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A NEW LOSS FUNCTION FOR THE SELECTION OF PRODUCER SPECIFICATION LIMITS

Scott L. Rosen and M. Jeya Chandra

Department of Industrial and Manufacturing Engineering
The Pennsylvania State University, University Park, Pa 16802

This paper presents a new approach to determining economical values for manufacturer lower and upper specification limits using a newly derived hybrid function for expected cost. The identification and use of specification limits are essential in protecting the producer from shipping defective products that pass unnoticed due to measurement error. This cost function is composed of four parts: a generic Taguchi loss function, a function for rework cost, a function for scrap cost, and a function that describes a variance to cost tradeoff. The expected cost function does not assume the process mean to be equal to the target value of the process nor is it restricted to being symmetrical. The cost function is implemented among different production systems and optimal values of manufacturer lower and upper specification limits are determined for each system.

Significance: Determination of the specification limits which optimize the expected cost without sacrificing the functional requirement is important. We include Taguchi's loss function, because it captures the external failure cost.

Keywords: optimum specification limits, loss function

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1.0 INTRODUCTION

The concept of a producer's specification limit first gained notoriety when manufacturers understood the need for a strategy behind protecting the producer from defective products. A defective product is defined as one whose quality characteristic falls outside the consumer's specification limits. Without tightening the specification limits on the producer's side, the possibility of shipping defective products increases because of measurement error (Eagle, 1954, Grubbs and Coon 1954). The methodology behind this early work was for the producer to first gain an understanding of the risk level involved in measurement error and to then tighten the producer specification limits accordingly. Then the decision as whether to withhold a product for rework or scrap was based strictly on the comparisons of the cost of correcting or replacing a product after it has shipped as opposed to before it has been shipped. This philosophy was known as the "goal post" philosophy.

The Taguchi loss Function (Kackar 1985, Taguchi 1986, Kapur and Wang 1987) took the concepts behind producer-consumer specifications a step further by assigning a cost to process variation. This is accomplished by quantifying an economic loss for quality characteristics that deviate from their intended target values, but are within customer specification limits. Taguchi defined this economic loss as "loss to society", and more specifically, he considered it loss to the customer Taguchi's loss function is derived by applying a second order Taylor series expansion around the target value of the process and assuming zero loss for a product at or outside the target value. Third and higher order terms are assumed insignificant to reduce the function into a continuous quadratic form. The loss function is defined in equation (1) and represents a mapping between loss to society and individual quality values.

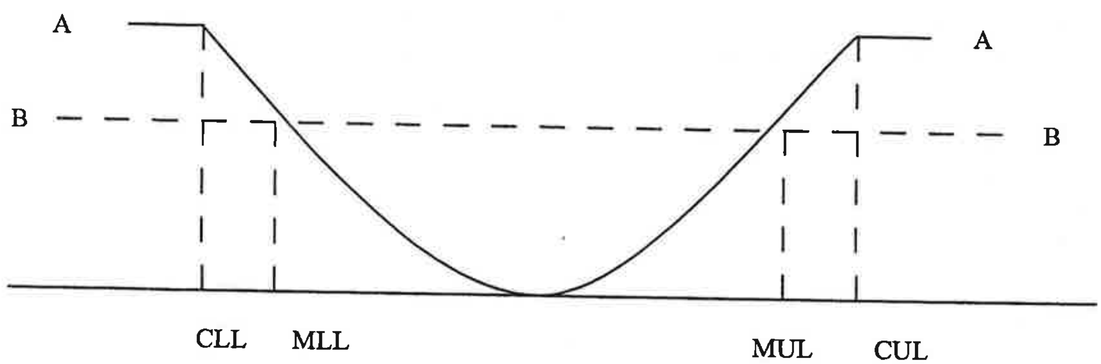
$$L(X) = K(X-\tau)^2, \qquad \ldots \qquad (1)$$

where X is the value of the quality characteristic, τ is the target value, and K is a proportionality constant. Since quality values are random variables, the expected value of the loss function is of interest. It can be calculated as

$$E[L(X)] = K[(\hat{\mu}_x-\tau)^2+\hat{\sigma}_x^2], \qquad \ldots \qquad (2)$$

where $\hat{\mu}_x$ is the estimate of the mean of X and $\hat{\sigma}_x^2$ is the estimate of the variance of X.

