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A COMBINED AHP-ENTROPY METHOD FOR DERIVING SUBJECTIVE AND OBJECTIVE CRITERIA WEIGHTS

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This paper investigates the problem of deriving subjective and objective criteria weights, by combining the AHP and Shannon's entropy method. The paper outlines the challenge of making preferential judgments based on non-homogeneous decision data and variant decision knowledge. Such decision complexity often leads to inaccurate assessment of criteria weights, and consequently reducing the credibility of decisions. The combined AHP-entropy method conforms to the type of decision data (qualitative or quantitative; deterministic or probabilistic) and to the degree of decision knowledge (none, partial, or full preferential judgments). An easy-to-apply spreadsheet-based application program of the unified AHP-entropy method is developed for deriving criteria weights, synthesizing decision elements, and ranking decision alternatives. A numerical example is used to clarify the method's application.

Significance: A unified approach is proposed to assist decision makers in deriving subjective and objective criteria weights for multiple, conflicting, and non-homogeneous decision criteria.

Keywords: Criteria Weights, Entropy, Analytical Hierarchy Process, Decision Support Systems.

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1. INTRODUCTION

Decision-making is an essential practice in controlling and managing technical or non-technical processes, where the expertise of a decision-maker or a team of specialists is utilized to assess decision alternatives and rank potential outcomes. However, the performance of the majority of real-world production and business systems is often based on multiple, interrelated, and potentially conflicting attributes. Hence, various Multi-Criteria Decision Making (MCDM) methods were proposed by researchers to help the decision-makers build and analyze complex decision models (Triantaphyllou, 2000; Yoon and Hwang, 1995; Ringuest, 1992). Common MCDM methods include Analytical Hierarchy Process (AHP) (Saaty, 1977), Simple Maulti-Attribute Rating Technique (SMART) (Edwards, 1986), and Multi-Attribute Utility Theory (MAUT) (Steuer, 1986). Most of these methods were later transformed into computer-based Decision Support Systems (DSS) (Alter, 1990; Eom, 1999). Consequently, MCDM has become a common engineering and managerial practice in different types and sizes of both manufacturing and service firms.

These widely used decision tools aim at breaking complicated decisions down into small pieces that can be analyzed individually and then recombined in a weighted-sum utility score. The key difference among these tools is mainly in the way the *scores* of individual attributes and their *weights* are assessed. AHP, in particular, provides various advantages to decision makers (Zahedi, 1986). These include making the selection process transparent, reducing decision complexity by organizing the decision in a structured hierarchical format, providing a framework to check and reconcile decision inconsistency, and facilitating information synthesis and sensitivity analysis. Thus, although several weaknesses in the AHP method have been reported by researchers (Belton and Gear, 1983; Dyer, 1990; Harker and Vargas, 1987; Triantaphyllou and Mann, 1995), AHP method is still common among managers and engineers for MCDM.

Assessing relative importance weights associated with attributes values is usually a key challenge to MCDM methods, including AHP (Lootsma, 1999). In AHP criteria weights are assessed *subjectively* given that the decision-maker can provide scaled preferences of pairs of decision criteria and alternatives with acceptable inconsistency. However, most real-world decision situations involve multiple difficulties in expressing preferential judgments (Corner and Buchanan, 1997; Zhang et al., 2004). Thus, for qualitative criteria and their categories, AHP assesses weights based on the judgments of the decision-maker (Basak and Saaty, 1993). This subjective judgment, however, may not be effective or accurate for quantitative criteria, where some or all pertinent decision data is available. This data if often expressed with technical, non-homogeneous, and deterministic or probabilistic measures of performance. This case is common in product development and manufacturing applications where relative importance is a function of operating conditions. Examples include material selection based on measures of surface finish, machinability, and yield stress or machine selection based on productivity (throughput), reliability (failure rate), flexibility (setup), and quality (defect rate). In such situations, decision makers need to assess criteria weights *objectively* based on pertinent decision data. The decision-maker's knowledge of various decision aspects and the characteristics of the overall decision context determine his/her ability to establish partial or full preference among various decision elements (Clemen, 1996).

The entropy method, which is taken from information theory (Shannon, 1948), can be used to derive criteria weights objectively from pertinent decision data in case preferential judgments are either partial or unavailable (Nijkamp, 1977; Zeleny, 1974). Using entropy, the weight assigned to a decision criterion is directly related to the average *intrinsic* information generated by a given set of alternative evaluations at that criterion, as well as to its subjective assessment (if available). The diversity of alternative evaluations at a certain decision criterion is expressed by viewing these evaluations as statistical events and associating a discrete probability distribution to their values. Entropy is then determined for the probability distribution from which the diversity is measured and normalized to derive the weights.

Practically, however, it is often hard to explain or make sense of the solution obtained with entropy (Jessop, 1990; Zeleny, 1982). Thus, as discussed in Pomerol and Romero (2000), the main advantage of entropy lies in its objectivity, where the evaluations of decision alternatives at a certain criterion determine its relative importance without direct involvement of the decision-maker. The greater the dispersion in the evaluations of the alternatives at the criterion, the higher the relative importance weight of the criterion (i.e., the criterion has a greater discrimination power between alternatives). Shannon (1948) discussed the axiomatic base of entropy as a measure of the uncertainty in information. Jaynes (1957) and Soofi (1990) explained the unique characteristics and application of the entropy concept. Some other aspects and applications of entropy were also presented in the literature (Hwang and Yoon, 1981; Jessop, 1995; Kojadinovic, 2004; Shipley et al., 1991; Soofi and Retzer, 1992).

