International Journal of Industrial Engineering, 16(3), 227-233, 2009.

Beyond Six Sigma – A Control Chart for Tracking Defects per Billion Opportunities (dpbo)

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In many processes and, in particular, those related to electronics packaging and assembly, the amount of possible defect counts per unit of product has become quite large. Because the classical attributes-based statistical process control (SPC) charts where defects are measured in counts – the *u* chart and *c* chart, for examples – were cumbersome to cope with such large scale defect possibilities, a defects per million opportunities (*dpmo*) chart was developed in the mid 1990s. Not long after this, it was mentioned by a researcher at Packard Bell that "world-class" in surface mount technology (SMT) – an assembly technique for electronics manufacturing – should indicate that the company is operating at 50*ppm* (or fewer) defect levels and suggested that the number could drop to 10*ppm*. Furthermore, it was hypothesized that the electronics industry may one day refer to defect levels in terms of parts-per-billion (*ppb*) – defect levels reflective of process capabilities better than six sigma levels. With this in mind, this paper presents a new control chart for attributes data measured in counts where the plot point per period is represented by defects per billion opportunities (*dpbo*). In addition to showing the plot point and control chart calculations, an example will be provided and analyzed to demonstrate its use.

Significance: This paper establishes an argument and presents a new control chart to track attributes data for processes that have process capabilities that exceed six sigma levels for products that are much more complex than those at the time traditional control charts were developed.

Keywords: SPC, attributes data, six sigma, *u* chart, *dpmo* chart, *dpbo* chart.

(Received: 15 April 2009; Accepted in revised form: 12 May 2009)

1. INTRODUCTION

Six sigma (6σ) processes are processes that can measure their output in single digits of parts-per-million (*ppm*) defect levels. Statistical Process Control (SPC) charts have been used since Shewart's work in the 1930s (see Shewhart (1931)). When appropriately applied, traditional control charts can be used to monitor the stability of a process and help track defect levels. Unfortunately, as demonstrated in a case study by McCoy et al. (2004), some practitioners may incorrectly apply control charts and, in so doing, may unknowingly – and hopefully, unintentionally – misrepresent quality data and process capability.

Whether used appropriately or not, traditional control charts were developed in an era in which products were not as complex as they are in today's age. Taking the electronics industry as but one example, some products (e.g., printed circuit boards (PCBs)) have thousands and others have even tens of thousands of opportunities for defects. When it is of importance to accurately track how many defects that occur per product (as opposed to just making a determination if the entire product is "good" or "bad"), the increased complexity of today's products drives the need for new control charting techniques. A Packard Bell researcher has suggested (Revelino, 1997) that "world-class" in electronics manufacturing would drop so much that defect levels will eventually be referred to in parts-per-billion (*ppb*). These defect levels will be better than that of 6σ and, due to the limited resolution of traditional control charts, new control charting techniques will be required.

2. BACKGROUND

Aside from more recent advances and techniques in SPC such as Engineering Process Control (EPC) and Automated Process Control (APC) – which, for the most part, are used synonymously – there are two classes of control charts. Control charts are classified as either variables control charts or attributes control charts.

Variables data are data that can be measured on a continuous scale (e.g., length, lumens, resistance, *ad infinitum*), subject to the accuracy of the measuring equipment. Control charts to monitor variables data trace their roots to work done

ISSN 1943-670X

by Shewhart in the 1930s. Most readers will likely be familiar with charts such as $\overline{X}$ (based on subgroup averages), *R* (based on subgroup ranges), and *s* (based on subgroup standard deviations). Of course there are variations to these charts. There are charts based upon individuals data (subgroups of size 1) and there are charts (*EWMA*, *CuSum*, and others) designed to detect process shifts smaller than those detected by the use of traditional variables control charts. While there are techniques to handle data that are non-normal, for the most part, variables control charts assume that the underlying process output is normally distributed.

Attributes data are data that can be measured by counts; i.e., using a discrete scale. Attributes are essentially characteristics that can be noted by their absence or presence. In the use of attributes data, practitioners are more interested, for example, if a part from the process is “good” or “bad” versus how much it deviates from a target or a specification range. Of course, there are other types of attributes, such as counting the number of defect types per part. The traditional attributes control charts, their use, subgroup size, and their underlying distributions, are shown in Table 1.

