

FUZZY GOAL PROGRAMMING APPROACH TO SOLVE THE EQUIPMENTS-PURCHASING PROBLEM OF AN FMC

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This paper focuses on the equipments purchasing decision for a Flexible Manufacturing Cell (FMC) that many managers pay more attention in order to increase the competitiveness of the company. The procedure of machine configuration always involves a multiple criteria decision making problem. To meet the four conflict and fuzzy goals: total number of machines, total floor space occupied by machines, total purchasing cost and total output of a specific part family in the equipments-purchasing problem of an FMC, a Fuzzy Goal Programming (FGP) model is proposed to solve the problem in this paper. A mixed integer programming model is built after the nonlinear constraints are transformed. Three suggestions-weights, threshold value and comparative relationship on the achieved level- are introduced when the importance and priority structure of goals are considered. Hence with those attributes of equipments and membership function of goals, the decision makers can apply this model to obtain the purchasing policy and the achieved level of each individual goal. Finally, an example is illustrated in order to demonstrate the proposed model.

Keywords: Equipment Purchasing, Fuzzy Goal, Goal Programming, Flexible Manufacturing Cell

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1. INTRODUCTION

To increase competitiveness, manufacturers often introduce advanced manufacturing technology such as the Flexible Manufacturing System (FMS). In FMS, the Flexible Manufacturing Cell (FMC) is an important module during the constructing stage. To implement the FMC, the first task is to select and purchase suitable equipments economically and effectively. From the manufacturer's viewpoint, many quantitative and qualitative factors such as purchasing cost, floor space, productivity, operation conditions, task and operating preference must be considered during the selection process. The importance of those factors always differs from case to case. Therefore, developing an effective, reliable and useful decision making model to buy suitable equipments is the key to the success of building an FMC.

In order to resolve decision-making problems, some models such as the multiple objectives, multiple attributes, and multiple criteria decision models have been used. The multiple objective decision making (MODM) consists of a set of conflicting goals that cannot be satisfied simultaneously. This method usually involves solving problems on a continuous space via mathematical programming model (Mehrdad, 1996). The multiple attribute decision making (MADM), on the other hand, deals with the problem of choosing an alternative from a set of candidate alternatives that are characterized in terms of certain attributes. The multiple criteria decision making (MCDM) model is generally used to solve the multiple objectives, multiple attributes or both problems (Ralph, 2000).

In order to determine the relative importance of those quantitative factors in the equipments-purchasing problem of an FMC, the floor space occupied by equipments should be considered first due to the expensive cost of land. In addition, the number of equipments affects the complexity of the FMC, especially the smoothness and effectiveness of the material handling activity. Also the total purchasing cost is limited by the business budget. On the other hand, the productivity required by the customers' demands and the precision of equipments relates to the variation of the quality during the manufacturing process. All these factors are important and must be taken into account in the purchasing procedure of the configuration of an FMC. To deal with the machine selection problem about FMC, Wang et al. (2000) and Wang and Chen (2002) have published two papers. Since the procedure of equipments configuration always involves a MCDM problem that relates to many factors, this paper will consider those factors as the objectives that must be reached. Hence, the equipments purchasing problem in an FMC becomes a multiple objective problem. Research shows that the problem can be solved by the Goal Programming (GP) originally developed by Charnes and Cooper (1977). A major limitation of GP, however, is that the aspiration levels and/or weights are imprecise or vague in the real world for decision makers. Besides, the input data such as resource or technical coefficient may not be determined precisely because of incomplete or unobtainable information in practice. The difficulties stated above can be overcome via fuzzy sets and theory (Zadah, 1965; Yager, 1977; Hannan, 1981; Ramik, 2000; Stewart, 2000). Therefore, a more practical decision making model such as

Fuzzy Goal Programming (FGP) that can deal with the multiple conflict and fuzzy goals should be applied for our equipments purchasing problem. The literature review is described in next section. The equipments purchasing problem is showed in section 3. According to the description in Section 3, the FGP model for the equipments purchasing problem is derived in Section 4. An example is given to illustrate the proposed approach in Section 5, followed by the results and discussion in Section 6.

2. LITERATURE REVIEW

In order to develop the approach for the equipments purchasing problem, much more literature were reviewed.