This paper attempts to provide a unified framework for MCDM with AHP and entropy where entropy is adopted to complement the functions of AHP based on the categorization of decision criteria. Although no complete work is yet presented, the idea of combining information theory with AHP is not new. Basak (2002) presents an example of utilizing entropy for selecting the most appropriate statistical model for the judgment data used in AHP. The main contribution of the proposed AHP-entropy method includes enabling decision makers to assess criteria weights both subjectively and objectively based on different types of decision data and under variant levels of decision knowledge, and enhancing the entropy application to derive weights from both deterministic and probabilistic alternative evaluations. Thus, AHP is utilized to derive subjective assessment of criteria weights, while entropy is used to derive objective criteria weights. These weights are used to establish a Multi-Attribute Value Function (MAVF) using a weighted-sum of criteria weights. This function is used to determine an overall utility score to each decision alternative and to rank these alternatives accordingly.

2. CRITERIA WEIGHTS DERIVATION

The structure of a typical MCDM problem can be represented with a Decision Matrix (DM). This matrix is made of a set of *n* decision alternatives that are evaluated based on *m* decision criteria. The objective of MCDM is to rank the set of decision alternatives ($i = 1, 2, \ldots, n$) and to determine the alternative at which the overall Multi-Attribute Value Function (MAVF) is maximized (Steuer, 1986). The MAVF of a decision alternative i is maximized by seeking maximum conceivable improvement to *all* decision attributes simultaneously (Butler et al., 2001). A tradeoff between these objectives is usually made to evaluate the utility value associated with solution alternatives. This tradeoff incorporates the contribution of each decision objective into an overall alternative evaluation. Therefore, assuming V_i represents the MAVF at an alternative i that is evaluated at m decision criteria $(y_1, y_2, \ldots, y_m)$, an alternative is selected so that:

$$Max\ \{V_i\ (y_1, y_2, \ldots, y_m) = \sum_{j=1}^{m} w_j\, v_i\ (y_j)\},\ 1 \leq i \leq n,\ 1 \leq j \leq m \qquad \ldots \qquad (1)$$

Where w_j is the weight of relative importance assigned to criterion j and $\sum_{j=1}^{m} w_j = 1$ for convenience.

The definition in (1) is primarily challenged by determining the set of criteria weights (W). Based on the type of decision data, weights of relative importance derived from subjective information are defined in terms of a set of subjective weights (W^s) while weights derived from objective information are defined in terms of a set of objective weights (W^o).

2.1 Deriving Subjective Weights with AHP

AHP is based on developing a hierarchal presentation of the decision-making problem and then analyzing this hierarchy through a series of pairwiase comparison judgments to express relative strength or intensity of hierarchy elements (Saaty, 1977; Saaty, 1980). Such judgments are then synthesized to derive priorities among criteria and alternative solutions. Thus, making decisions with AHP can be described in a four-phase procedure: problem structuring, structure evaluation, analyses (computations), and synthesis. Problem structuring includes an unambiguous definition of decision goal, criteria, and alternatives, developing problem hierarchy, and collecting pertinent data and information on criteria and alternatives. The evaluation phase involves pairwise comparison judgments of criteria and alternatives at each criterion using Saaty's nine-point integer ratio scale, where a ratio scale of 1 indicates *equality* between two terms and 9 indicates an *absolute importance*. Other intermediate values between 1 and 9 are interpreted correspondingly. The pairwise comparison of each two elements (i and j) results in a reciprocal $n \times n$ matrix A, where $a_{ij} = 1$ on the diagonal and $a_{ji} = 1/a_{ij}$. Hence, only $n(n-1)/2$ pairwise comparisons are required to establish the comparison matrix. Transitivity of the relative importance among i and j elements leads to comparison consistency. AHP's analyses phase includes computing subjective relative importance weights (W^s) by solving the matrix eigenvalue for criteria and for alternatives and checking the consistency of the pairwise comparison judgments using a Consistency Ratio (CR). Consistency means that the decision-maker is exhibiting coherent judgment in specifying the pairwise comparison of the criteria or alternatives. If the level of inconsistency is acceptable (CR $\leq$ 10%), the synthesis phase combines the ratings (weights) of criteria and alternatives to compute an overall priority rating to each decision alternative using a weighted-sum of criteria and alternatives weights. The alternative with highest combined weight is considered the best among alternatives set. Figure 1 depicts the details of AHP procedure.