Table 1. Shewhart Control Charts for Attributes Data

Chart	Use	Subgroup Size	Distribution
p	Assess proportion defective (entire product is good or bad)	> 1	Binomial
q	Assess proportion good (complement of the *p* chart)	> 1	Binomial
np	Assess the number of defective parts in a subgroup (scaled version of *p* chart)	> 1	Binomial
c	Assess number of defects per part (i.e., each part can have multiple defects)	= 1	Poisson
u	Assess number of defects per part when sample size > 1	> 1	Poisson

The *c* and *u* charts are utilized when a product can have multiple defect opportunities. Consider an automobile for example. The automobile can have a defective paint application, could have a defective drive train, etc. If the interest is in determining what the total number of defects per automobile is, then the use of a *c* or *u* chart – depending upon sample size per period – would be appropriate. On the other hand, if we were only interested in determining if, as a whole, the automobile was “good” or “bad”, then the use of the other charts (*p, q, np*, and other variations not discussed herein) would be appropriate.

2.1 Calculations for the *u* Chart

Most readers with a general understanding of classical SPC techniques will be familiar with the control chart calculations for the charts discussed to this point. However, the control chart calculations for the *u* chart will now be presented because it serves as the basis for two other charts (*dpmo* and *dpbo*) that will be later described.

In order to calculate the control limits for the u chart, we need the following:

- k = number of subgroups
- n_i = size of subgroup i ($i = 1, 2, \ldots, k$); subgroups are typically constant, but may be allowed to vary
- c_i = count of defects in subgroup i ($i = 1, 2, \ldots, k$)

Given the above, the point to plot on this chart is the average number of defects per unit, typically denoted as u_i, and is calculated per each subgroup, i, as follows:

$$u_i = \frac{c_i}{n_i} \tag{1}$$

Once all subgroups are gathered, the centerline for the *u* chart is $\overline{u}$ and is calculated as follows:

$$\overline{u} = \frac{\text{total defects found in all subgroups}}{\text{total number of items inspected}} = \frac{\sum_i c_i}{\sum_i n_i} \tag{2}$$

Now that the centerline is established, the control limits can be calculated. As in other control limits, they are ±3 standard deviations of the process output. As process output for defect counts is assumed to follow a Poisson distribution (see Table 1), the control limits are the following:

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{n_i}} \tag{3}$$

3. THE *dpmo* CHART

The defects per million opportunities (*dpmo*) chart is a relatively new attributes control chart, certainly as compared to Shewhart charts. One of the first references to the *dpmo* chart can be found in relation to an electronics manufacturing application by Ngo (1995). Although this reference is more than a decade old, most recently published quality control textbooks (see Montgomery (2005), DeVor et al. (2007), and Besterfield (2008) for examples) still do not include the *dpmo* chart in their review of attributes control charts.

The *dpmo* chart is particularly useful in the monitoring of electronics manufacturing operations or any other process that has products with large numbers of defect opportunities. For another practical application of the *dpmo* chart, see Santos et al. (1997) and see Yepez et al. (2008) for other SPC applications for electronics manufacturing. As electronics products become more complex, the number of defect opportunities per product has increased tremendously in recent years. Electronics products (e.g., backplanes, complex motherboards for server systems, etc.) can have as many as thousands of opportunities for defects per circuit board. The defects can be traced to improper solder joints (potentially thousands on a PCB), missing components, improperly placed components, and others.

3.1 Calculations for the *dpmo* Chart

Just as the *np* chart is a scaled version of the *p* chart, the *dpmo* chart is a scaled version of the *u* chart. The *u* chart assumes a few defect opportunities per product, but the *dpmo* chart assumes there are a substantial number of defect opportunities per product.