The motivation behind the use of the Taguchi continuous loss function is to incur a loss to society for parts that meet specification limits and are shipped to consumers, but do not have a quality characteristic that is at the target value. Implicitly, each product incurs some loss to society measured by the loss function and if this loss exceeds the cost of corrective action that the producer can take before shipment, then the producer should undertake some rework or corrective action. Using the Taguchi loss function, the manufacturer's specification limits are determined by locating the point at which the cost of corrective action is equivalent to the loss of society. The figure on the next page presents a graphical representation of using the Taguchi loss function to determine optimal manufacturer specification limits.



A = cost of shipping part outside of CLL or CUL

B =constant cost of corrective action

Figure 1: Determining Producer Specification Limits with Taguchi Loss Function

The locations for the manufacturer's upper and lower specification limits (MLL, MUL) are the corresponding point on the x-axis where the constant cost of corrective action intersects the Taguchi loss function. This approach is only appropriate under a rework procedure in which the reworked product's quality characteristic is always equal to its target value.

Fathi (1990) derives a more sophisticated loss function by performing some mild extensions to Taguchi's work and defining more detailed and valid assumptions. The Taguchi loss function utilized an assumption of perfect corrective procedure, which implies that the quality characteristic of a reworked product will exactly equal the target value. Intuitively, it is evident that error can result from this impractical assumption. Therefore, the Fathi loss function replaces this assumption by assuming that a reworked product will have the same probability density function as the original. The Fathi loss function does not modify the assumption of a constant value of corrective action, but acknowledges the error in the loss function due to this assumption. Another limitation of the Fathi loss function is that there is no discrimination in cost between reworking or scrapping a product.

There are other adaptations of the Taguchi Loss function for producer specification limit allocation. Kapur et al. (1990) suggests a component tolerance allocation model, which utilized the Taguchi loss function as the objective function. The model is constrained by the requirement for observing individual assembly tolerances. Vasseur et al. (1997) models the manufacturing cost as a function of process precision to determine tolerances. The impact of process precision to loss to society was modeled in a Taguchi loss function. Both the manufacturing cost and loss to society are combined for a total cost function. The total cost function also includes a parameter for the proportion of defects, which enables scrap cost to be incorporated into the model. However, this model only allows for parts to be scrapped and did not take into account the possibility of rework.

The Taguchi loss function is also employed when product quality characteristics follow an exponential distribution (Chen 1999). The use of the exponential distribution is suitable for longer-the-better type quality characteristics. (Chen 1999) also provides a modification to the work of Kapur and Wang for economic design of specification limits with an exponential quality characteristic.

Jeang (1997, 2001) discusses the integration of parameter and tolerance design optimization for minimizing cost. Within this two-phase process he introduces a new additive cost function for tolerance design, which incorporates ideas from the Taguchi loss function. This additive cost function can be broken down into three main components, depending on the ranges of the quality characteristic, which are (i) the product's quality value lies within the customer's specification limits (ii) when the quality value lies outside the customer specification limits and in an non-repairable region and (iii) the quality value lies outside the customer specification limits but in a repairable region. Material cost, manufacturing cost, and inspection cost were all inclusive in each of the three components to the cost function. The Jeang loss function is able to distinguish between rework costs and scrap costs, which is a main drawback of the Fathi loss function. The repair cost is assumed to be a function of the width of the tolerance range.

The accuracy of implementing the Taguchi loss function to determine economical producer specification limits is currently diminished by unrealistic assumptions. The previous works contain a debatable assumption on the assignment of a constant cost of corrective action. The assumption made by the Taguchi loss function and Fathi loss function is that the cost of corrective action is entirely independent of the value of the quality characteristic. The cost of corrective action incorporated by the Jeang loss function is dependent on the tolerance alone. It was also assumed that the cost of rework for a product is affected by its deviation from the target and must be incorporated into the cost function.

