International Journal of Industrial Engineering, 32(1), 1-15, 2025

OPTIMIZATION OF INVENTORY REPLENISHMENT UNDER ASYMMETRIC STOCK-OUT AND INVENTORY HOLDING COSTS

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Perishable products are an essential part of commerce. Shelf-life characteristics are usually not modeled in traditional inventory models. This study proposes an inventory replenishment model for perishable products with an asymmetric cost structure for holding and stock-out costs. The modeling phase involves the shelf-life characteristics of products. Shelf life is essential due to sustainability concerns, costs, and service levels due to perished products. In contrast to classical safety stock models, where stock-out costs increase linearly, the proposed model utilizes incrementally increased fixed costs for holding costs in a conflicting cost structure. It incorporates the shelf-life of the products, calculates the probability of perishing, and formulates accurate waste and total costs using an asymmetrical cost structure. The model is applied to a real dataset to assess the performance and compare it with the traditional approach. The performance of the proposed model is better, with a total cost reduction of 45.33%. Additionally, the model demonstrated a 17.21% increase in service level. The sensitivity analysis further underlined the robustness of the proposed model across various demand scenarios and shelf-life conditions. The main research gap addressed by this study is the lack of consideration for shelf-life characteristics and asymmetric cost structures in traditional inventory models. By integrating these factors, this research provides a more accurate and cost-effective approach to inventory management for perishable products, enhancing sustainability and service levels. This study's findings can help businesses optimize inventory strategies, reduce waste, and improve operational efficiency.

Keywords: Inventory model; Perishable; Asymmetric Costs; Optimization; Metaheuristics.

(Received on March 27, 2024; Accepted on September 25, 2024)

1. INTRODUCTION

Perishable products have drawn the attention of recent academic studies as they cover an essential part of commerce. 54% of total store sales and 57% of the total inventory of the supermarkets are perishable products (NSSS, 2005). In line with the study, about 15% of perishable products are lost in supermarkets due to spoilage and damage (Ferguson & Ketzenberg, 2005). In a broader view, products that deteriorate in price, such as electronics and fast fashion, are not covered in the given numbers. The importance of perishable products increases with this additional aspect. Another area affected by such perished products is sustainability. In terms of sustainability, products that perish or lose value over time are concerns for the environment. Valuable resources that are a part of the product, such as workforce, raw materials, water, and similar resources, are lost when products perish. Although recycling is an option to benefit from perished products, deteriorated products are still a waste of resources. Also, increased costs, lower service levels, relevant costs of stock-out situations, and expedited orders to cover such stock-out positions are costs associated with perishable products. Effective management of inventory would have multiple objectives. Among others, customer satisfaction is associated with service level and revenue, which are essential goals (Ovezmyradov & Kurata, 2019). As given in the literature review section, with the increased concerns about perishable products, much research focuses on perishable products.

Safety stock is coupled with traditional approaches for inventory replenishment to overcome variability and achieve service levels for an inventory system. The goal of safety stock is to allocate some resources in terms of inventory to counter unexpected demand or lead-time variability. Customer satisfaction is critical to the sustainable performance of a company. Customers should not be affected by changes, as their primary concern is to have the product ready at the desired time. Safety stock levels will increase with high variability coupled with high service levels. Increased stock levels would increase the possibility of products perishing, particularly those with short shelf lives. Perished products would lower the inventory level

DOI: 10.23055/ijietap.2025.32.1.10035 **ISSN 1072-4761**

than expected and increase the number of perished products. This perishing would increase the total cost and lower the realized service level. This two-sided effect would undermine the company's customer satisfaction and financial performance while undermining sustainability.

In the classical models of safety stock, when costs are considered, the stock-out costs increase linearly. The study assumes that stock-out costs occur asymmetrically. The stock-out costs increase in a quadratic function in line with stock-out situations, and also, in a conflicting cost structure, the proposed model uses incrementally increased fixed costs of holding costs. Holding costs will be incremental, and a fixed cost will occur based on some predetermined limits. This fixed cost is modeled based on the additional costs of opening a new warehouse, hiring additional workers, and renting a new forklift when a specific threshold is achieved. As a result, the cost will be asymmetric.

The proposed study aims to optimize inventory replenishment decisions based on asymmetric costs. The proposed model considers shelf-life, calculates the probability of perishing, and formulates the accurate waste and total cost using an asymmetrical cost structure. Total cost covers a realistic approach, incorporating a fixed cost approach for holding costs and increased stock-out costs—the proposed model is based on the optimization of the asymmetrical cost-based inventory optimization model (ACSIOM). Comparisons are shown in a real case study using data received from a company by employing the proposed approach and traditional (R, S) model with safety stock.