In order to calculate the control limits for the *dpmo* chart, we need the following:

- k = number of subgroups
- n_i = size of subgroup i ($i = 1, 2, \ldots, k$); subgroups are typically constant, but may be allowed to vary
- c_i = count of defects in subgroup i ($i = 1, 2, \ldots, k$)
- Number of defect opportunities per product

Given the above, the average number of defects per unit, dpu_i, is calculated per each subgroup in a similar fashion as the plot points for the *u* chart. However, dpu_i, shown below, is not the plot point for the *dpmo* chart:

$$dpu_i = \frac{c_i}{n_i} \tag{4}$$

The plot point for this chart is $dpmo_i$, for each subgroup, i. The plot point is found by the following calculation which is a scaled version of the number of defects per unit provided in reference to one million defect opportunities:

$$dpmo_i = \frac{dpu_i}{\text{number of defect opportunities}} \, x \, 10^6 \tag{5}$$

Once all subgroups are gathered, the centerline for the *dpmo* chart is $\overline{dpmo}$ and is calculated as the average of all the *dpmo* values as follows:

$$\overline{dpmo} = \frac{\sum_i dpmo_i}{k} \tag{6}$$

Now that the centerline is established, the control limits can be calculated. The control limits for the *dpmo* chart will also be based upon the Poisson distribution as the purpose is to still measure defect counts which are assumed to be distributed in a Poisson fashion; but, as expected, the *dpmo* chart limits are a scaled version of the *u* chart's as follows:

$$\overline{dpmo} \pm 3\sqrt{\frac{\overline{dpmo}\ x\ 10^{6}}{n_i\, x\,(\text{number of defect opportunities})}} \tag{7}$$

And the above can be simplified to the following:

$$\overline{dpmo} \pm 3000\sqrt{\frac{\overline{dpmo}}{n_i\ \ x\,(\text{number of defect opportunities})}} \tag{8}$$

4. THE *dpbo* CHART

At this point, it may be obvious to the reader that the *dpbo* chart represents another scaling of the *u* chart. In this case, and keeping in line with the earlier statement by Revelino, the scale/resolution of a *u* chart is just too large to be of practical use for dealing with defect levels at six sigma or beyond (i.e., very low defect levels with applications that have high opportunities for defects). Like the *dpmo* chart, the *dpbo* chart is of benefit to the variety of manufacturing operations that have products with very large numbers of defect opportunities such as the multitude of organizations in the various levels of the electronics packaging industry – from Level 0 (semiconductor fabrication) through Level 2 (printed circuit board assembly).

4.1 Calculations for the *dpbo* Chart

Aside from $dpbo_i$, the variables in this chart are the same as those appearing for use in the *dpmo* chart discussion, above. The calculations to follow have been provided in an unpublished presentation (Santos, 2008). The plot point for this chart is $dpbo_i$ and is calculated as follows, where dpu_i is also calculated as in Equation 4:

$$dpbo_i = \frac{dpu_i}{\text{number of defect opportunities}}\ x\ 10^{9} \tag{9}$$

The centerline is $\overline{dpbo}$ and is calculated as follows:

$$\overline{dpbo} = \frac{\sum_i dpbo_i}{k} \tag{10}$$

The control limits for the *dpbo* chart are the following:

$$\overline{dpbo} \pm 3\sqrt{\frac{\overline{dpbo}\ x\ 10^{9}}{n_i x\,(\text{number of defect opportunities})}} \tag{11}$$

The above limits simplify to the following:

$$\overline{dpbo} \pm 94{,}868.33\sqrt{\frac{\overline{dpbo}}{n_i\ \ x\,(\text{number of defect opportunities})}} \tag{12}$$

4.2 Example of *dpbo* chart

To illustrate the *dpbo* chart, a hypothetical example – but one indicative of a typical PCB assembly process – will be utilized. This hypothetical example is intentionally designed to be of questionable (i.e., less than 6σ) quality levels so as to compare the *dpbo* chart with the *dpmo* chart and to demonstrate that it is, in fact, a scaled version of the *dpmo* (and, though not demonstrated, the *u*) chart.

For this example, the following are known:

- 100 PCB assemblies are inspected each day (n_i = 100 for all days)
- Each assembly has 3,000 opportunities for defects
- 24 days are used to establish the control limits (k = 24)

Table 2 lists, for each day, the total number of defects found (per 100 assemblies), the *dpu* values, the *dpmo* values, and the *dpbo* values.