Also, many authors do not differentiate between the cost of reworking and scrapping a product in their work. Therefore, it is not possible to infer when it is cost beneficial for a product to be reworked or scrapped. Both a rework and a scrap cost should be incorporated into the cost function. Another common and limiting assumption through these works is that the process mean and the target value are the same. This assumption is generally made to simplify the model, but it is not always a valid assumption. The proposed loss function in this paper is an extension of the Taguchi loss function that provides the follow extensions. It incorporates a cost of rework, dependent on the quality characteristic, distinguishes the cost between reworking and scrapping a product, and does not assume that the process mean is equal to the target value.

2.0 MODEL DESCRIPTION

The following section provides a description of our proposed hybrid cost model for determining producer tolerance limits. The mathematical form of the cost function is a seven-piece function that includes costs for scraps, reworks, loss to society, and a variance cost. The loss to society component of the function follows the same quadratic form as the traditional Taguchi loss function. This section first provides the modifications and extensions of previous work on the Taguchi loss function for tolerance allocation. Then, the assumptions used in this model are provided.

The first extension is the removal of the constant cost of rework assumption, which is embedded in much of the previous literature. The cost associated with reworks is calculated by a linear function of the distance between the quality characteristic and the target value. This cost is set equal to zero at the target value and increases linearly to a maximum value at the final point where the product can be reworked. All products whose quality characteristics fall outside the upper and lower rework limits will incur a scrap cost. A cost associated with the loss to society for parts that fall within the manufacturer's specification limits is calculated using Taguchi's loss function. It is assumed that the quality characteristic is normally distributed.

The variance will be incorporated as a decision variable in the cost function. There is always a trade off between cost and variance in a manufacturing process. Obviously, the process variance decreases the expected cost of the process increases but the exact shape of this mathematical relationship is specific to the actual manufacturing process. It is assumed that this relationship is K/σ^2 where K is a constant to be chosen by the user. Users should replace this function with one that more accurately represents the relationship between cost and variance in their specific process.

In this paper it is be assumed that the probability density function (PDF) of a reworked part will be the same as the PDF of the characteristic before rework. The rework cost of the process will then include not only the cost to rework the part and its loss to society, but also the entire expected cost because the process starts anew when rework occurs since identical PDFs are assumed for a new and rework process. The final extensions made in this paper are the removal of the assumption that the process mean and the target value are the same and the assumption of symmetry of the cost function.

To facilitate the understanding of each of the different costs included in this hybrid cost function a graphical representation of the function is provided in Figure 3. The y axis of the graph represents the cost per part and the x axis represents the value of the quality characteristic. Abbreviations of the different limits and costs depicted on the graph are provided in Figure 2 below. Each limit and cost is also employed as a variable in the derivation of the expected cost model and are referenced using their corresponding abbreviations during the rest of the paper.

It should be noted that there are controllable and uncontrollable input variables to the expected cost model. The manufacturer can decide on its lower and upper specification limit as well as its process variance. However, it is assumed by this research that the manufacturer cannot control its rework and scrap cost as well as the functional upper and lower limit of the product. Furthermore, the manufacturer has no control over the customer's upper and lower specification limit or the cost of shipping a product falling outside of the customer specification limit.

Controllable Input Variables	
MLL	Manufacturer's lower specification limit
MUL	Manufacturer's upper specification limit
σ^2	Process variance
Uncontrollable Input Variables	
CLL/ CUL	Customer upper and lower specification limit
SCLL/ SCUL	Cost incurred by the manufacturer when a product falling outside of the customer specification limits is shipped
FLL/FUL	Function upper and lower limit. A product that falls outside of this limit cannot be reworked and must be scrapped
SFLL/ SFUL	Cost incurred by the manufacturer when a product that falls outside of the functional limits is scrapped.
RFLL/ RFUL	Rework cost incurred by the manufacturer when a product that falls within the functional limits but outside the manufacturer's limits.