2.1 Machine Selection Problems

Some researches have focused on cost measurement for the requirements and the purchasing problems of automated facilities (Knott and Getto, 1982; Kimms, 2000). Miller and Davis (1977) reviewed the literature on the machine requirements problem and deduced some important factors, such as production output requirements, machine output rates, machine scrape factors, and the available work time. Kusiak (1987) built machine selection models subjected to the machining type, machine dimensions, horsepower and other facility constraints such as the available AGV's number and time. Behnezhad and Khoshnevis (1988) enhanced the machine requirement model by adding market demand and inventory amount. Most of the models mentioned above for the machine requirement problem have been conducted on the traditional mathematical programming model. However, many of the practical difficulties such as unobtainable data or ambiguous data tended to be overlooked.

2.2 The FGP Models

A fuzzy programming approach for linear programming problems with several objectives was developed by Zimmermann (1978). Narasimhan (1980) proposed a complex method for dealing with the goal programming problem with fuzzy goal and discussed an approach to deal with fuzzy priority in 1980. In the subsequent research, Hannan (1981, 1982) introduced a simplified procedure to formulate a FGP problem as an equivalent single linear programming problem with 2K goal-related constraints and pointed out the distinction between the fuzzy goal programming and fuzzy multi-criteria programming. Ignizio (1983) documented and briefly reviewed the history of fuzzy goal programming.

The methodology based on the use of a nested hierarchy of priorities for each goal was proposed by Rubin and Narasimhan (1984). Tiwari et al. (1986) demonstrated a computational algorithm for solving an FGP problem with symmetrical triangular membership functions of fuzzy goals and preemptive priority structure. Subsequently, these authors introduced an additive model which used arithmetic addition to aggregate the fuzzy goals to construct the relevant decision function (Tiwari et al., 1987). Based on their model (Tiwari et al., 1986; Chen 1994) provided a modified solution procedure to reduce the number of sub-problems. Yang and Ignizio (1991), Rao et al., (1992), Roy and Maiti (1998) have examined the fuzzy nonlinear goal programming problem. In addition, Kim and Whang (1998) provided a tolerance approach to solve an FGP problem with unequal weight unbalanced triangular membership functions. Also, Wang and Fu (1997) proposed a method to solve the FGP problem with preemptive structure via utilizing a penalty cost. Pal and Moitra (2003) offered an alternative idea, a multi-stage DP model, in order to solve GP problems with preemptive priority for achieving of the highest degree of each of the membership function. Arora and Gupta (2009) presented an interactive FGP approach for bilevel programming problems with the characteristics of DP.

2.3 Empirical Literature Related FGP Models

Many applications have shown that FGP approaches are suitable and practically useful. These applications are useful in many different fields such as the selection of the sequence of timer harvests applied over time to a forest (Pickens and Hof, 1991), the stochastic transportation problem under budgetary constraint (Chalam, 1994), the control problem (Stewart, 1992), the solid waste management (Chang and Wang, 1996, 1997) and the water quality management problem (Lee and Wen, 1997). Other investigations include the expert system (Rasmy et al., 2002) and portfolio selection (Parra et al. 2001). As for the application for production and operation management, Pendharker (1997) applied a fuzzy linear programming model for production planning in coal mines, while Arikan and GÜngör (2001) investigated the multi-objective project network problem, and Kumar et al. (2004) provided a fuzzy goal programming approach for vendor selection in a supply chain. Recently, Biswas and Pal (2005) used the fuzzy goal programming technique to model and solve the land-use planning problem in agricultural system. Due to the importance of the machine selection problem in building an FMC, this paper implements the machine purchasing problem constrained by machine speed, utility and precision for an FMC via the fuzzy goal programming model with four fuzzy goals: number of machines, total floor space, total purchasing cost, and total productivity.

3. PROBLEM DESCRIPTION

The purchasing dilemma involves a decision-making problem for selecting suitable kinds of equipments from the market. At first, a screening step is performed to identify those equipments that can provide the requested processing functions from equipments suppliers. Then, equipments are chosen as candidates to meet the required precision to promise the variation of part quality. Subsequently, the four objectives: total purchasing cost, total floor space, total number of equipments and the amount of total output for all parts for equipments-purchasing problem are considered. The first three objectives are desired to have a minimum value, while the last objective is contradictory to the first three objectives. Moreover, each objective has assigned a rough and ambiguous target value by the manager. That is to say, an FGP model is suggested to choose the feasible equipments that are needed and satisfy those objectives as closely as possible.

