 can be obtained from the observations of experiment, from equations (2)-(5), the expected values and variances of random variables X and Y can be solved by

$$\mu_X = \frac{R \cdot \Phi^{-1}(p_1 + p_2) - R_1 \cdot \Phi^{-1}(p_2)}{\Phi^{-1}(p_1 + p_2) - \Phi^{-1}(p_2)} \quad \cdots \quad (6)$$

$$\sigma_X = \frac{R_1 - R}{\Phi^{-1}(p_1 + p_2) - \Phi^{-1}(p_2)} \quad \cdots \quad (7)$$

$$\mu_Y = \frac{R \cdot \Phi^{-1}(q_1 + q_2) - R_2 \cdot \Phi^{-1}(q_1)}{\Phi^{-1}(q_1 + q_2) - \Phi^{-1}(q_1)} \quad \cdots \quad (8)$$

$$\sigma_Y = \frac{R - R_2}{\Phi^{-1}(q_1 + q_2) - \Phi^{-1}(q_1)} \quad \cdots \quad (9)$$

From the above analysis, we propose an approach to the digital dynamic systems where output is classified into four classes, as described in the following:

Step 1. Evaluate the loss coefficients K_{11}, K_{12}, K_{21} and K_{22}, and then calculate the four error rates p_1, p_2, q_1 and q_2.

Step 2. Compute the expected values and variances of random variables X and Y of signals “Input 1” and “Input 2”, and then build the quality loss function using equation (1) for each test.

Step 3. Find error rates p_1', p_2', q_1' and q_2', and threshold value R' after leveling operation by mathematical programming for minimizing the quality loss, and then calculate the SN ratio by following equation for each test.

$$\eta_{af} = -10 \cdot \log\left(K_{11} \cdot \frac{p_1'}{1 - p_1' - p_2'} + K_{12} \cdot \frac{p_2'}{1 - p_1' - p_2'} + K_{21} \cdot \frac{q_1'}{1 - q_1' - q_2'} + K_{22} \cdot \frac{q_2'}{1 - q_1' - q_2'} \right) \quad \cdots \quad (10)$$

Step 4. Construct the main effects of factor levels table according to the SN ratio and then determine those factors that have a strong effect on the quality characteristic of interest.

Step 5. Compute the predicted average SN ratio of optimal parameter settings, and then find the error rates, threshold value and quality loss after leveling operation.

4. INDUSTRY APPLICATION

Case 1: Digital receiving

The effectiveness of the proposed optimization procedure is demonstrated in a digital receiving case presented by Taguchi (1992). RS-232 is a complete interface standard developed by the Electronics Industry Association. A high level voltage (+3V to +15V) for the driver output is defined as is defined as logic 0 and is historically referred to as “Space”. A low level voltage (-3V to -15V) for the driver output is defined as is defined as a logic 1 and is referred to as “Mark”. If the received pulse is near zero positive voltage, it is referred to as “Bad Space”. If the received pulse is near zero negative voltage, it is referred to as “Bad Mark”. The current digital receiving status is as shown in Table 3.

Seven factors with two levels each were selected and assigned to an L_8 orthogonal array for improvement. The current conditions are the first levels. For each test, 10,000 spaces and marks each were transmitted and the observed status was as shown in the Table 4.

Table 3. Current digital receiving status

Input \ Receiving	Good Space	Bad Space	Bad Mark	Good Mark	Total (%)
Space (S)	99.8735	0.1221	0.0044	0.0000	100.0000
Mark (M)	0.0000	0.0030	0.0845	99.9125	100.0000
Total (%)	99.8735	0.1251	0.0889	99.9125	200.0000