The study aims to optimize the order quantities under asymmetric stock-out and inventory costs using particle swarm optimization, covering the new order replenishment decision model based on asymmetric stock-out costs, incremental fixed costs, and variable costs. The objective function is to minimize the total costs. The application using a total cost structure that minimizes the total cost may perform better than the traditional approach of (R, S), period-or-review, and replenishment up to the level with additional safety stock. Based on the details of the proposed model, this research paper's main contributions are as follows:

- The study uses asymmetrical approaches to assess the effect of lead times, risk period, variability, and shelf-life on the total cost.
- The study uses an asymmetrical approach for stock-out costs based on quadratically increasing costs due to service level reduction.
- An incremental approach is used for holding costs. The costs cover fixed costs and variable costs.
- Comparing the traditional model with the proposed model in terms of total cost is demonstrated by a real-world example.
- Employment of a metaheuristic known as particle swarm optimization to optimize the stock levels based on risks
- Complying with the service level while simultaneously reducing the amount of inventory and the overall cost is a difficult task to accomplish. These are the functions that the model that has been proposed aims to fulfill.

The sections of the study are as follows. The literature review is given in section 2 regarding the topics associated with safety stock, inventory costs, developed models, and associated studies. The methodology in section 3 covers the theoretical formulation of the proposed model ACSIOM. The model's underlying assumptions, formulations, and relevant details are given in the same section. The proposed model is applied in a real-life case in the application in section 4. The application is performed in the mentioned section, and the proposed and traditional models are compared. Additionally, a summary of the findings of this study, as well as its limitations and potential areas for further investigation, are presented in the conclusion.

2. LITERATURE REVIEW

The literature review is relevant to the proposed model in the following sub-sections. The first sub-sections are associated with perishable products, as the proposed model covers the products that deteriorate over time. The second sub-section covers the asymmetric optimization models and the use of metaheuristics in asymmetric optimization. In the last sub-section, the literature review is conducted on applying asymmetric optimization for inventory management of perishable products.

2.1 Perishable Products

Perishable products inventory management is complex and deserves intense management for a sound application (Holmström, 1997). Inventory mismanagement caused a loss of millions of dollars in European grocery stores due to spoiled products after their shelf-life (Beck, 2004). This aspect of effective inventory management is realized, and as a result, studies emerged to model the perishability of products. To model the perishability of products, exponentially decaying perishability is modeled (Ghare & Schrader, 1963). Different studies have also extended the perishability characteristics of products. Covert and Philip (1973) modeled the perishability of products as subject to the continuous deterioration of perishable items. The accurate inventory management of perishable products may become complicated when the inputs and objectives are extensive. Expert systems are implemented that successfully control distribution operations according to weather conditions and can significantly reduce energy consumption and costs (Accorsi *et al.*, 2017). The importance of perishable products also drew the attention of researchers for literature reviews (Chaudhary *et al.*, 2018; Goyal & Giri, 2001; Li *et al.*, 2010; Sharma, 2016). The primary outcome of the studies underlines the importance of perishable products and the necessity of integration for effective inventory management.

2.2 Asymmetric Optimization

Asymmetric costs exist in business life. Such models provide an accurate representation of real-life scenarios. To name a few, the cleaning time in the dye industry. The switch from light to dark colors and vice-versa are different. It is shorter for the first case and much longer for the second alternative due to the risk of contamination. Similarly, in a well-known case of a traveling salesman problem, travel from one location to another may not be equal in terms of time or cost. In terms of time, due to traffic regulations and traffic density, the time may not be equal.

Similarly, the cost of transportation is sensitive to different inputs such as market conditions, inclined and relevant fuel consumption, and taxes. As a result, asymmetric optimization is an important aspect of recent studies. Multiple studies focused on accurate modeling and solving asymmetric problems (Odili *et al.*, 2021; Qu *et al.*, 2021).

In a recent study, asymmetric inventory investment is optimized in response to sales changes. In the study, asymmetric inventory investment provides valuable information for accurately forecasting future sales growth. The decision makers' sales forecasts are positively associated with the asymmetry involved in the modeling. Also, in the model, inventory information is important in understanding managers' resource adjustment and utilization decisions that have implications for forecasting future demand. The study provides evidence of the incremental usefulness of asymmetric inventory investment (Hwang *et al.*, 2021). To the best of our research, studies in asymmetric optimization of inventory models are limited. The limited number of studies is important to underline the requirement of asymmetric costs in the proposed model.