3.1 Assumptions

In implementing the FGP model for the equipments purchasing decision, the following assumptions are required.

- (i). There is only one specific part family to be produced in the FMC.
- (ii). The manufacturing operations of the part family are known.
- (iii). The information on all the candidates and different type equipments are obtainable.
- (iv). The FMC to be constructed does not perform assembly or disassembly operations.
- (v). There is no defective parts considered during the manufacturing process in this planning stage.

3.2 Notations

There are some notations used in this model that should be defined before the formulation procedure.

G_t : The t^{th} goal function.

L_t : The lower tolerance limit for the t^{th} fuzzy goal.

M : A very large number, i.e., a big- M .

U_t : The upper tolerance limit for the t^{th} fuzzy goal.

c_{jk} : The cost of the k^{th} kind of equipment used in operation j .

g_t : The aspiration level of the t^{th} fuzzy goal.

m_{jk} : The total number of equipments of the k^{th} kind of equipment purchased for operation j .

n : The number of different parts to be processed in the FMC.

o_{ijk} : The processing time when the part i is processed on the k^{th} kind of equipment used in operation j . This can be calculated from the spindle speed and traversing speed of the equipment.

q_i : The output of the part i .

r : The number of operations to be processed in the manufacturing cell.

s_{jk} : The floor space for the k^{th} kind of equipment used in operation j .

t_j : The number of kinds of equipments that can be chosen for each manufacturing operation j .

u_{jk} : The available machining time of the k^{th} kind of equipment used in operation j , measured according to the utilization that relates to the reliability and maintenance plan of the equipment.

y_{jk} :
$$y_{jk} = \begin{cases} 1, & \text{if } k^{\text{th}} \text{ kind of machine used in operation } j \text{ is to be purchased} \\ 0, & \text{otherwise} \end{cases}$$

μ_t : The grade of membership or achievement level of goal t .

The decision variables are m_{jk} , q_i , y_{jk} and μ_t in our equipments-purchasing problem.

4. METHODOLOGY

From the description in Section 3, one can derive the four goals for the equipments-purchasing problem as:

- (i). The total equipments number is approximately less than or equal to g_1 .
- (ii). The total equipments floor space needed is approximately less than or equal to g_2 .
- (iii). The total purchasing cost is approximately less than or equal to g_3 .
- (iv). The integrated productivity of constructed FMC is approximately greater than or equal to g_4 .

According to Zimmermann (1978), a linear membership function μ_t for the t^{th} fuzzy goal $G_t \leq g_t$ can be expressed as

$$\mu_t = \begin{cases} 1 & , \text{ if } G_t \leq g_t \\ \frac{U_t - G_t}{U_t - g_t} & , \text{ if } g_t \leq G_t \leq U_t \\ 0 & , \text{ if } G_t \geq U_t \end{cases}$$

Whereas in the case of fuzzy goal, $G_t \geq g_t$ can be expressed as

$$\mu_t = \begin{cases} 1 & , \text{ if } G_t \geq g_t \\ \frac{G_t - L_t}{g_t - L_t} & , \text{ if } L_t \leq G_t \leq g_t \\ 0 & , \text{ if } G_t \leq L_t \end{cases}$$

Then we formulate and propose the FGP model in Section 4.1.

4.1 FGP Model

To accommodate the above fuzzy goals, the additive objective function model is proposed by Tiwari et al. (1987). Then, the model for equipments-purchasing problem is formulated as:

$$\text{Maximize } V(\mu) = \sum_{t=1}^4 \mu_t \quad \dots \quad (1)$$

$$\text{Subject to } \mu_t = \frac{U_t - G_t}{U_t - g_t}, t = 1, 2, 3 \quad \dots \quad (2)$$

$$\mu_4 = \frac{G_4 - L_4}{g_4 - L_4} \quad \dots \quad (3)$$

$$G_1 = \sum_{j=1}^r \sum_{k=1}^{t_j} m_{jk} \quad \dots \quad (4)$$

$$G_2 = \sum_{j=1}^r \sum_{k=1}^{t_j} s_{jk} m_{jk} \quad \dots \quad (5)$$

$$G_3 = \sum_{j=1}^r \sum_{k=1}^{t_j} c_{jk} m_{jk} \quad \dots \quad (6)$$

$$G_4 = \sum_{i=1}^n q_i \quad \dots \quad (7)$$

$$y_{jk} \sum_{i=1}^n q_i o_{ijk} \leq m_{jk} u_{jk}, k = 1, \dots, t_j, j = 1, \dots, r \quad \dots \quad (8)$$

