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COMMENT ON "OPTIMAL SHELF-SPACE STOCKING POLICY USING STOCHASTIC DOMINANCE UNDER SUPPLY-DRIVEN DEMAND UNCERTAINTY"

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The paper authored by Amit et al. [Amit, R. K., Mehta, P., & Tripathi, R. R. "Optimal shelf-space stocking policy using stochastic dominance under supply-driven demand uncertainty", European Journal of Operational Research, 246(1), pp. 339-342 (2015)] proposes a newsvendor model in which demand depends on displayed inventory. Under stochastic dominance conditions, they claimed that the profit function is concave at its extremum points. In the present study, it is expressed that this statement is not generally precise.

Keywords: Inventory, newsvendor, stock dependent demand, stochastic dominance

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In their paper, Amit et al. (2015) proposed a single period model where demands depended on the inventory level. It is worth glancing over the stated model of Amit et al. together with its assumptions. Now, this is how it goes; a random variable represented by θ, which in this context stands for demand, has a probability density function shown by $\phi(\theta, s)$ and a cumulative probability function shown by $\Phi(\theta, s)$. The distribution depends on the initial stock level indicated by s. In this way, Amit et al. assumed that the demand distribution with higher inventory levels dominates the demand distribution with lower inventory ($\frac{\partial \Phi(\theta,s)}{\partial s} \le 0, \qquad \forall s, \theta$). It could be readily noticed that this is a first-order stochastic dominance.

The expected profit function is:

$$\Pi(s) = p\left[\int_0^s \theta\phi(\theta,s)d\theta + \int_s^\infty s\phi(\theta,s)d\theta\right] - h\int_0^s (s-\theta)\phi(\theta,s)d\theta - b\int_s^\infty (\theta - s)\phi(\theta,s)d\theta - cs, \tag{1}$$

where p is the unit selling price, c is the unit purchase cost, and h is the holding cost for each unsold inventory at the end of the period, and b is the shortage cost for each demand that encounters stock out. The authors later obtained s^* by complying with the first optimality condition ($\frac{\partial \Pi(s)}{\partial s} = 0$); in their proposition (2.5), they claimed that $\Pi(s)$ is concave at s^*.

Their proof starts by assuming s^* as the stocked quantity where $\forall s < s^*\ \pi(\theta, s) = p\theta - cs - h(s-\theta)$ is an increasing function of θ. This assumption is incorrect because of the presence of s^* instead of θ. Therefore the proof procedure is completely wrong. However, the correct expression for s which represents the initial stock level is:

$$\pi(\theta, s) = \begin{cases} p\theta - cs - h(s-\theta), & \theta \le s \\ ps - cs - b(\theta - s), & \theta > s \end{cases} \tag{2}$$

The above mistake sufficiently supports our claim. To state clearer, consider the following examples. Let $p=7, c=3, h=4, b=7$. Assume that the demand has a uniform distribution on the interval $[0, 10+15\sqrt{s}]$ in which s is the initial stock level. For the stated distribution, first-order stochastic dominance holds.