Table 4. Factor assignment and experimental data

Expt. No.	Layout							Input Pulse	Types of Received Pulse				Total
	A	B	C	D	E	F	G		Good Space	Bad Space	Bad Mark	Good Mark	
1	1	1	1	1	1	1	1	Space	9678	310	12	0	10000
								Mark	0	6	238	9756	10000
2	1	1	1	2	2	2	2	Space	9892	104	4	0	10000
								Mark	0	3	72	9925	10000
3	1	2	2	1	1	2	2	Space	9971	28	1	0	10000
								Mark	0	1	42	9957	10000
4	1	2	2	2	2	1	1	Space	9009	956	35	0	10000
								Mark	0	52	1426	8522	10000
5	2	1	2	1	2	1	2	Space	9874	119	7	0	10000
								Mark	0	5	85	9910	10000
6	2	1	2	2	1	2	1	Space	9957	42	1	0	10000
								Mark	0	0	32	9968	10000
7	2	2	1	1	2	2	1	Space	9963	35	2	0	10000
								Mark	0	3	49	9948	10000
8	2	2	1	2	1	1	2	Space	7942	1980	78	0	10000
								Mark	0	110	2820	7070	10000

Suppose that the error rate of the received pulse is "Bad Space" is p_S and error rate of the received pulse is "Bad Mark" is p_M while input pulse is "Space"; the error rate of the received pulse is "Bad Space" is q_S and error rate of the received pulse is "Bad Mark" is q_M while input pulse is "Mark". Let K_{SS} present the loss coefficient of judging the received pulse is "Bad Space" and K_{SM} present the loss coefficient of judging the received pulse is "Bad Mark" while input pulse is "Space"; K_{MS} present the loss coefficient of judging the received pulse is "Bad Space" and K_{MM} present the loss coefficient of judging the received pulse is "Bad Mark" while input pulse is "Mark". Hence, the total quality loss L is expressed as

$$L = K_{SS} \cdot \frac{p_S}{1-p_S-p_M} + K_{SM} \cdot \frac{p_M}{1-p_S-p_M} + K_{MS} \cdot \frac{q_S}{1-q_S-q_M} + K_{MM} \cdot \frac{q_M}{1-q_S-q_M} \quad \cdots \quad (11)$$

From Table 4, the input/output table in terms of error rate is tabulated in Table 5. Suppose the random variables of input pulse are "Space" and "Mark" are normal distributions $N\left(\mu_X, \sigma_X^2\right)$ and $N\left(\mu_Y, \sigma_Y^2\right)$, then the following equalities hold.

$$P\left(R < X < 3\right) = p_S \quad \cdots \quad (12)$$

$$P\left(X < R\right) = p_M \quad \cdots \quad (13)$$

$$P\left(Y > R\right) = q_S \quad \cdots \quad (14)$$

$$P\left(-3 < Y < R\right) = q_M \quad \cdots \quad (15)$$

Suppose the original thresholds are $R_1 = 3$, $R = 0$ and $R_2 = -3$, then from equations (12)-(15), we can find the parameters of normal distributions for input pulses are "Space" and "Mark", shown as Table 6.

Table 5. Input/output table in terms of error rate

Expt. No.	Layout A	B	C	D	E	F	G	Input Pulse	Types of Received Pulse: Good Space	Bad Space	Bad Mark	Good Mark	Total
1	1	1	1	1	1	1	1	Space	0.9678	0.0310	0.0012	0.0000	1
								Mark	0.0000	0.0006	0.0238	0.9756	1
2	1	1	1	2	2	2	2	Space	0.9892	0.0104	0.0004	0.0000	1
								Mark	0.0000	0.0003	0.0072	0.9925	1
3	1	2	2	1	1	2	2	Space	0.9971	0.0028	0.0001	0.0000	1
								Mark	0.0000	0.0001	0.0042	0.9957	1
4	1	2	2	2	2	1	1	Space	0.9009	0.0956	0.0035	0.0000	1
								Mark	0.0000	0.0052	0.1426	0.8522	1
5	2	1	2	1	2	1	2	Space	0.9874	0.0119	0.0007	0.0000	1
								Mark	0.0000	0.0005	0.0085	0.9910	1
6	2	1	2	2	1	2	1	Space	0.9957	0.0042	0.0001	0.0000	1
								Mark	0.0000	0.0000	0.0032	0.9968	1
7	2	2	1	1	2	2	1	Space	0.9963	0.0035	0.0002	0.0000	1
								Mark	0.0000	0.0003	0.0049	0.9948	1
8	2	2	1	2	1	1	2	Space	0.7942	0.1980	0.0078	0.0000	1
								Mark	0.0000	0.0110	0.2820	0.7070	1