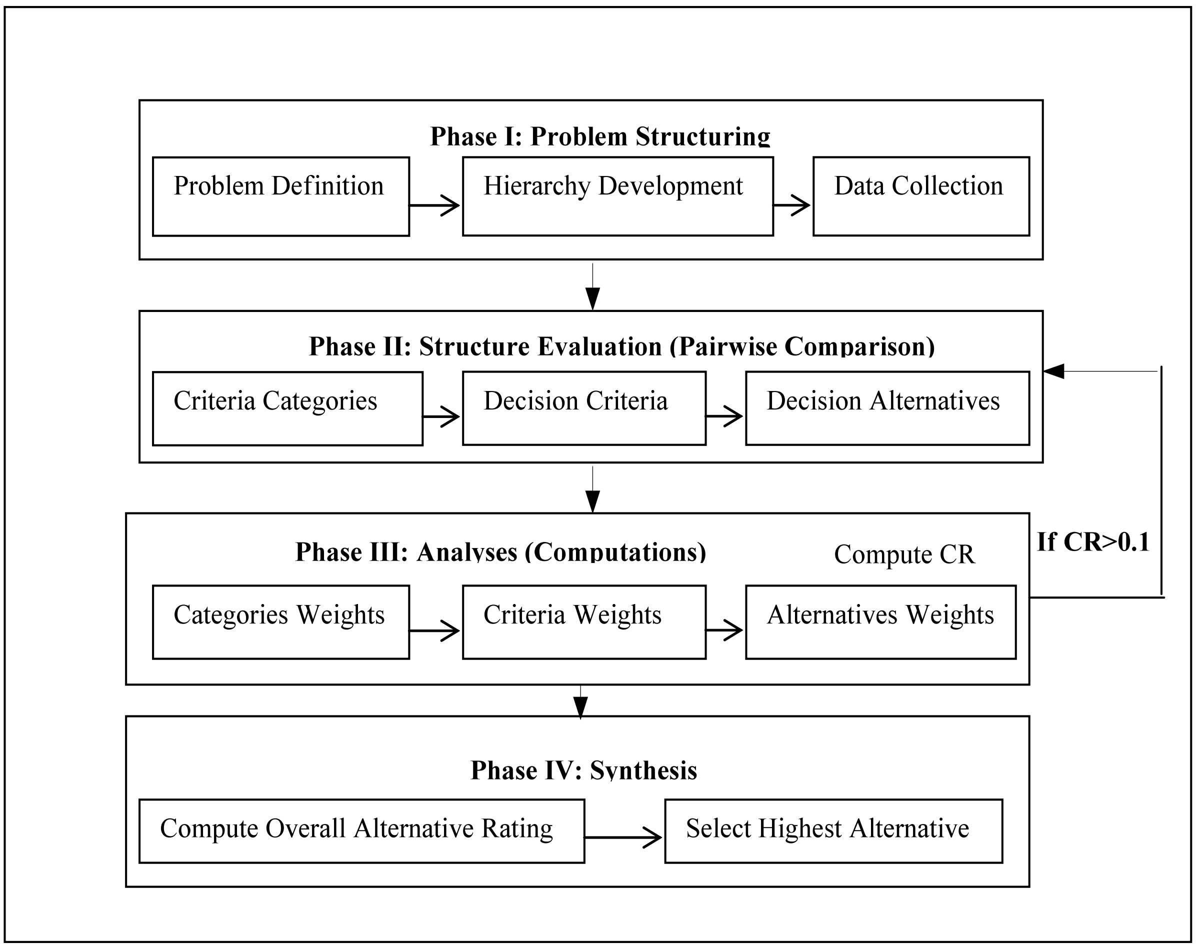

Figure 1. AHP procedure

2.2 Deriving Objective Weights with Entropy

The term Entropy is used in thermodynamics to describe a quantity accompanying a change from thermal to mechanical energy (Van-Wylen and Sonntag, 1976). In information theory, entropy measures the *uncertainty* associated with random phenomena of the expected information content of a certain message (Shannon and Weaver, 1963). Thus, the entropy of a random variable X is defined in terms of its probability distribution P and can be shown to be a good measure of randomness or uncertainty. If X takes a finite number of possible values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$

respectively, the entropy of the probability distribution is given by Shannon (1948) as:

$$E\,(p_1, p_2, \ldots, p_n) = -K \sum_{i=1}^{n} p_i \ln p_i \text{ , where } K \text{ is a positive constant.} \quad \ldots \quad (2)$$

It is noted that $E\,(p_1, p_2, \ldots, p_n)$ takes its maximum value when the uncertainty in distribution outcomes is maximized, i.e. when all outcomes have the same probability $p_i = 1/n$.

In MCDM, entropy of a weight distribution is similar to the entropy of a probability distribution, which establishes a useful rationale for utilizing the definition of entropy in objective criteria weights assessment. As discussed in Hwang and Yoon (1981), a criterion does not function much when all the alternatives have similar outcomes for that criterion. Hence, the entropy method is particularly useful for investigating *contrasts* between sets of data when no preference among criteria can be established. These sets of data can be pictured as a set of solution alternatives in the Decision Matrix (DM), where the performance of the *i*th alternative on the *j*th criterion is defined as y_{ij}. That is:

$$DM = \begin{bmatrix} y_{11} & y_{12} & \ldots & y_{1m} \\ y_{21} & y_{22} & \ldots & y_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ y_{n1} & y_{l2} & \ldots & y_{nm} \end{bmatrix} \quad \ldots \quad (3)$$