Figure 2: Table of Variables

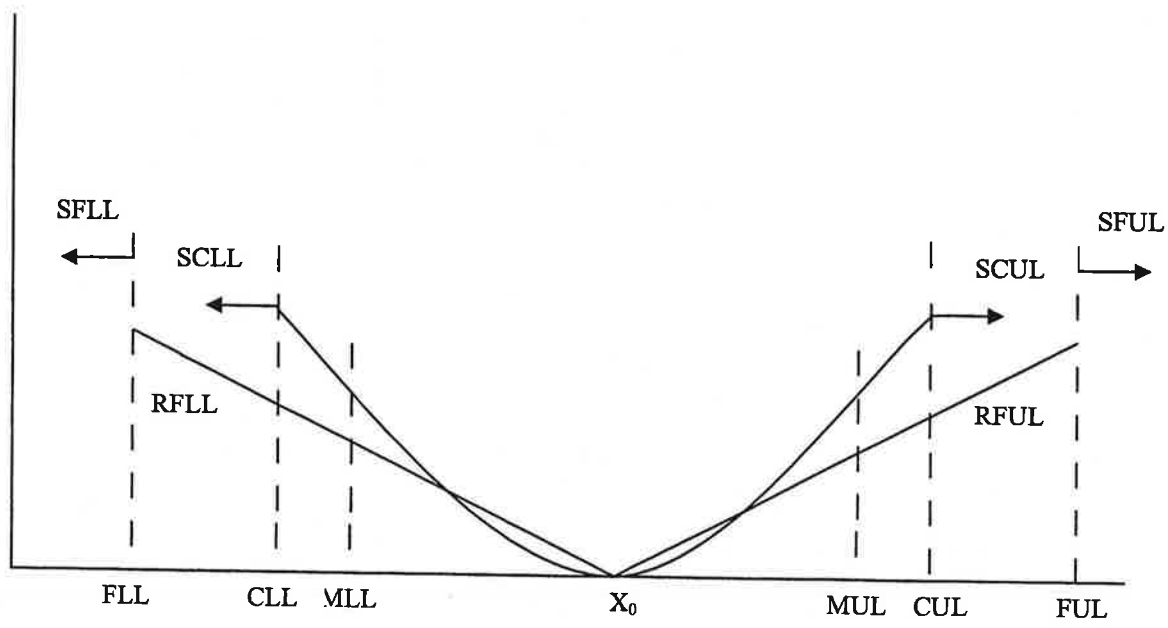


Figure 3: Graph of the Hybrid Cost Function

The mathematical representation of the proposed hybrid cost function is a seven-piece function that includes costs for scraps, reworks, loss to society, and a variance cost. It is assumed that the PDF used in this function $f_x(x)$ is the normal distribution PDF. The first two pieces of the cost function are consistent with the Taguchi loss function and pertain to the expected loss to society. These pieces cover the area under the cost function from MLL to the target value and from the target value to MUL. The loss to society pieces are given below.

$$k_1 \int_{MLL}^{x_o} f_x(x)(x-x_o)^2\,dx + k_2 \int_{x_o}^{MUL} f_x(x)(x-x_o)^2\,dx \quad \ldots \quad (3)$$

The next two pieces of the expected cost function pertain to the expected cost of having to perform rework on that product. Products are reworked if they fall outside the manufacturer's specification limit and within the reworkable limits (FLL / FUL). The cost of rework increases linearly starting from zero at the target value to RFUL at FUL and RFLL at FLL. This linear function upholds the assumption stated earlier that the cost of rework increases as the product's quality characteristic moves away from the target value. The rework cost pieces are provided below.

$$\int_{FLL}^{MLL} [RFLL(x_o - x)/(x_o - FLL)] + E[C(X)]]f_x(x)dx$$

$$+ \int_{MUL}^{FUL} [RFUL(x_o - x)/(FUL - x_o)] + E[C(X)]]f_x(x)dx \quad \ldots \quad (4)$$

The next two pieces relate to the scrap cost. If the product falls outside the functional lower limits it is scrapped. There is one constant scrap cost (*SFLL / SFUL*), independent of the distance between the product's quality characteristic and the functional limit. The scrap cost pieces are provided below

$$SFLL \int_{-\infty}^{FLL} f_x(x)dx + SFUL \int_{FUL}^{\infty} f_x(x)dx \quad \ldots \quad (5)$$