Table 6. The parameters of normal distributions for input pulse

Expt. No.	Layout A	B	C	D	E	F	G	Space μ_X	Space σ_X	Mark μ_Y	Mark σ_Y
1	1	1	1	1	1	1	1	7.67701	2.52893	-7.65967	2.36491
2	1	1	1	2	2	2	2	9.52981	2.84235	-10.30272	3.00230
3	1	2	2	1	1	2	2	11.62026	3.12455	-10.22215	2.74862
4	1	2	2	2	2	1	1	5.73737	2.12744	-5.06931	1.97847
5	2	1	2	1	2	1	2	10.02162	3.13700	-10.67303	3.24356
6	2	1	2	2	1	2	1	10.22215	2.74862	-10.02695	2.57723
7	2	2	1	1	2	2	1	12.32337	3.48110	-11.84164	3.45075
8	2	2	1	2	1	1	2	4.54236	1.87845	-3.93596	1.71848

Suppose the loss coefficients are $K_{SS}=1$, $K_{SM}=2$, $K_{MS}=2$ and $K_{MM}=1$. The error rates, threshold, quality loss and SN ratio for each test can be obtained by steps 1-3, shown as Table 7.

Table 7. The error rates and SN ratio after leveling operation for each test

Expt. No.	Layout	Error rates after leveling operation $p_S{}'$	$p_M{}'$	$q_S{}'$	$q_M{}'$	Threshold R'	Loss	SN ratio (η)
1	Ignored	0.03131	0.00089	0.00083	0.02357	-0.22532	0.06006	12.21442
2		0.01047	0.00033	0.00036	0.00714	-0.14007	0.01917	17.17349
3		0.00279	0.00011	0.00009	0.00421	0.05094	0.00743	21.29170
4		0.09455	0.00455	0.00394	0.14386	0.18789	0.29310	5.32978
5		0.01202	0.00058	0.00060	0.00840	-0.17111	0.02304	16.37586
6		0.00423	0.00007	0.00007	0.00313	-0.20607	0.00767	21.15045
7		0.00345	0.00025	0.00024	0.00496	0.19307	0.00943	20.25339
8		0.19551	0.01029	0.00814	0.28486	0.19278	0.69803	1.56126

From Table 7, the main effects of factor levels according to the SN ratio are listed in Table 8 and the optimal factor settings are $B_1C_2D_1F_2$.

Table 8. Factor effects (SN ratio)

Level \ Factor	A	B	C	D	E	F	G
Level 1	14.00235	16.72856	12.80064	17.53384	14.05446	8.87033	14.73701
Level 2	14.83524	12.10903	16.03695	11.30375	14.78313	19.96726	14.10058
difference	0.83289	4.61953	3.23631	6.23010	0.72867	11.09693	0.63643

The predicted average SN ratio is

$$\widehat{\mu}_{\eta} = \overline{T} + \left(\overline{B}_1 - \overline{T}\right) + \left(\overline{C}_2 - \overline{T}\right) + \left(\overline{D}_1 - \overline{T}\right) + \left(\overline{F}_2 - \overline{T}\right) = 27.01022\ (d\mathrm{B}) \quad \cdots \quad (16)$$

Hence, the total quality loss before leveling operation for the optimal factor settings is $10^{-27.01022/10} = 0.00199$, and the predicted error rates are $\widehat{p}_S = 0.0011553$, $\widehat{p}_M = 0.0000024$, $\hat{q}_S = 0.0000012$ and $\hat{q}_M = 0.0008159$ for input pulses are "Space" and "Mark", respectively. The random variables of input pulses "Space" and "Mark" are normal distributions $N\left(8.9857, 1.9648^2\right)$ and $N\left(-9.0506, 1.9210^2\right)$.

The error rates and threshold after leveling operation are $\widehat{p}_S{}' = 0.0011560$, $\widehat{p}_M{}' = 0.0000017$, $\hat{q}_S{}' = 0.0000017$, $\hat{q}_M{}' = 0.0008154$ and $R' = -0.1295$. The quality loss reduces to 0.00198. The comparison of current and optimal factor settings is tabulated in Table 9.