2.3 Optimization via Metaheuristics in Inventory Management

Metaheuristics are widely used optimization methods to solve problems. Exact methods guarantee that the found solution is the best in a set of feasible solutions. Although metaheuristics models do not guarantee the solution is the optimum solution, the time needed to solve using metaheuristics is acceptable for Np-Hard or Np-Complete problems. As a result, metaheuristics are also widely used in inventory models.

Recent studies employed metaheuristics in integrated models. Total cost minimization is a widely used objective function for inventory problems. The model can be complex and challenging to solve using exact methods. Metaheuristics are used widely in operations research. As a result, they have wide applications. To name a few examples, total cost optimization due to lost sales, cranes in a port, and cold-chain logistics are examples of optimization using GA (Liu & Li, 2024; Ng *et al.*, 2006; Xu *et al.*, 2019). The problem involves the simultaneous application of assortment, shelf planning, and perishable products. The metaheuristics used for this problem is a GA approach (Sajadi & Ahmadi, 2022). A recent study employed a metaheuristic model in optimizing inventory management under stochastic demand (Tan *et al.*, 2024). The study employed a grey-wolf optimization model to solve the problem. These studies and the literature review show that metaheuristics are widely used for optimizing inventory models.

Pirabán *et al.* (Pirabán *et al.*, 2019) evaluated the studies published between 2005 and 2019. They identified the methods used to optimize the management of bloodstock inventory in healthcare facilities, identifying that the main metaheuristics used by the reference papers are variable neighborhood search (VNS) (Hemmelmayr *et al.*, 2010; Zahiri *et al.*, 2018), simulated annealing (SA)(Eskandari-Khanghahi *et al.*, 2018; Rabbani *et al.*, 2017), self-adaptive differential evolution algorithm (SDE) (Zahiri *et al.*, 2018), genetic algorithm (GA) (Ayer *et al.*, 2018), and symbiotic organism search algorithm (SOS) (Govender & Ezugwu, 2018). Another corresponds to an experiment by researchers from the Universitat Oberta de Catalunya – Spain, in collaboration with researchers from the Univ. Southampton - England; Boston Univ. – USA; and Euncet Business Sch, Terrassa, Spain, who works on the agri-food supply chain, with a problem known in the literature as the perishable inventory routing problem (PIRP) with stochastic demands. In order to find a solution, Onggo *et al.* (2019) modeled a mixed integer program and proposed a heuristic algorithm to solve it. This algorithm incorporates Monte Carlo simulation into an iterated local search. According to the results of the experiments, the proposed algorithm can enhance the initial solution while maintaining reasonable computation times. Consequently, the researchers discovered that it was simple to implement and extend to other domains where a multi-period PIRP with stochastic demands might be encountered.

Another inventory-related problem is one of the main challenges facing retail units, which is determining order quantities for different types of products, each with a specific expiration date, so that the system cost, including the cost of shortages, is minimized. In search of a solution to this problem, researchers from Iran, France, and Denmark collaborated to model a new multi-product, multi-period replenishment problem for a warehouse management system based on First-Expired-First-Out (FEFO). Sazvar *et al.* (Sazvar *et al.*, 2016) proposed a non-linear model that is first converted into a linear model and then solved by applying two evolutionary algorithms, the GA and PSO, in which the design parameters are defined using the Taguchi method. According to Sazvar *et al.* (2016), the computational outcomes illustrated the suitability of the suggested model for handling perishable goods. Furthermore, the efficiency of the proposed metaheuristic was confirmed by comparing the findings.

Product recovery and reuse, according to North Carolina State University researchers, is an effective strategy that aids businesses in addressing sustainability's economic and environmental aspects at the same time. In actuality, managing the inventory of reusable goods in a network that includes several products, several merchants, and a single supplier is a brand-new challenge. In this sense, Sadeghi *et al.* (2023) equate the problem using a mathematical model of non-linear programming. The researchers use the grey wolf optimizer (GWO) and the whale optimization algorithm (WOA) as two new metaheuristics. Their performance is validated by the exact sequential quadratic programming (SQP) algorithm. The Taguchi method of experiment design is used to calibrate the parameters of the algorithms. A detailed analysis is performed by solving numerical problems at scales ranging and applying numerous comparison metrics. The results analysis reveals a substantial disparity among the algorithms, with GWO exhibiting superior performance in resolving the given problem. Additionally, both algorithms perform well in the solution space search.

To the best of our research, some studies aim to optimize inventory replenishment under asymmetric cost assumptions, as given in the literature review. The problem is extended to cover real-life cases with the coverage of shelf-life. Perishability is an essential part of sustainability concerns. A model is proposed, and its applicability is analyzed under real-life cases.