The last component of the expected cost function pertains to the cost associated with maintaining a particular process variance and is provided below.

$$K / \sigma^2 \quad \ldots \quad (6)$$

Aggregating all seven pieces together yields the complete mathematical representation for the proposed cost function

$$\begin{aligned} E[C(X)] = {} & k_1 \int_{MLL}^{x_o} f_x(x)(x-x_o)^2\,dx + k_2 \int_{x_o}^{MUL} f_x(x)(x-x_o)^2\,dx \\ & + \int_{FLL}^{MLL} [RFLL(x_o - x)/(x_o - FLL)] + E[C(X)]]f_x(x)dx \\ & + \int_{MUL}^{FUL} [RFUL(x_o - x)/(FUL - x_o)] + E[C(X)]]f_x(x)dx \\ & + SFLL \int_{-\infty}^{FLL} f_x(x)dx + SFUL \int_{FUL}^{\infty} f_x(x)dx + K/\sigma^2 \end{aligned} \quad \ldots \quad (7)$$

where $k_1 = SCLL/(x_o - CLL)^2$: proportionality constant for the Taguchi loss function
$k_2 = SCUL/(CUL - x_o)^2$: proportionality constant for the Taguchi loss function

After the expected cost equation was formulated the next step was to manipulate the equation in order to transfer the expected cost to the left hand side. This results in

$$E[C(X)] = \frac{\begin{bmatrix} k_1 \int_{MLL}^{x_o} f_x(x)(x-x_o)^2 dx + k_2 \int_{x_o}^{MUL} f_x(x)(x-x_o)^2 dx \\ + \int_{FLL}^{MLL} [RFLL(x_o - x)/(x_o - FLL)] f_x(x) dx \\ + \int_{MUL}^{FUL} [RFUL(x_o - x)/(FUL - x_o)] f_x(x) dx \\ + SFLL \int_{-\infty}^{FLL} f_x(x) dx + SFUL \int_{FUL}^{\infty} f_x(x) dx + K/\sigma^2 \end{bmatrix}}{1 - \int_{FLL}^{MLL} f_x(x) dx - \int_{MUL}^{FLL} f_x(x) dx} \quad \ldots \quad (8)$$

After using the normal probability function and the following substitutions,

$$\begin{aligned} & w = x - x_o \\ & \mu_w = \mu_x - x_o \\ & z = (w - \mu_w)/\sigma_x \\ & \Rightarrow dw = \sigma_x dz \text{ and } dw = \sigma_w dz \text{ since } \sigma_w = \sigma_z \end{aligned} \quad \ldots \quad (9)$$

we arrive at the following result

$$E[C(X)] =$$

$$\frac{\begin{bmatrix} k_1 \sigma_x \{ \sigma_x^2 \int_{(MLL-\mu_w)/\sigma_x}^{(x_o-\mu_w)/\sigma_x} z^2 e^{-z^2/2} / \sqrt{2\Pi} dz + 2\sigma_x \mu_w \int_{(MLL-\mu_w)/\sigma_x}^{(x_o-\mu_w)/\sigma_x} z e^{-z^2/2} / \sqrt{2\Pi} dz + \mu_w^2 \int_{(MLL-\mu_w)/\sigma_x}^{(x_o-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \} \\ + k_2 \sigma_x \{ \sigma_x^2 \int_{(x_o-\mu_w)/\sigma_x}^{(MUL-\mu_w)/\sigma_x} z^2 e^{-z^2/2} / \sqrt{2\Pi} dz + 2\sigma_x \mu_w \int_{(x_o-\mu_w)/\sigma_x}^{(MUL-\mu_w)/\sigma_x} z e^{-z^2/2} / \sqrt{2\Pi} dz + \mu_w^2 \int_{(x_o-\mu_w)/\sigma_x}^{(MUL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \} \\ + (\sigma_x^2 * RFUL)/(FUL - x_o) * \{ \int_{(MUL-\mu_w)/\sigma_x}^{(FUL-\mu_w)/\sigma_x} z e^{-z^2/2} / \sqrt{2\Pi} dz + \int_{(MUL-\mu_w)/\sigma_x}^{(FUL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \} \\ + (\sigma_x^2 * RFLL)/(x_o - FLL) * \{ \int_{(FLL-\mu_w)/\sigma_x}^{(MLL-\mu_w)/\sigma_x} z e^{-z^2/2} / \sqrt{2\Pi} dz + \int_{(FLL-\mu_w)/\sigma_x}^{(MLL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \} \\ + \sigma_x SFLL \int_{-\infty}^{(FLL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz + \sigma_x SFUL \int_{(FUL-\mu_w)/\sigma_x}^{\infty} e^{-z^2/2} / \sqrt{2\Pi} dz \end{bmatrix}}{1 - \sigma_x \int_{(FLL-\mu_w)/\sigma_x}^{(MLL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \} - \sigma_x \int_{(MUL-\mu_w)/\sigma_x}^{(FUL-\mu_w)/\sigma_x} e^{-z^2/2} / \sqrt{2\Pi} dz \}} \quad \ldots \quad (10)$$