Table 9. Comparison of current and optimal factor settings

Input \ Receiving	Current Factor Settings				
	Good Space	Bad Space	Bad Mark	Good Mark	Loss
Space	0.998735	0.001221	0.000044	0.000000	0.00222
Mark	0.000000	0.000030	0.000845	0.999125	
Input \ Receiving	**Optimal Factor Settings**				
	Good Space	Bad Space	Bad Mark	Good Mark	Loss
Space	0.9988423	0.0011560	0.0000017	0.0000000	0.00198
Mark	0.0000000	0.0000017	0.0008154	0.9991829	

Case 2: Evaluation of the performance of tests for the detection of HBsAg

The qualitative analysis for the detection of a virus in medicine often causes a patient's doubt about inspection accuracy that is due to the critical value (threshold) varying with different laboratories and manufacturers. Randrianirina et al. (2008) evaluated four rapid immunochromatographic assays – Determine™ HBsAg, Virucheck® HBsAg, Cypress HBsAg Dipstick® and Hexagon® HBsAg – for human hepatitis B surface antigen (HBsAg) detection in human serum.

A collection of reference serum samples (91 HBsAg positive and 109 HBsAg negative) stored at -80°C was used. The tests evaluated were (1) Determine™ HBsAg rapid immunochromatographic test (storage temperature: 2-30 °C), which requires 50 µl of serum and gives a readout in 15 min; (2) Virucheck® HBsAg (storage temperature: 4-30 °C), which requires 100 µl of serum and gives a readout in 15 min; (3) Cypress HBsAg Dipstick® (storage temperature: 2-30 °C), requiring 250-500 µl of serum and gives a readout in 10 min; (4) Hexagon® HBsAg (storage temperature: 2-30 °C), that necessitates from 250 to 500 µl of serum and gives a readout in 20 min. All these tests give visual readout. If no red bar appeared in the test window, then the test was considered to be negative. If any red coloration was visible in the test window, the sample was considered to have tested positive. If no bar was observed in the control window, the test was rejected. The results of four immunochromatographic assays with reference serum samples for HBV infection are listed in Table 10.

Table 10. Results of evaluation for HBV infection

Immunochromatographic assays	Input \ Output	Negative	Positive	Total
Determine™ HBsAg	Negative	109	0	109
	Positive	2	89	91
Virucheck® HBsAg	Negative	107	2	109
	Positive	4	87	91
Cypress HBsAg Dipstick®	Negative	105	4	109
	Positive	3	88	91
Hexagon® HBsAg	Negative	105	4	109
	Positive	4	87	91

It is quite obvious that the cost of false-positive is different to the cost of false-negative. Let's assume the loss coefficients are K_1=1 (false-positive) and K_2=5 (false-negative). The input/output table in terms of sensitivity and specificity are presented in Table 11 and the data in Table 11 can be converted into those in Table 12 for processing one accurate output.

Table 11. Input/output table in terms of sensitivity and specificity

Immunochromatographic assays	Input \ Output	Negative	Positive	Total
Determine™ HBsAg	Negative	1.00000 ($1-p_D$)	0.00000* (p_D)	1
	Positive	0.02198 (q_D)	0.97802 ($1-q_D$)	1
Virucheck® HBsAg	Negative	0.98165 ($1-p_V$)	0.01835 (p_V)	1
	Positive	0.04396 (q_V)	0.95604 ($1-q_V$)	1
Cypress HBsAg Dipstick®	Negative	0.96330 ($1-p_C$)	0.03670 (p_C)	1
	Positive	0.03297 (q_C)	0.96703 ($1-q_C$)	1
Hexagon® HBsAg	Negative	0.96330 ($1-p_H$)	0.03670 (p_H)	1
	Positive	0.04396 (q_H)	0.95604 ($1-q_H$)	1

* As an approximation, it is assumed that $p_D = 1/(2\cdot 109)$ suggested by Taguchi (1992) when $p_D = 0$.