2.4 Particle Swarm Optimization

As given in 2.3, metaheuristics have been widely used for optimization since their introduction, and different methods have been used. Some examples of metaheuristics are simulated annealing, genetic algorithms, particle swarm optimization, and tabu search. PSO was developed in 1995 based on the behavior of swarms (Kennedy & Eberhart, 1995). The model uses a swarm of particles representing a candidate solution. Metaheuristics are widely used to solve different types of problems, and PSO is a promising model for optimization (Shami *et al.*, 2022). This simplicity allows the wide use of PSO for optimization problems.

Due to its ease of understanding and applicability, it is applied to different models. PSO can be integrated into hybrid approaches. A recent model using type-2 fuzzy sets is integrated with PSO for predicting problems (Mai *et al.*, 2021). A similar hybrid approach is used for the Multi-Criteria Inventory Classification problem (Cui *et al.*, 2021). The study showed the high performance of the hybrid approach with a numerical analysis. PSO is applied in a stock portfolio selection model. The portfolio analysis used an integrated fuzzy approach with PSO (Narang *et al.*, 2022). This subsection shows that PSO is a novel method to optimize the problems. Also, flexibility in the application allows for its wide use in inventory management, as in the proposed study.

The research gap of the proposed work lies not only in the methodological approach but also in the incorporation of new assumptions often overlooked in previous research. These assumptions make our model more practical and applicable to real-world scenarios in perishable inventory management. The study identified and addressed the following research gaps in the proposed study that aim to cover according to the literature review. From the methodological approach, the following research gaps are aimed to be fulfilled.

Incorporation of Shelf-life Characteristics: Traditional inventory models typically assume infinite or constant shelf-life for products. Our model, however, explicitly incorporates the varying shelf-life characteristics of perishable products. This inclusion allows for a more accurate estimation of waste and total costs, reflecting real-world conditions more closely.

Asymmetric Cost Structure: Most existing models assume symmetric cost structures, where the costs of holding inventory and stock-outs are treated similarly. Our model introduces an asymmetrical cost structure, where holding costs and stock-out costs are not only different but also vary incrementally. This approach captures the true cost implications more effectively, especially when stock-outs lead to significantly higher penalties than excess inventory.

The study's goal is to propose a methodological approach to cover the gap and propose practical improvements and goals. They can be summarized as follows;

Optimizing Inventory Levels: Our model optimizes inventory levels more accurately by accounting for product shelf-life and asymmetric cost structures. This optimization reduces holding costs and stock-out occurrences, as our case study results demonstrate.

Reducing Waste: The model's ability to predict the probability of perishing products and integrate this into the cost calculations helps in minimizing waste. This reduction is significant for sustainability and cost-efficiency in inventory management.

3. PROPOSED MODEL

The details of the proposed model will be given in the following section. The model aims to optimize order quantities to minimize the total cost. The model employs perishable products based on expected demand and shelf-life. Total cost is calculated based on asymmetric cost structure and costs associated with perished products. The aim is to improve the traditional models that employ lead time and demand variations and make calculations based on the traditional approach of the (R, S) model. (R, S) is a control system also known as a periodic review or order-up-to-level. Periodic review is denoted as R, and S represents the level of inventory that should be replenished in each review period. The main goal is to optimize the order data to calculate the model's total cost structure. In the sub-sections of Section 3, the details of the proposed model will be given. The following subsection gives general information and assumptions of the model. In subsection 3.2, the model is explained.

3.1 General Information of the Model and Assumptions

The objective of the proposed approach is to minimize the total cost covering holding costs, asymmetric stock-out costs, and perishable product inventories in an inventory system. The following are the model's underlying presumptions. A two-stage supply chain with suppliers and sellers is used in the model. There are no intermediary steps in the supply chain. The suggested model's goal is to minimize the total cost. The supply is not constrained as a model component, and the lead times are consistent with the model. The following are some of the assumptions and notations that were considered when developing the model. The following assumptions and notations are utilized in the study that Muriana (2016) conducted. The following assumptions are used for the proposed model.

- The object's shelf-life is deterministic, constant, and equal to p.
- *rp* represents the risk period. The risk period equals the sum of the lead time (lt_x) and review period (rv_x).
- The first-in-first-out principle (FIFO) is applied in the application.

The notations for the proposed model are given below.