In deterministic decision data, each alternative evaluation y_{ij} is expressed in terms of a unique value. In probabilistic decision data, each alternative evaluation is expressed in terms of a discrete or continuous probability distribution. This is often the case for technical performance measures such as productivity, reliability, and quality. Normal distribution is a common measure of performance that indicates both performance mean and standard deviation. In the proposed method, each probability distribution is transformed into a crisp value (discriminating parameter) to measure the diversity of alternative evaluations and to allow for entropy application. The selected discriminating parameter (y) is determined from the parameters of the probability distribution. As shown in Table 1, the parameter y is defined so that no negative values are produced while the differences between distributions are indicated. In case of normal distribution, a weighting factor (α) is used to combine the mean and standard deviation and provide the decision maker with further flexibility in balancing the impact of distribution mean and variability. Similarly, α is used at the triangular distribution to balance the impact of mode and range and at the uniform distribution to balance the impact of median and range. For other distributions, the expected value is used as a discriminating parameter. The values of y are used to fill the decision matrix defined in (3).

Table 1. Discriminating parameters at four probability distributions

Alternative distribution	Distribution parameters	Discriminating parameter
Normal	$N\,(\bar{x}, s)$	$y = \alpha \bar{x} + (1-\alpha)s$
Exponential	$E(\lambda)$	$y = \lambda$
Triangular/Uniform	$T(x_{min}, x_m, x_{max})$	$y = \alpha x_m + (1-\alpha)\,(x_{max} - x_{min})$
Binomial	$B(n, p)$	$y = np$

To measure entropy, the entries of DM in (3) are represented in a probability distribution, p_{ij}. The probability value (p_{ij}) for each entry in the DM is determined by normalizing criteria performance values at each decision alternative. That is:

$$p_{ij} = y_{ij} \Big/ \sum_{i=1}^{n} y_{ij} \text{ , } \forall\, i, j \quad \ldots \quad (4)$$

Using the normalized values from Equation (4), the p_{ij} matrix is formed as follows:

$$p_{ij} = \begin{bmatrix} p_{11} & p_{12} & \ldots & p_{1m} \\ p_{21} & p_{22} & \ldots & p_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ p_{n1} & p_{l2} & \ldots & p_{nm} \end{bmatrix} \quad \ldots \quad (5)$$

Consequently, the entropy E_j for a set of outcomes (n decision alternatives) at a decision criterion j is defined as:

$$E_j = -K \sum_{i=1}^{n} p_i \ln p_i \text{ , } \forall j \qquad \text{...} \qquad (6)$$

Where K is a constant with a value of $1/ln$n at *the* maximum entropy (E_{max}), which guarantees that $0 \leq E_j \leq 1$.

The degree of diversification d_j of the information provided by a decision criterion j is defined as:

$$d_j = 1 - E_j, \forall j \qquad \text{...} \qquad (7)$$

An attribute in which all alternatives have similar values has little discriminatory power (small d_j values) and thus should be given less weight. The objective criteria weight set W^o can be defined as follows:

$$w_j^{\,o} = \frac{d_j}{\sum_{j=1}^{m} d_j}, \forall \ j \qquad \text{...} \qquad (8)$$

3. UNIFIED AHP-ENTROPY MODEL

Figure 2 depicts a prototype of the unified AHP-entropy approach. The model starts with developing a decision hierarchy, based on Figure 1. The hierarchy involves both qualitative and quantitative decision elements. An AHP decision module is utilized for assessing subjective weights of criteria categories (if applicable) and for qualitative decision criteria. To increase the accuracy of judgments, the approach suggests that a non-integer scale be used in developing the AHP's pairwise comparisons. For quantitative criteria, three decision modules are used depending on the level of decision knowledge. For the case of full preferences judgments, the AHP decision module is used to derive weights subjectively. When no preferences judgments exist, entropy method is used to provide objective assessments of criteria's weights. A combined AHP-entropy module is used for assessing weights of quantitative criteria where the decision-maker has prior partial preferences among decision criteria. AHP is first used to derive the set of subjective criteria weights (W^s) and entropy is then applied to derive a set of objective criteria weights (W^o) based on the contrast of decision data. The final set of criteria weights (W) is determined from both objective and subjective weights as follows:

$$w_j = \frac{w_j^{\,o}\, w_j^{\,s}}{\sum_{j=1}^{m} w_j^{\,o}\, w_j^{\,s}}, \forall \ j \qquad \text{...} \qquad (9)$$

Alternative evaluation on decision criteria is expressed using scores of relative importance (individual utilities). This evaluation can be either "quantified" or "not quantified". AHP pairwise comparison is used to provide alternatives weights at the "not quantified" alternatives. For "quantified" ones, there is no need to develop AHP's pairwise judgments. Instead, a normalized vector (NV) of data is used when data proportionality in the alternative evaluation is direct, i.e., higher criterion values increase the alternative's relative importance. A normalized vector of data *reciprocals* is used when the data proportionality is inverse, i.e. higher criterion values lower the alternative's relative importance. The derived criteria weights and the individual utilities of decision alternatives are combined into a composite weighted-sum utility value (V) based on Equation (1) and a rank (R) for each decision alternative is then determined accordingly.