Table 2. Example Data for *dpmo* and *dpbo* Control Chart Calculations

Day	Defects	*dpu*	*dpmo*	*dpbo*
1	19	0.19	63.33	63333.33
2	19	0.19	63.33	63333.33
3	22	0.22	73.33	73333.33
4	19	0.19	63.33	63333.33
5	21	0.21	70.00	70000.00
6	17	0.17	56.67	56666.67
7	29	0.29	96.67	96666.67
8	13	0.13	43.33	43333.33
9	15	0.15	50.00	50000.00
10	17	0.17	56.67	56666.67
11	16	0.16	53.33	53333.33
12	17	0.17	56.67	56666.67
13	17	0.17	56.67	56666.67
14	15	0.15	50.00	50000.00
15	23	0.23	76.67	76666.67
16	22	0.22	73.33	73333.33
17	27	0.27	90.00	90000.00
18	17	0.17	56.67	56666.67
19	20	0.20	66.67	66666.67
20	22	0.22	73.33	73333.33
21	20	0.20	66.67	66666.67
22	23	0.23	76.67	76666.67
23	30	0.30	100.00	100000.00
24	24	0.24	80.00	80000.00
		Averages	67.22	67222.22

To demonstrate the calculations, consider the first day (subgroup). In Day 1, 19 defects were found. Using Equation 4, dpu_1 = 19/100 = 0.19. The $dpmo_1$ value is 63.33 (= $10^6 \cdot 0.19/3000$). Of course, $dpbo_1$ is 63,333.33 (= $10^9 \cdot 0.19/3000$). Based upon the results in Table 2 and in Equation 8, the control limits for the *dpmo* chart are approximately the following:

- UCL = 112
- Centerline = 67
- LCL = 22

The control limits for the *dpbo* chart are approximately the following, as based upon Table 2 and Equation 12:

- UCL = 112,130
- Centerline = 67,222
- LCL = 22,315

To illustrate the control charts, Figure 1 for the *dpmo* chart and Figure 2 for the *dpbo* chart are presented.