$$m_{jk} \leq M y_{jk}, k = 1, \dots, t_j, j = 1, \dots, r \quad \dots \quad (9)$$

$$m_{jk} \geq y_{jk}, k = 1, \dots, t_j, j = 1, \dots, r \quad \dots \quad (10)$$

$$\sum_{k=1}^{t_j} y_{jk} \geq 1, j = 1, \dots, r \quad \dots \quad (11)$$

$$y_{jk} \in \{0, 1\} \quad \dots \quad (12)$$

$$m_{jk} \in \text{integer} \quad \dots \quad (13)$$

$$q_i \in \text{integer}, i = 1, \dots, n \quad \dots \quad (14)$$

$$\mu_t \leq 1, t = 1, \dots, 4 \quad \dots \quad (15)$$

$$\mu_t \geq 0, t = 1, \dots, 4 \quad \dots \quad (16)$$

In the model shown above, equation (1) denotes that the objective function intends to reach a maximum total satisfaction with all goals. Equations (2) and (3) indicate the linear membership function of each goal if target and tolerance value are given. Equations (4) to (6) show the demands of total number of equipments, total spending floor space, and total purchasing cost carried from an alternative. Equation (7) represents that the total productivity could be possible. Equation

(8) constrains the total productivity to confirm the available machining time. The chosen constraints are given in equations (9) to (11), and those equations relating to the attributes of all decision variables are shown in equations (12) to (16).

The FGP model presented above is a mixed nonlinear integer-programming problem. The problem comes from the product term of $q_i y_{jk}$ in the productivity constraint in equation (8). To deal with the mixed nonlinear integer programming problem, a heuristic annealing algorithm based on mean field theory was proposed by Chen et al. (1997). Yet, since solving the nonlinear and combinatorial optimization problem directly is very difficult and may not promise an optimal solution, a preprocessing procedure is needed to reformulate the model.

4.2 Model Reformulation

To overcome the difficulty resulting from equation (8), we first relax the output amount q_i into a continuous variable; equation (14) then becomes

$$q_i \geq 0, i = 1, \dots, n \quad \dots \quad (17)$$

Although the output amount should be a discrete number in practice, it is acceptable to treat it as a continuous number for a sufficiently large output amount. Then, we define $v_{ijk} = q_i y_{jk}$ to deal with the product of $q_i y_{jk}$. Therefore, equation (8) is replaced by

$$\sum_{i=1}^n v_{ijk} o_{ijk} \leq m_{jk} u_{jk} \quad \forall j, k \quad \dots \quad (18)$$

Also the additional equations $\dots \quad (19)$

$$\begin{cases} v_{ijk} \leq q_i \\ v_{ijk} \leq M y_{jk} \\ v_{ijk} \geq q_i - (1 - y_{jk})M \end{cases} \quad \forall i, j, k$$

are added. Now the nonlinear problem is conquered and the model is transformed into a mixed integer-programming problem.

Then the purchasing constraint in equation (9) needs to be considered further. For an upper tolerance limit of the total number of equipments is assigned, the constraint (9) can be replaced by equation (20).

$$m_{jk} \leq U_1 y_{jk}, k = 1, \dots, t_j, j = 1, \dots, r \quad \dots \quad (20)$$

After transferring the above constraints, the FGP model for the equipments-purchasing problem can be reformulated as in the following.

Maximize $V(\mu) = \sum_{t=1}^4 \mu_t$

- Subject to Equations (2) ~ (7)
- Equations (10) ~ (13)
- Equations (15) ~ (17)
- Equations (18) ~ (20)

Therefore, solving the reformulated FGP model, in fact, is a task to solve a mixed integer programming problem that can be solved by a commonly used software package such as LINDO. Consequently, the problem of equipments-purchasing in an FMC can be modeled and solved.