A spreadsheet-based application program is developed for the prototype of the AHP-entropy method in Figures 3. The program comprises five easy-to-use decision modules: decision hierarchy, AHP, entropy, AHP-entropy, and MAVF. Using the program, a generic 6-step procedure is followed when applying the AHP-entropy method to real-world cases:

Step 1: Develop decision hierarchy based and collect data as shown in Figure 1.
Step 2: Apply AHP module as discussed in Section 2.1 and for the following cases:
- a- Assessing weights of criteria categories (if applicable).
- b- Deriving subjective weights for qualitative criteria.
- c- Deriving subjective weights for quantitative criteria where the decision-maker can establish full pairwise preference.

Step 3: Apply entropy module as discussed in Section 2.2 to derive objective weights of quantitative criteria where decision preferences cannot be established.

Step 4: Apply a combined AHP-entropy module for assessing criteria weights of quantitative criteria where partial decision preference can be established (use formula 9).

Step 5: For assessing individual utilities (scores) of decision alternatives:

a- Apply AHP to assess weights for non-quantified decision alternatives (if applicable).

b- Apply direct normalization to assess weights for quantified alternatives with direct proportionality.

c- Apply reciprocal normalization to assess weights for quantified alternatives with inverse proportionality.

Step 6: Synthesize all weights assessments in a composite weighted-sum using the MAVF module and rank decision alternatives accordingly (use formula 1).

3.1 Numerical Example

This section presents an example of applying the unified AHP-entropy approach in supporting a machine shop decision. The case example is based on Weber (1993), where a shop manager is considering the following alternatives to improve an existing conventional milling machine:

- Retrofit the existing mill with a power feed, digital readout of positions, and numerical control (NC) features.
- Buy a new numerical control mill with an interactive graphic controller and computer-aided design (CAD).
- Replace the mill with a machining center (MC) and programmable tool changer.

The three alternatives (NC, CAD, and MC) are evaluated based on three criteria categories: monetary (M), performance (P), and strategic (S). Monetary criteria include initial machine cost (IC), yearly operation and maintenance cost (OMC), and yearly training cost (TC). Engineering performance criteria include probability distributions of production rate (PR) in terms of units produced per hour, machine reliability in terms of mean-time-between-failure (MTBF) in hours, machine setup time (ST) in minutes, and scrap rate (SR) in kilograms per day. Finally, strategic considerations include manufacturing flexibility (F), product quality (Q), business growth (G), and implementation risk (R). Table 2 provides the pertinent data for the quantitative criteria (i.e., deterministic monetary measures and probabilistic performance measures).

Table 2. Decision data at the quantitative criteria

Criteria Category	Decision Criteria	Decision Alternatives		
		NC	CAD	MC
Monetary (M)	IC ($)	12,000	25,000	120,000
	OMC ($/year)	2,000	4,000	15,000
	TC ($/year)	3,000	8,000	20,000
Performance (P)	PR (unit/hour)	*N(8,2)*	*N(12,3)*	*N(40,5)*
	MTBF (hours)	*E(2)*	*E(6)*	*E(10)*
	ST (minute)	*T(10,30,50)*	*T(15,20,25)*	*T(1,3,5)*
	SR (kg/day)	*B(np = 200)*	*B(np = 75)*	*B(np = 20)*

The AHP-entropy method categorizes decision criteria based on data availability and decision knowledge. Classification in this case is made at the category level, where AHP is applied to derive subjective criteria weights for the three criteria categories and for the four strategic criteria. For monetary criteria, AHP is used to derive initial subjective weights based on the partial judgments of the decision-maker. Entropy objective weights are then developed to enhance AHP's initial weights for monetary criteria. The probabilistic and technical nature of machine performance criteria has made it difficult for the decision-maker to provide preferential judgments among their non-homogeneous measures. Hence, criteria weights are derived objectively with entropy module. Table 3 shows the used decision module ate ach case.

Table 3. Modules of deriving criteria weights

Decision Category	**Criteria Type**	**Decision Data**	**Decision Knowledge**	**Decision Module**
Strategic (S)	Qualitative	N/A	Full Judgment	AHP
Monetary (M)	Quantitative	Deterministic	Initial preference	AHP-Entropy
Performance (P)	Quantitative	Probabilistic	No preference	Entropy