Table 12. Input/output table in terms of processing one accurate output

Immunochromatographic assays	Input \ Output	Negative	Positive	Total
Determine™ HBsAg	Negative	1	0.00461	1.00461
	Positive	0.02247	1	1.02247
Virucheck® HBsAg	Negative	1.00000	0.01869	1.01869
	Positive	0.04598	1	1.04598
Cypress HBsAg Dipstick®	Negative	1	0.03810	1.03810
	Positive	0.03409	1	1.03409
Hexagon® HBsAg	Negative	1	0.03810	1.03810
	Positive	0.04598	1	1.04598

Hence, the total quality loss L is expressed as

$$L = K_1 \cdot \frac{p}{1-p} + K_2 \cdot \frac{q}{1-q} \quad \cdots \quad (17)$$

The following equality holds for minimizing the total quality loss by adjusting the threshold value.

$$K_1 \cdot \frac{p'}{1-p'} = K_2 \cdot \frac{q'}{1-q'} = \sqrt{K_1 \cdot K_2 \cdot \frac{p}{1-p} \cdot \frac{q}{1-q}} \quad \cdots \qquad (18)$$

Thus, the rates of false-negative and false-positive p' and q' after adjustment are given by:

$$p' = \sqrt{K_1 \cdot K_2 \cdot \frac{p \cdot q}{(1-p)(1-q)}} \Bigg/ \left(K_1 + \sqrt{K_1 \cdot K_2 \cdot \frac{p \cdot q}{(1-p)(1-q)}} \right) \quad \cdots \qquad (19)$$

$$q' = \sqrt{K_1 \cdot K_2 \cdot \frac{p \cdot q}{(1-p)(1-q)}} \Bigg/ \left(K_2 + \sqrt{K_1 \cdot K_2 \cdot \frac{p \cdot q}{(1-p)(1-q)}} \right) \quad \cdots \qquad (20)$$

The rates of false-positive and false-negative after leveling operation, quality loss and SN ratio for each assay can be obtained by equations (19-20), shown as Table 13.

Table 13. Results of evaluation for HBV infection

Immunochromatographic assays	False-positive	False-negative	Threshold R'	Quality loss	SN ratio (η)
Determine™ HBsAg	0.02225	0.00453	$0.77121 \cdot R_D$	0.04551	13.41895
Virucheck® HBsAg	0.06152	0.01294	$0.73818 \cdot R_V$	0.13110	8.82392
Cypress HBsAg Dipstick®	0.07457	0.01586	$0.80573 \cdot R_C$	0.16116	7.92730
Hexagon® HBsAg	0.08557	0.01837	$0.76438 \cdot R_H$	0.18716	7.27779

From Table 13, the Determine™ HBsAg test appears to be the most suitable for Madagascar based on the SN ratio, which is consistent with the results of Randrianirina et al.

5. CONCLUSIONS

Analyzing the sensitivity and specificity from Table 11, the differences between the tests were not significant to evaluate which is better between Virucheck® HBsAg and Cypress HBsAg Dipstick®. Although the sensitivity (95.604 %) of Virucheck® HBsAg test is lower than the sensitivity (96.703 %) of Cypress HBsAg Dipstick® test, the specificity (98.165 %) of Virucheck® HBsAg test is better than the specificity (96.330 %) of Cypress HBsAg Dipstick® test. Traditional evaluation of the performance of tests in medicine is based on the sensitivity and specificity separately. The SN ratio can evaluate the performance of the sensitivity and specificity simultaneously. From Table 13, the performance of Virucheck® HBsAg test is better than Cypress HBsAg Dipstick® test by SN ratio.

Robust design is conventionally used for off-line quality control and the digital communication systems, chemical separation processes, detections of virus or toxicity, etc., where the signal factor and the quality characteristic are digital, are examples of the digital dynamic system. To reduce the quality loss occurred by errors in digital dynamic system, it is important to calibrate by error rates of so-called leveling operation and calculate the SN ratio after leveling operation. The SN ratio recommended by Taguchi is based on the errors with the same loss coefficient to optimize the digital dynamic systems. However, the losses due to the errors are not equal in practice. This paper proposes a general model for optimizing parameter design and selecting threshold value for the digital dynamic systems where the output is classified into four classes. The implementation and the effectiveness of the proposed approach are illustrated through two case studies.

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