In addition, a least output requirement must be given to satisfy the customers' demands such that a constraint such as equation (21) must be added.

$$q_i \geq q_0, i = 1, \dots, n \quad \dots \quad (21)$$

4.3 FGP Model with Unequal Importance

For decision making, different levels of importance of those goals, or priority structure among those goals, may exist and must be dealt with in many real world cases (Chen and Tsai, 2001). Three alternatives can be used for the equipments purchasing problem:

- (i). Use different weights to represent different levels of importance. Generally, these weights can be obtained by using group decision techniques such as the Delphi method and AHP(Lin and Yang, 1996) etc. Let w_t be the weight of goal t . Accordingly, the objective function can be established to maximize $\sum_{t=1}^4 w_t \mu_t$.
- (ii). Use a threshold value to ensure the achievement level. We can set threshold values to limit the minimum requirement for those goals, such as $\mu_3 \geq \mu_0$ to force the total purchasing cost for all bought equipments, where μ_0 is the threshold value of the grade of satisfaction.
- (iii). Use a comparative expression. Differing from the deterministic values, the fuzzy characteristics may exist in the measurement of importance. To describe the preemptive or preemptive relationship among those goals, an easier way that uses $\mu_i > \mu_j$ or $\mu_i \geq \mu_j + \mu_0, 0 \leq \mu_0 < 1$ to indicate that goal i is more important than goal j .

There are some benefits by setting $\mu_i \geq \mu_0$ or $\mu_i > \mu_j$ to represent the different importance of goals. The first benefit is that the above setting can avoid the inconsistency between the rank of achieved level of goals and the weights set. For example, the weight value of goal i is larger than the weight value of goal j but the relationship of achieved levels of goal i and j may be contrary. Furthermore, the method of using the threshold value or comparative expression is more simple and convenient than the method that directly introduces the membership function of weight into the solving procedure.


To explore the solution of the problems discussed above, an example will be provided in the following section.

5. INDUSTRY APPLICATION

In order to demonstrate the model described in the previous section, a numerical example abstracted from a real world case is shown in this section. Assuming that a factory needs an FMC that configures with machining center, CNC lathe, broaching equipments and hobbing machine to process a specific part family, the project manager is requested to purchase the required machines. The part family of spindle includes six different parts that are described in Table 1, and the part p_1 is shown in Table 1 also.

Table 1. Parts of the spindle family

Part number	Description
p_1	ψ 160 spindle
p_2	ψ 160 spindle for fast speed motor
p_3	ψ 160 single-speed spindle
p_4	ψ 160 double-speed spindle
p_5	ψ 190 single-speed spindle
p_6	ψ 190 double-speed spindle



P_1 . ψ 160 spindle

After screening the equipments database, some alternative equipment is listed in Table 2. According to the maximum spindle speed and/or maximum traversing speed of equipments, the operating time needed for one piece of each part is estimated and shown in Table 2 simultaneously. The signal “N/A” denotes that the part does not need to be processed by that equipment.

In addition, the target and bound values that decide the membership function of each goal are determined by decision

makers and presented in Table 3. The weights ($w_1 = 0.8, w_2 = 0.6, w_3 = 1.0, w_4 = 0.9$) that represent different levels of importance of goals are also listed.

Table 2. Attribute values of each equipment

	Equipment type													
	A		B			C			D		E			
Kinds of equipment for each equipment type	A1	A2	A3	B1	B2	B3	C1	C2	C3	C4	D1	D2	E1	E2
Floor space needed														
Length (m)	7.17	7.17	4.45	3.29	2.06	2.06	3.02	3.29	2.73	1.99	2.50	2.70	1.35	1.50
Width (m)	5.70	5.70	3.76	1.50	1.43	1.43	1.44	1.50	1.98	1.19	2.00	1.50	1.10	1.5
Area (m ²)	40.869	40.869	16.732	4.935	2.9458	2.9458	4.3488	4.935	5.4054	2.3681	5	4.05	1.485	1.725
Purchasing cost: 10 thousand dollars (US)	20.0	16.7	18.3	5.0	4.5	4.2	8.3	7.3	6.0	6.7	3.3	4.0	3.3	3.0
Max. spindle speed (rpm) or Max. traversing speed (m/min)	6000	4500	4000	6000	6000	6000	4500	4200	3000	3000	6	6	8	8
Available machining time: min per working day	1368	1368	1368	1368	1368	1368	1368	1368	1368	1368	1296	1296	1296	1296
Operating time needed for one piece of part i : min														
$i=1$	0.30	0.35	0.32	N/A	N/A	N/A	3.10	3.10	3.10	3.10	N/A	N/A	5.00	4.90
$i=2$	0.30	0.35	0.30	1.10	1.30	1.20	3.20	3.20	3.40	3.30	5.80	5.80	4.75	4.75
$i=3$	0.22	0.25	0.20	0.80	1.00	0.90	2.80	2.80	2.90	2.85	N/A	N/A	4.50	4.60
$i=4$	0.25	0.30	0.28	N/A	N/A	N/A	2.70	2.80	2.90	2.80	5.20	5.30	N/A	N/A
$i=5$	0.50	0.55	0.52	1.30	1.30	1.35	3.30	3.40	3.50	3.40	N/A	N/A	5.10	5.20
$i=6$	0.40	0.45	0.42	N/A	N/A	N/A	3.20	3.25	3.30	3.25	5.40	5.45	N/A	N/A