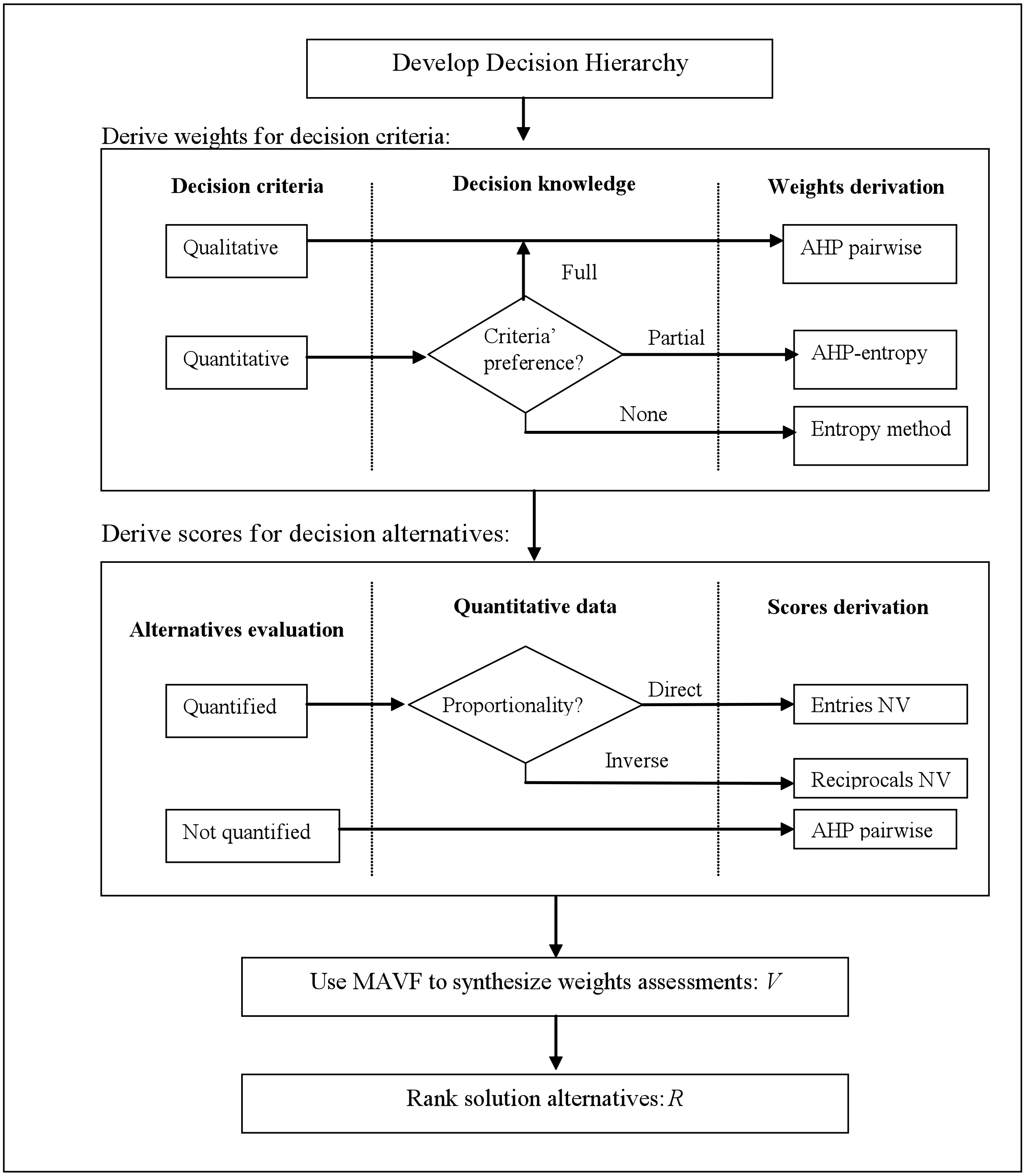

Figure 2. A Unified AHP-Entropy Model

We now apply the AHP-Entropy's six-step procedure to select an alternative that improves the existing milling machine:

Step1: Developing decision hierarchy:

In this step, the machine shop decision is structured into an AHP hierarchy to provide by decomposing the problem into smaller sub-structures that are easier to analyze and evaluate. As shown in Figure 3, the hierarchy of the machine shop example includes four levels: the decision goal, criteria categories, decision criteria, and decision alternatives. The overall goal is to improve the existing conventional milling operation to meet the growing need in demand, quality, and performance. The hirerarchy then presents the elements of the three criteria categories. The three alternatives to a conventional mill (NC, CAD, and MC) are evaluated at each criterion within the three subgroups.

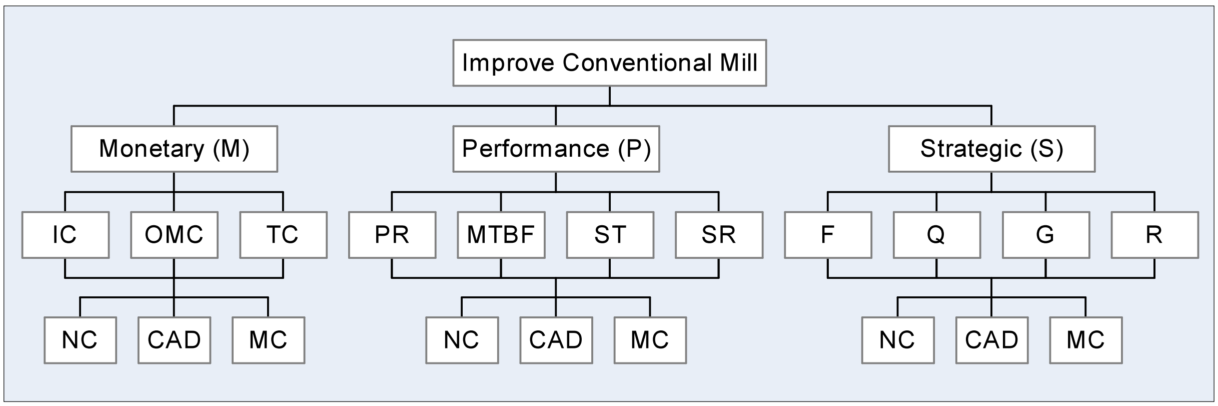

Figure 3. The Machine Shop Decision Hierarchy

Step 2: Applying AHP module:

AHP module is first used to derive weights to the three criteria categories (M, P, and S). The defined criteria categories depict the split of the decision-maker's bias among different aspects of interest, where the decision-maker specifies a preference for financial, technical, and strategic aspects. The decision-maker has decided that strategic considerations are 4 times as important as performance criteria and 2 times as important as monetary criteria. Using non-integer scale ratios in order to increase the accuracy of judgments, monetary criteria are set to be 1.5 times as important as performance criteria. Table 4 shows the *3×3* AHP pairwise comparison matrix and the weights resulted from AHP calculations. The values of consistency index (CI), random index (RI), and consistency ratio (CR) are also shown to check the judgments consistency. Since CR is less than the 10% threshold, the decision inconsistency level was found acceptable.