After integration the expected cost function yields the following expression

$$E[C(X)] = \frac{\left[\begin{array}{l} k_1\sigma_x^2[(((x_o-\mu_w)e^{-\{(x_o-\mu_w)/\sigma_x\}^2*1/2})/\sqrt{2\Pi})-(((MLL-\mu_w)e^{-\{(MLL-\mu_w)/\sigma_x\}^2*1/2})/\sqrt{2\Pi}) \\ -\sigma_x(F_z\{(x_o-\mu_w)/\sigma_x\}-F_z\{(MLL-\mu_w)/\sigma_x\})] \\ +k_1\sigma_x\mu_w^2[F_z\{(x_o-\mu_w)/\sigma_x\}-F_z\{(MLL-\mu_w)/\sigma_x\}] \\ +[((2k_1\sigma_x^2\mu_w)/\sqrt{2\Pi})*(e^{-\{(MLL-\mu_w)/\sigma_x\}^2*1/2}-e^{-\{(x_o-\mu_w)/\sigma_x\}^2*1/2}))] \\ +k_2\sigma_x^2[(((MUL-\mu_w)e^{-\{(MUL-\mu_w)/\sigma_x\}^2*1/2}/\sqrt{2\Pi})-(((x_o-\mu_w)e^{-\{(x_o-\mu_w)/\sigma_x\}^2*1/2})/\sqrt{2\Pi}) \\ -\sigma_x(F_z\{(MUL-\mu_w)/\sigma_x\}-F_z\{(x_o-\mu_w)/\sigma_x\})] \\ +k_2\sigma_x\mu_w^2(F_z\{(MUL_o-\mu_w)/\sigma_x\}-F_z\{(x_o-\mu_w)/\sigma_x\}) \\ +((2k_2\sigma_x^2\mu_w)/\sqrt{2\Pi})*(e^{-\{(x_o-\mu_w)/\sigma_x\}^2*1/2}-e^{-\{(MUL-\mu_w)/\sigma_x\}^2*1/2}))] \\ +[(\sigma_x^2*RFUL)/(\sqrt{2\Pi}(FUL-x_o))*(e^{-\{(MUL-\mu_w)/\sigma_x\}^2*1/2}-e^{-\{(FUL-\mu_w)/\sigma_x\}^2*1/2}) \\ +(\sigma_w\mu_w RFUL)/(FUL-x_o)*(F_z\{(FUL-\mu_w)/\sigma_x\}-F_z\{(MUL-\mu_w)/\sigma_x\})] \\ +[(\sigma_x^2*RFLL)/(\sqrt{2\Pi}(x_o-FLL))*(e^{-\{(FLL-\mu_w)/\sigma_x\}^2*1/2}-e^{-\{(MLL-\mu_w)/\sigma_x\}^2*1/2}) \\ +(\sigma_w\mu_w RFLL)/(x_o-FLL)*\{F_z\{(MLL-\mu_w)/\sigma_x)\}-F_z\{(FLL-\mu_w)/\sigma_x)] \\ +[\sigma_w SFLL(\{F_z\{(FLL-\mu_w)/\sigma_x)\}-F_z\{-\infty\})+\sigma_w SFUL(F_z\{\infty\}-F_z\{(FUL-\mu_w)/\sigma_x)\})] \end{array}\right]}{1-[\sigma_x(\{F_z\{(FLL-\mu_w)/\sigma_x\})-\sigma_x(F_z\{(MLL-\mu_w)/\sigma_x\})]-[\sigma_x(\{F_z\{(FUL-\mu_w)/\sigma_x\})-\sigma_x(F_z\{(MUL-\mu_w)/\sigma_x\})\})]}$$