Table 3. Target and bound values of each goal

	Goals			
	Number of equipments	Total floor space	Total purchasing cost	Total output
Target value	$g_1 = 6$	$g_2 = 50$	$g_3 = 50$	$g_4 = 1200$
Upper/Lower bound	$U_1 = 8$	$U_2 = 70$	$U_3 = 70$	$L_4 = 500$
Importance	$w_1 = 0.8$	$w_2 = 0.6$	$w_3 = 1.0$	$w_4 = 0.9$

In the case that the weights are deterministic, the value of weights can be decided subjectively. While in the fuzzy case, the preemptive relationship is required to set as $\mu_3 > \mu_4 > \mu_1 > \mu_2$.

Then, the FGP model is applied to solve the above equipments-purchasing problem. In the subsequent subsections, two cases are illustrated and discussed in Section 4.3. The first case compares the situations based on whether or not there are distinct weights exist. The other case presents the effect of threshold value and priority of goals. Both cases are performed with two conditions depending on whether the least output requirement ($q_i \geq q_0, i = 1, \dots, 6$) is presented or not. The solutions are analyzed in the following.

5.1 Solutions with Given Weights

Solving the problem of the first case, the optimal solutions and purchasing policy are summarized in Tables 4 and 5. From

Table 4 the four total degrees of achievements are found to be 3.1003, 2.4118, 2.6407 and 2.1228. To observe those solutions we find that:

- (i). Those values of total degree of achievement (2.4118, 2.1228) with the least output are smaller than the values (3.1003, 2.6407) without the least output. This means that the increased constraints result in lower total satisfaction.
- (ii). When setting the unequal weights, as the achieved level of the most important goal (goal 3) increases from (0.9350, 0.9950) to (0.9700, 0.9950), the total achieved levels decreases from (3.1003, 2.4118) to (2.6407, 2.1228). The result implies that the decision maker can emphasize the importance of some specific goals by setting larger weight, while the total achieved level of all goals may not be retained.
- (iii). Those values of total output with the least output are smaller than the values without the least output.

Table 4. Optimal solutions with equal and unequal weights

Variables	Conditions			
	Equal weights and no special requirement	Equal weights and least output requirement	Unequal weights and no special requirement	Unequal weights and least output requirement
V : total degree of achievement	3.1003	2.4118	2.6407	2.1228
μ_1 : degree of achievement of goal 1	0.5000	0.5000	0.5000	0.5000
μ_2 : degree of achievement of goal 2	0.6653	0.3245	0.6178	0.3245
μ_3 : degree of achievement of goal 3	0.9350	0.9950	0.9700	0.9950
μ_4 : degree of achievement of goal 4	1.0000	0.5923	1.0000	0.5923
G_1 : value of goal 1 achieved	7.0000	7.0000	7.0000	7.0000
G_2 : value of goal 2 achieved	56.6941	63.5098	57.6441	63.5098
G_3 : value of goal 3 achieved	51.3000	50.1000	50.6000	50.1000
G_4 : value of goal 4 achieved	1200.0000	914.6428	1200.0000	914.6428
q_1 : daily output of part 1	0.0000	100.0000	186.6667	100.0000
q_2 : daily output of part 2	0.0000	100.0000	0.0000	100.0000
q_3 : daily output of part 3	0.0000	237.8504	0.0000	237.8504
q_4 : daily output of part 4	186.6667	276.7924	0.0000	276.7924
q_5 : daily output of part 5	1013.3333	100.0000	1013.3333	100.0000
q_6 : daily output of part 6	0.0000	100.0000	0.0000	100.0000