Table 4. AHP subjective weights of criteria categories

	M	P	S	Relative Weight	Consistency Check	
Monetary (M)	1.00	1.50	0.50	26.3 %	C.I.	0.005
Performance (P)	0.67	1.00	0.25	15.9 %	R.I.	0.382
Strategic (S)	2.00	4.00	1.00	57.8 %	C.R.	0.012

AHP is also applied to derive subjective weights for the four strategic considerations. The decision-maker used non-integer AHP scale to increase the accuracy of judgments. Table 5 shows the *4×4* AHP pairwise comparison matrix and the resulting weights. Since CR is less than the 10% threshold, decision inconsistency level was found acceptable.

Table 5. AHP subjective weights for qualitative strategic criteria

	F	Q	G	R	Weight	Consistency Check	
Flexibility (F)	1.00	0.75	1.50	2.00	28.4 %	C.I.	0.005
Quality (Q)	1.33	1.00	2.00	3.00	39.1 %	R.I.	0.946
Growth (G)	0.67	0.50	1.00	1.00	17.6 %	C.R.	0.006
Risk (R)	0.50	0.33	1.00	1.00	14.8 %		

Step 3: Applying AHP-entropy module:

For the three homogeneous monetary criteria (IC, OMC, and TC), the decision-maker has the following partial prior preference: IC is 2 times as important as both OMC and TC while OMC and TC are equally important. Table 6 shows AHP pairwise comparison and the weights resulting from AHP calculations with consistency check.

Table 6. AHP subjective weights for monetary criteria

	IC	OMC	TC	Weight	Consistency	
Initial Cost (IC)	1.00	2.00	2.00	50.0 %	C.I.	0.000
Operation & Maintenance Cost (OMC)	0.50	1.00	1.00	25.0 %	R.I.	0.382
Training Cost (TC)	0.50	1.00	1.00	25.0 %	C.R.	0.000

Entropy method is then used to determine objective criteria weights for the monetary criteria by establishing a decision matrix from the values of IC, OMC, and TC at the three decision alternatives (NC, CAD, and MC). Entropy results to monetary criteria are summarized in Table 7, which shows each criterion's entropy (E_j), degree of diversification (d_j), and objective relative importance weight ($W^o{}_j$).

Table 7. Entropy objective weights for monetary criteria

Criteria	IC	OMC	TC
E_j	0.632	0.710	0.781
d_j	0.368	0.290	0.219
$w^o{}_j$	0.420	0.331	0.250

Equation (9) is used for adjusting AHP-based weights of monetary criteria using Entropy-based weights by combining both subjective and objective judgments. Table 8 summarizes adjusted weights for monetary criteria.

Table 8. AHP-entropy adjusted weights for monetary criteria

Criteria	Entropy Weights	AHP Weights	Adjusted Criteria Weights
IC	0.420	0.500	0.591
OMC	0.331	0.250	0.233
TC	0.250	0.250	0.176

Step 4: Applying entropy module:

The entropy module is used to derive objective weights for the technical data category which includes four probabilistic performance criteria (PR, MTBF, SR, and ST). Table 9 summarizes the probabilistic alternative evaluations at the three performance criteria.

Table 9. Probabilistic alternative evaluations

	NC	**CAD**	**MC**
PR (unit/hour)	*N(8,2)*	*N(12,3)*	*N(40,5)*
MTBF (hours)	*E(2)*	*Expo(6)*	*E(10)*
ST (minute)	*T(10,30,50)*	*T(15,20,25)*	*T(1,3,5)*
SR (kg/day)	*B(np = 200)*	*B(np = 75)*	*B(np = 20)*

The discriminating parameters defined in Table 1 are first determined from the probability distributions of performance data in Table 9 to establish the decision matrix and to apply the entropy method. Using $\alpha = 0.5$ at normal and triangular distributions, the obtained decision matrix is as follows:

$$DM = \begin{bmatrix} PR & MTBF & ST & SR \\ 5 & 2 & 35 & 200 \\ 7.5 & 6 & 15 & 75 \\ 22.5 & 10 & 3.5 & 20 \end{bmatrix} \quad \ldots \quad (10)$$

Applying the entropy solution to the matrix in Equation (10) leads to the results in Table 10.

Table 10. Entropy objective weights of technical performance criteria

Criteria	PR	MTBF	ST	SR
Ej	0.812	0.853	0.740	0.723
dj	0.188	0.147	0.260	0.277
wj	0.215	0.169	0.298	0.318

Step 5: Assessing alternatives' individual utilities.