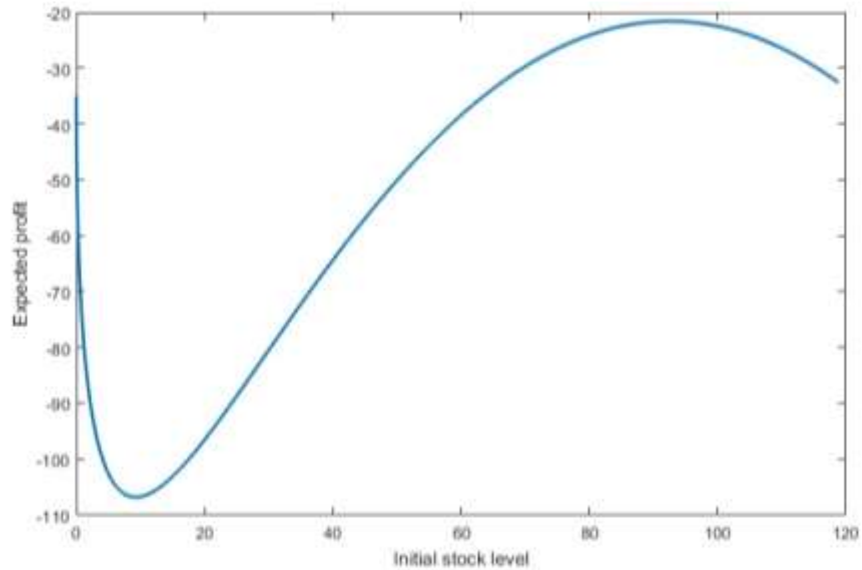

Figure 1. The expected profit as a function of the stock level; $D \sim U[0,10+15\sqrt{s}], p=7, c=3, h=4, b=7$

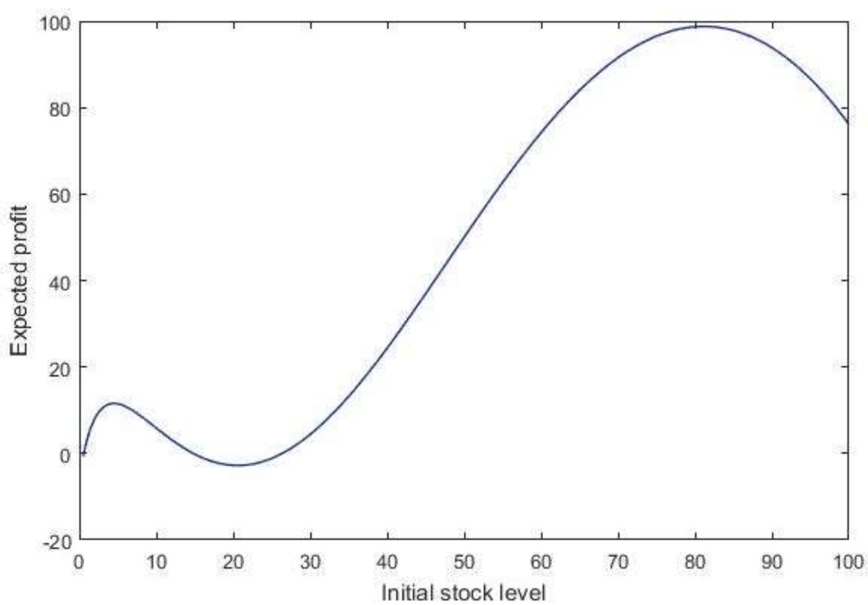

Figure 2. The expected profit as a function of the stock level; $D \sim U[0, f(s)], p=15, c=6, h=1, b=4$

In Figure 1, we plot the expected profit function with respect to the initial inventory. It could be readily observed that the expected profit function is not strictly concave, and two extremum points exist where the first is a local minimum. In the above simple example, demand has a uniform distribution on the interval $[0, f(s)]$ in which $f(s)$ increases at a decreasing rate.

In the second example, assume *f(s)* increases as the inventory increases, but its rate fluctuates. Let $f(s) = 50 + 60 \cdot \tan^{-1}(s - 50), p = 15, c = 6, h = 1, b = 4$. Note that first-order stochastic dominance holds. In Figure 2 expected profit function with respect to the initial inventory is plotted. The expected profit function is not concave, and also, there are three extremum points: a local minimum and two maximum points with positive expected profit.

Note that in their model, Amit et al. (2015) presented an extension to previous works function (see, e.g., Balakrishnan et al. (2008) as one of the most outstanding researches in the area), including shortage cost and more general demand distribution. In their model, profit function is strongly nonlinear, therefore trapped into a local maximum; one may desist searching, considering that profit function is strictly concave. As shown in Figure 2, such a local maximum can lead to a very bad policy.

In most stock-dependent demand models, it is assumed that inventories stimulate demand. Proposed first-order stochastic dominance represents the stimulation effect. Now let describe the saturation condition: assume $p = \Phi(\theta, s)$ is the cumulative distribution of demand conditional on s. under saturation condition $\theta = \Phi^{-1}(p, s)$ increases at a decreasing rate with s. it can be easily observed that in the second example, saturation condition does not hold. Exploring the literature, it could be observed that Balakrishnan et al. (2008) proved that the objective function is concave under demand stimulation and saturation conditions when shortage cost is not considered.

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