Table 5. Optimal purchasing policy with equal and unequal weights

Purchasing policy	Conditions			
	Equal weights and no special requirement	Equal weights and least output requirement	Unequal weights and no special requirement	Unequal weights and least output requirement
Type A equipment	$y_{11}^* = 1$ $m_{11}^{**} = 1$	$y_{11} = 1$ $m_{11} = 1$	$y_{11} = 1$ $m_{11} = 1$	$y_{11} = 1$ $m_{11} = 1$
Type B equipment	$y_{23} = 1$ $m_{23} = 1$	$y_{22} = 1$ $m_{22} = 1$	$y_{23} = 1$ $m_{23} = 1$	$y_{22} = 1$ $m_{22} = 1$
Type C equipment	$y_{34} = 1$ $m_{34} = 3$	$y_{32} = 1$ $m_{32} = 2$	$y_{34} = 1$ $m_{34} = 3$	$y_{32} = 1$ $m_{32} = 2$
Type D equipment	$y_{42} = 1$ $m_{42} = 1$	$y_{42} = 1$ $m_{42} = 2$	$y_{41} = 1$ $m_{41} = 1$	$y_{42} = 1$ $m_{42} = 2$
Type E equipment	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$

* denotes that the k -th kind of j -th type equipment is purchased.

** denotes that the purchasing number of equipment of the k -th kind of j -th type equipment.

For the purchasing policy, the values of m_{jk} 's in Table 5 show that more type C and D equipments are purchased than other types of equipments. By examining the operating time needed for each part as listed in Table 3, this is reasonable and

can avoid the bottle-neck process. In Table 2, the results indicate that the operating times processed by equipment types C and D are generally longer than those by the other equipment types.

From the above discussion, it can be seen that this FGP model can implement the purchasing task and analyze the finishing situation of all considered goals. Then the other case is examined in the next subsection.

5.2 Solutions with Threshold Value and Priority for Goals

Here the problem with fixed threshold value and priority of goals is solved and the solutions and purchasing policy are summarized in Table 6 and 7 respectively. Some of the results shown in Table 6 are described in the following.

- (i). Firstly, both the minimum grades of satisfaction of the total purchasing cost and total output are assumed to be 0.8. Then from Table 6, the goal of the total purchasing cost is satisfied by 93.5 and 90 percentage, while the goal of the total output is fully achieved.
- (ii). On the other hand, when ordering the goals with priorities and ranking those grades of satisfaction by $\mu_3 > \mu_4 > \mu_1 > \mu_2$, the solutions are obtained in columns 4 and 5 of Table 6. Comparing columns 4 and 5 in Table 6 with those in Table 4, the results show that the values of achieved level of each goal with priority structure agree with the decision maker’s preemptive while using weights value does not (when seeing column 4 in Table 4, $\mu_1 < \mu_2$ for the setting the condition $w_1 > w_2$).

When comparing the optimal purchasing policy in Table 7 with that in Table 5, the difference is not significant, except for the kinds of types B and C equipment.

From the demonstration presented above, the project manager could execute the equipments-purchasing problem for an FMC using our FGP model. With the attributes of equipments and strategic target values and bound value of goals, the optimal purchasing policy and the satisfying level of those conflicting goals can be obtained.