In this step, we categorize decision data based on the method used for evaluating the three decision alternatives (NC, CAD, and MC). AHP pairwise comparison is used for determining individual utility values (weights) for the three decision alternatives at the four qualitative strategic considerations (i.e., F, Q, G, and R). For quantitative criteria, the provided data is used to directly determine the corresponding relative weights using normalization. For IC, OMC, TC, ST, and SR where data proportionality is inverse (i.e., higher cost, setup time, and scrap rate result in lowering the alternative's relative importance), a normalized vector of data reciprocals is used to derive alternatives' weights. For PR and MTBF where data proportionality is direct (higher throughput and MTBF increase the alternative's relative importance), normalized vector of data is used to derive alternatives' weights. Table 11 summarizes methods of assessing the evaluations (individual utilities) of the three decision alternatives (i.e., NC, CAD, and MC) at each one of the eleven decision criteria. Results are summarized in Table 12.

Table 11. Methods of assessing alternatives at decision criteria

Decision criteria	**Decision data**	**Proportionality**	**Alternatives evaluations**
F, Q, G, and R	Qualitative	N/A	AHP- pairwise comparison
IC, OMC, TC, ST, and SR	Quantitative	Inverse	Normalized Vector of Data Reciprocals
PR and MTBF	Quantitative	Direct	Normalized Vector of Data

Table 12. Alternatives' evaluations at decision criteria

Decision Criteria	Criteria Weights	Alternatives Weights at Decision Criteria		
		NC	CAD	MC
Flexibility (F)	28.4 %	17.0 %	23.0 %	60.0 %
Quality (Q)	39.1 %	20.0 %	36.0 %	44.0 %
Growth (G)	17.6 %	22.0 %	26.0 %	56.0 %
Risk (R)	14.8 %	43.0 %	32.0 %	25.0 %
Initial Cost (IC)	59.1%	63.3 %	30.4 %	6.3 %
O & M Cost (OMC)	23.3%	61.2 %	30.6 %	8.2 %
Training Cost (TC)	17.6%	65.6 %	24.6 %	9.8 %
Production Rate (PR)	21.5 %	12.9 %	22.6 %	64.5 %
Uptime (MTBF)	16.9 %	11.1 %	33.3 %	55.6 %
Setup Time (ST)	29.8 %	8.0 %	12.0 %	80.0 %
Scrap Rate (SR)	31.8 %	7.3 %	19.5 %	73.2 %

Step 6: Synthesizing and determining alternatives' ranking:

In this final step, weights in Tables 4, 5, 8, 10, and 11 are combined to determine the overall rating of decision alternatives (i.e., NC, CAD, and MC). The linear additive MAVF shown in Equation (1) is used to assign a weighted-sum rating (R) to each decision alternative. Alternatives are then ranked accordingly. As shown in Table 13, the machine center (MC) alternative has received the highest composite weight among the set of alternatives, hence it is ranked 1. The NC alternative is ranked 2 and the CAD alternative is ranked 3. Consequently, the machine shop manager is expected to select the MC alternative to replace the existing conventional milling machine.

Table 13. Final rating of decision alternatives

Alternative	Overall rating	Rank
NC	0.3138	2
CAD	0.2814	3
MC	0.4048	1

4. CONCLUSION

This paper has presented a unified AHP-entropy approach for deriving subjective and objective criteria weights for multiple, interrelated, and conflicting decision criteria at different types of decision data and variant level of decision knowledge. The structured MCDM approach is comprised of decision modules that enable decision-makers to rank a set of decision alternatives at different decision situations. An AHP module was used to derive subjective weights for qualitative decision elements where preferential judgments can be established. An entropy module was used to derive objective weights for quantitative decision elements. An AHP-entropy module was used to derive weights for quantitative elements at which partial preferential judgments can be established. To assess individual utilities of decision alternatives, AHP pairwise comparisons were used to rate non-quantified decision alternatives and normalization (direct or inverse proportionality) was used for quantified alternatives. The approach was implemented into a spreadsheet-based application program that can be utilized as an easy-to-apply Decision Support System (DSS). The approach was applied to a machine shop example that includes qualitative and quantitative, probabilistic and deterministic decision data. The developed AHP-Entropy spreadsheet DSS is also applicable to other real-world decisions that involve technical data and qualitative information. This includes material selection, buying computers and pieces of equipment, making online-orders, sourcing decisions, and supplier selection. For example, a facility location decision is often made based on quantitative data such as transportation cost and qualitative criteria such as quality of life for workers. Such situations highlight the validity and effectiveness of the approach compared to other approaches in the literature. Implications for practitioners when using the proposed approach include collecting accurate data on quantified decision alternatives and criteria. Such data is critical to accurate derivation of objective weights using the Entropy method. For deriving subjective weights, it is recommended to adopt a team-based pairwise comparison to come up with realistic and representative assessments. In cases where decision-makers can provide subjective weights on quantitative criteria, its better to derive objective weights with Entropy and combine them with the qualitative judgments (subjective weights) to arrive at a comprehensive assessment. It is also recommended to simplify the decision structure (hierarchy) by focusing the decision model on key categories, alternatives, and criteria.

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BIOGRAPHICAL SKETCH

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