... (11)

where $F_z(x) = \int_{-\infty}^{x} e^{-z^2/2}/\sqrt{2\Pi}\,dz$

There are a couple of assumptions used to facilitate the integration. First, the decision variables will never be chosen by the user that result in the upper limit of an integral being smaller than its lower limit. This assumption would only fail if the user of this function decided on a non-traditional setting for one of their decision variables. For instance setting the lower manufacturer specification limit above the target value would break this assumption. Secondly, it is assumed that the upper and lower limits of each integral were of the same sign.

3.0 APPLICATION OF THE EXPECTED COST MODEL

The proposed expected cost function consists of 3 decision parameters, σ^2 , MLL and MU*L* which are to be configured by the user. There is an acknowledged significance to the effect of the variability of σ^2 , MLL and MUL on the resulting level of expected cost. It is the task of the manufacturer to configure these parameters properly for the minimization of expected cost. However, the effect of the parameters FLL/ FUL, RFLL/ RFUL, and SFLL/ SFUL on the minimal expected cost is not as apparent as it is for the decision parameters. Though these parameters are assumed to be fixed, in certain situations it is feasible that extensive effort by the manufacturer could provide a change to these parameter values. Therefore, a meaningful example application of the proposed expected cost model is an investigation as to how much these input parameters affect the quality of the expected cost minimum.

Therefore, it is of interest to experiment with the expected cost model to investigate the uncertain sensitivity to the variability of these input parameters on the minimal level of the expected function achieved under a decision space of σ^2 , MLL and MUL values. An analysis of variance (ANOVA) is applied to test the significance of the level of fixed parameter values FLL/ FUL, RFLL/ RFUL, and SFLL/ SFUL on the resulting minimum expected cost.

A 2^3 full factorial experiment is implemented to perform the analysis of variance for the *FLL*/ FUL, RFLL/ RFUL, and SFLL/ SFUL parameters by considering these parameters to be symmetrical. For each combination level of these parameters, an optimization routine is executed to determine the minimum expected cost achieved by a combination of the decision variables σ^2 , *MLL* and *MUL*. Optimization of the expected cost function is performed by a steepest descent search using finite difference gradient estimations of the expected cost function. The forward finite difference estimate is used for an estimate $\hat{g}(\theta) = [\hat{g}_1(\theta), \hat{g}_2(\theta), ...\hat{g}_n(\theta)]^T$ of the gradient of $E[C(X)]$ and is given below.

$$\hat{g}_i(\theta) = \frac{E[C(X+ce_i)] - E[C(X)]}{c} \quad \text{for } i = 1 \text{ to } n \quad \ldots \quad (12)$$

where c is some small positive scalar and $e_i = (0,0,\ldots1,\ldots..0)$ and denotes a vector with a one in the ith position and zeros in the remaining positions for $i = 1,\ldots,n$.

The example process used for this experiment consists of a target value of 115 and a mean of 114. The constant K for the variance cost is set to 2000. A reasonable hypothetical value for *CLL/ CUL* is assigned by considering these values to be the midpoints between *FLL* and *MLL* and *MUL* and *FUL*, respectively. Values for SCLL and SCULL are both set at 200 and k_1 and k_2 are obtained with the use of the equations $k_1 = SCLL/(x_o - CLL)^2$ and $k_2 = SCUL/(CUL - x_o)^2$. The high and low levels of each experimental parameter are given in the figure below.

	FLL/ FUL	RFLL/ RFUL	SFLL/ SFUL
Low	(100, 130)	(20, 20)	(40, 40)
High	(80, 150)	(50, 50)	(70, 70)

Figure 4: High and Low levels for the Experimental Design

The figure below presents the minimum expected cost achieved for each combination level of the full factorial experiment.