Table 6. Optimal solutions when threshold and priority are applied

Variables	Conditions			
	With threshold values ($\mu_3 > 0.8, \mu_4 > 0.8$) and no special requirement	With threshold values ($\mu_3 > 0.8, \mu_4 > 0.8$) and least output requirement	With priority $\mu_3 > \mu_4 > \mu_1 > \mu_2$ and no special requirement	With priority $\mu_3 > \mu_4 > \mu_1 > \mu_2$ and least output requirement
V : total degree of achievement	3.1003	2.3628	2.6107	2.4119
μ_1 : degree of achievement of goal 1	0.5000	0.0000	0.5000	0.5000
μ_2 : degree of achievement of goal 2	0.6653	0.4628	0.3207	0.3245
μ_3 : degree of achievement of goal 3	0.9350	0.9000	0.8950	0.9950
μ_4 : degree of achievement of goal 4	1.0000	1.0000	0.8950	0.5923
G_1 : value of goal 1 achieved	7.0000	8.0000	7.0000	7.0000
G_2 : value of goal 2 achieved	56.6941	60.7441	63.5862	63.5098
G_3 : value of goal 3 achieved	51.3000	52.0000	52.1000	50.1000
G_4 : value of goal 4 achieved	1200.0000	1200.0000	1126.5000	914.6429
q_1 : daily output of part 1	0.0000	100.0000	0.0000	100.0000
q_2 : daily output of part 2	0.0000	100.0000	0.0000	100.0000
q_3 : daily output of part 3	1200.0000	100.0000	1126.5000	414.6429
q_4 : daily output of part 4	0.0000	100.0000	0.0000	100.0000
q_5 : daily output of part 5	0.0000	528.0734	0.0000	100.0000
q_6 : daily output of part 6	0.0000	271.9266	0.0000	100.0000

Table 7. Optimal purchasing policy when threshold and priority are applied

Purchasing policy	Conditions			
	With threshold values ($\mu_3 > 0.8, \mu_4 > 0.8$) and no special requirement	With threshold values ($\mu_3 > 0.8, \mu_4 > 0.8$) and least output requirement	With priority $\mu_3 > \mu_4 > \mu_1 > \mu_2$ no special requirement	With priority $\mu_3 > \mu_4 > \mu_1 > \mu_2$ and least output requirement
Type A equipment	$y_{11}^* = 1$ $m_{11}^{**} = 1$	$y_{12} = 1$ $m_{12} = 1$	$y_{12} = 1$ $m_{12} = 1$	$y_{11} = 1$ $m_{11} = 1$
Type B equipment	$y_{23} = 1$ $m_{23} = 1$	$y_{23} = 1$ $M_{23} = 1$	$y_{23} = 1$ $m_{23} = 1$	$y_{22} = 1$ $m_{22} = 1$
Type C equipment	$y_{34} = 1$ $m_{34} = 3$	$y_{34} = 1$ $M_{34} = 3$	$y_{31} = 1$ $m_{31} = 3$	$y_{32} = 1$ $m_{32} = 2$
Type D equipment	$y_{42} = 1$ $m_{42} = 1$	$y_{42} = 1$ $M_{42} = 2$	$y_{41} = 1$ $m_{41} = 1$	$y_{42} = 1$ $m_{42} = 2$
Type E equipment	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$	$y_{52} = 1$ $m_{52} = 1$

* denotes that the k -th kind of j -th type equipment is purchased.

** denotes that the purchasing number of equipment of the k -th kind of j -th type equipment.

6. RESULTS AND DISCUSSION

To build an FMC successfully, an effective and useful decision making tool for equipments purchasing policy is crucial. When the four conflicting and fuzzy goals are referred, a fuzzy goal programming has been suggested to meet as closely as possible the four conflicting goals in this paper.

In order to demonstrate the proposed model, an industry example is given in Section 5. After the decision makers have the referred data such as historical cost or company’s strategy, they can get these weights by using group decision techniques such as the Delphi method and AHP, etc. While the goals are asked to be achieved with minimum values or there is priority among those goals, the purchasing policies can be gotten (see table 7). Therefore, with those attributes of equipments, strategic target and bound values of goals, the decision makers can apply our model to obtain the purchasing policy and the attained percentage of each individual goal. Furthermore, the decision makers do not need to worry about how to obtain a deterministic target value or deterministic weight value due to incomplete data or linguistic representation. On the basis of the results of the demonstrated example, we conclude that the suggested model is suitable for the equipments purchasing problem under the assumptions we defined.

On the other hand, this model can provide decision makers more decision flexibility. They can adjust the priority between the goals and the threshold value of goals if the solutions are not satisfied. Sometimes a rearrangement of the weights or priority is required if the company strategy changes. It is not difficult, however, to rerun the mixed integer programming model that we construct for the equipments-purchasing problem of an FMC by OR software such as the LINDO, LINGO and GINO etc. And for a machine selection or equipments purchasing problem, this model proposed in this paper is not the only method. Some other models such as Dynamic Fuzzy Goal Programming can be tried.

In conclusion, the FGP model proposed in this paper provides a useful decision tool for equipments purchasing in the purchasing stage to configure an FMC.

7. REFERENCES

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