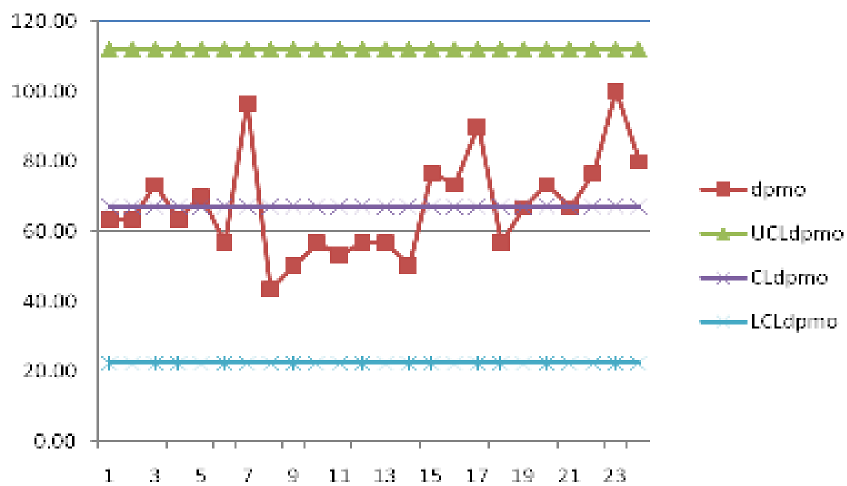

Figure 1. *dpmo* Chart for the Example Problem

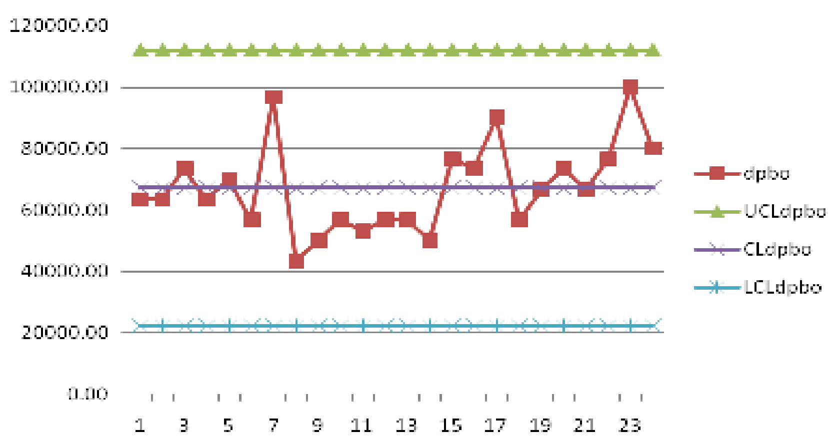

Figure 2. *dpbo* Chart for the Example Problem

Analysis of the *dpbo* chart is no different than the analysis of other attributes control charts. For this example, there are no plot points outside of the ±3σ bands. As such, it may be chosen to accept these control limits subject to periodic review. While there are no points outside the ±3σ control limits, there is an interesting situation in Days 8 through 14. Each of these plot points is below the centerline. Since these represent better-than-average defect levels as compared to the rest of the subgroups, it may be of interest (in a realistic situation) to investigate to determine if there are assignable causes for this.

An earlier point should be revisited that was discussed with this hypothetical example. This example intentionally does not reflect a 6σ process. So while the process appears to be in control, efforts should be made to reduce the defect levels. An additional comment can be made regarding high (whether out-of-control or not) *dpbo* values. Consider $dpbo_{23}$ which is the highest in this data set. Day 23 has the highest defect count of 30; even though this is in-control, one might be interested in determining why the value is high. What is not apparent from the data (and this is true whether we are using any of the other scaled charts – *dpmo* or *u*) is whether defects are indicative of all the parts in a subgroup (i.e., spread out among the 100 PCBs) or do these defects come from 1 or a few number of the PCBs on Day 23. If representative of the entire day, then investigation should be made to determine why that day was appreciably different from any other. On the other hand, the process, for that day, could be relatively similar to any other day, but the defects may have come from one bad (or very few) PCB(s) and perhaps the defects are traced more to supplied parts, than to process settings.

5. DISCUSSION

From an examination of the example presented, it is apparent that the chart limits and plot points for the *dpbo* chart are 1000x their respective values for the *dpmo* chart – as they should be. The *dpbo* has been presented as a scaled version of not only the *dpmo* chart, but also of the *u* chart. The interested reader might then pose the question (which is similar to one posed in a recent graduate engineering class covering SPC concepts (Santos, 2009): Why don't we just get the *dpmo* control limits and multiply those by 1000 instead of performing all of the calculations of the *dpbo* chart? The answer is simple – the author does not propose the use of both charts in realistic applications, only the use of one chart. Two charts were developed for demonstration and comparison purposes. Thus, if the *dpbo* chart is to be used, it is not suggested to be used in addition to the *dpmo* chart, but to be used instead of the *dpmo* chart.

The benefit from the use of the *dpbo* chart will ultimately become evident as processes with large opportunities for defect counts reach high quality (i.e., low defect) levels that are measured in terms of *ppb*, as opposed to *ppm*. These are defect levels that are truly better than 6σ.

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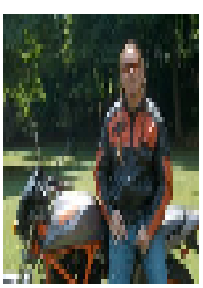

BIOGRAPHICAL SKETCH

Daryl Santos is a Professor in the Systems Science and Industrial Engineering Department and Director of the university-wide Freshman Engineering Program (the Engineering Design Division) at Binghamton University (a.k.a. SUNY Binghamton) in New York State, USA. He earned his Ph.D. ('93) and M.S. ('90) from the Department of Industrial Engineering at the University of Houston. His B.S. ('87) in Operations Research and Industrial Engineering was obtained at Cornell University, Ithaca, NY. Since his arrival at Binghamton in 1994, Dr. Santos has worked closely with the Integrated Electronics Engineering Center and local and national industries to successfully resolve electronics manufacturing issues in the following areas: production scheduling, engineering economics, and quality control. Dr. Santos is currently utilizing his process development, control, and improvement background as part of the team charged with demonstrating the feasibility of a continuous, roll-to-roll (R2R) process for flexible electronics at the Center for Advanced Microelectronics Manufacturing (CAMM) facility. The CAMM is a national microelectronics manufacturing R&D center led by Binghamton University in partnership with Endicott Interconnect Technologies and Cornell University.