FLL/ FUL	RFLL/ RFUL	SFLL/ SFUL	Optimal $E[C(X)]$
-1	-1	-1	251.18
1	-1	-1	241.23
-1	1	-1	262.48
1	1	-1	247.81
-1	-1	1	311.86
1	-1	1	301.94
-1	1	1	317.74
1	1	1	311.52

Figure 5: Optimal Expected Cost for Each Combination Level

From this experiment an analysis of variance can be performed to investigate the effects of the parameters FLL/ FUL, RFLL/ RFUL, and SFLL/ SFUL on the optimal expected cost. The figure below presents the analysis of variance results.

Source	Sum of Squares	DF	Mean Square	F-statistic	F_0 (.01)
FLL/ FUL	207.67	1	207.67	44.39	21.2
RFLL/ RFUL	138.94	1	138.94	29.70	21.2
SFLL/ SFUL	7221.62	1	7221.62	1543.66	21.2
Error	18.71	4	4.68		

Figure 6: Analysis of Variance Results

The analysis of variance reveals that each parameter has a significant effect on the optimal expected cost with a critical value of .01. It is also evident that a linear model captures the majority of the effects of the variability of the parameters demonstrating the insignificance of the interaction and quadratic terms. This is beneficial for the analysis since negating the interaction effects provides degrees of freedom (DF) for the estimation of the mean squared error.

Furthermore, the analysis of variance reveals that in increase in the (FLL, FUL) range results in a decrease in the minimum expected cost value. Also a decrease in the level of RFLL/ RFUL and SFLL/ SFUL resulted in a decrease in the minimum expected cost value. The majority of the total sum of squares resulted from the SFLL/ SFUL parameter, which again corresponds to the cost to scrap a part above or below the functional upper and lower limit. It is intuitive from the model derivation that this value would have the strongest effect on the minimum cost per part value

4.0 SUMMARY

In this paper, the upper and lower specification limits were derived using a newly derived hybrid function for expected cost. This cost function includes Taguchi loss function, rework cost, scrap cost and a variance- to cost- tradeoff. The cost function is implemented among different production systems and optimal values of manufacturer lower and upper specification limits are determined for each system. It is one way to consider costs while determining the specification limits.

5.0 REFERENCES

1. Chen, C. (1999). Specification Limit under a Quality Loss Function. Journal of Applied Statisitcs, 26: 903-908.
2. Eagle, A. (1954). A Method for Handling Errors in Testing and Measuring. Industrial Quality Control, 11: 10-15.
3. Grubbs, F. and Coon, H. (1954). On Setting Test Limits Relative to Specification Limits. Industrial Quality Control, 11: 15-20.
4. Jeang, A. (1997). An Approach of Tolerance Design for Quality Improvement and Cost Reduction. International Journal of Production Research, 35: 1193-1211.
5. Jeang, A. (2001). Combined Parameter and Tolerance Design Optimization with Quality and Cost. International Journal of Production Research, 39: 923-952.
6. Kackar, R. (1985). Off-line Quality Control, Parameter Design, and the Taguchi method. Journal of Quality Technology, 17: 176-188.
7. Kapur, K. and Wang, C. (1987). Economic Design of Specifications Based on Taguchi's Concept of Quality Loss Function. In Proceedings of ASME Symposium on Quality: Design, Planning and Control (Ed: R.E. DeVor and S.G. Kapoor). American Society of Mechanical Engineers, Boston, MA , pp. 23-26.
8. Kapur, K., Raman, S., and Pulat, P. (1990). Methodology for Tolerance Design Using Quality Loss Function. Computers and Industrial Engineering, 19: 254-257.
9. Taguchi, G. (1986). Introduction to Quality Engineering. UNIPUB/Kraus International Publications, White Plains, NY and American Supplier Institute Inc., Dearborn, MI.
10. Vasseur, H., Hurfess, T., and Cagan, J. (1997). Use of a Quality Loss Function to Select Statistical Tolerances. Transactions of the American Society of Mechanical Engineers, 119: 410-416.