σ_{rpx} : The standard deviation of demand in the risk period for product x
σ_{slxy} : The standard deviation of revised service level for product x per period y
$\mu_{sl_{xy}}$: Arithmetic mean value of revised service level for product x per period y
C_{soxy} : Stock-Out cost for product x in period y
d_{xy} : The demand for product x in period y
h_{xy} : The inventory holding cost per unit value per period y for product x
fcs_x : Fixed cost of storage for product x
fcv_{ix} : Incremental Threshold for Warehousing Fixed Cost for product x
k_x : Safety factor for product x
k_{px} : Disposal Cost coefficient of product x
k_{sx} : Storage cost coefficient for product x
k_{so} : Stock-out cost coefficient based on value
i_x : Fixed storage threshold values for product x
lt_x : Lead Time for product x
j : Inventory threshold level indices
n : Number of products
$P(d_{xy})$: Probability of the demand for product x in period y
$P_x(sh)$: The average inventory of product x
$P_{sl}(x)$: Probability of x during shelf-life
rp : Risk period
rv_x : The review period for product x
oh_{xy} : On-hand inventory of product x in period y
Q_{xy} : Order quantity of product x in period y
s_{xy} : Sales quantity of product x in period y
sl_x : Service level for product x
sl_{rx} : Service level revised for product x
ss_{px} : Stock perished for product x
ss_x : Safety stock for the product x
ss_{xy} : Safety stock for the product x per period y

t : Time unit
TC : Total cost
TC_f : Total financial cost of inventory
TC_h : Total holding cost
TC_s : Total storage cost
TC_p : Total cost of perished products
TC_{px} : Total cost of perished products for product x
TC_{so} : Total cost of a stock-out
TS : Total inventory
so : Stock-Out
x : Index of products
v_x : Cost of the item x
y : Indice of period y

Products perish after the given shelf-life. The higher inventory level will increase the probability of perishing. The higher inventory level represents a higher probability of having lower sales than inventory on hand. Having total sales lower than the on-hand inventory of products with a limited shelf-life increases the quantity of perished products. Based on this information, the amount of perished products is proportional to the amount of inventory on hand and shelf-life.

Safety stock will be calculated according to Equation (1) and Equation (2). The safety stock is calculated based on the safety stock factor multiplied by the standard deviation of demand during the risk period.

$$ss_x = k_x * \sigma_{rpx} \tag{1}$$

where
ss_x = safety stock for the product x
k_x = safety factor of the service level for product x
σ_{rpx} = standard deviation of demand during rp for product x

where

$$rp_x = \mathrm{lt}_x + rv_x \tag{2}$$

Equation (1) shows that safety stock increases with the variation of demand, lead time, and arithmetic mean of lead time and service level. Service level denotes a specified probability of no stock-outs per replenishment cycle (Silver *et al.*, 1998). The service level will be the z-score in statistics. Details of the safety stock are given in a recent study (Rosado *et al.*, 2016). The classical approach for safety stock mainly considers variable lead time and demand. Lower sales increase the likelihood of products perishing as the model does not consider shelf-life. The short shelf-life also has another impact on inventory management and its objectives. The quantity of perishable goods decreases the amount of usable safety stock after the shelf-life is reached and products perish, which negatively impacts customer service quality (Silver *et al.*, 1998). Among the primary inputs that the ACSIOM takes into consideration are the service level, risk period, standard deviation, and demand. The proposed model includes a reassessment of this matter and an updated calculation for the safety stock.

The decision model that is proposed in Section 3.2 takes into account the quantity of products that have expired while being considered. Because the calculation of total cost also serves as a model for safety stock, the additional cost is crucial (Silver *et al.*, 1998). The proposed model that considers waste quantity and value is presented in Section 3.2. The products perish when the total demand during shelf-life exceeds the inventory at the beginning of period y. Total sales are equal to demand if inventory is greater or demand. The relevant equations are given in Equation (3).

$$s_{xy} \ = \begin{cases} \mathrm{d}_{xy} & \text{if } \mathrm{oh}_{xy} \geq d_{xy} \\ oh_{xy} & otherwise \end{cases} \tag{3}$$

As given in Equation (4), the equation represents that the satisfied demand during the shelf-life can not exceed the inventory level. When the total demand is lower than the order quantity, the products perish at the end of a shelf-life. Based on this calculation, stock-out or perished products may occur. Based on Equation (4) and Equation (5), stock-out probability can be calculated. Equation (4) represents the stock that will perish as the shelf-life is exceeded. $d_{xy}(\sum_x \sum_y P(d_{xy}))$ this model represents the multiplication of all probability of a demand occurrence multiplied by the quantity. Based on this

Equation (4), an expected demand is given. Equation (5) represents the probability that total demand exceeds the revised inventory level. We assume that based on normal distribution, a demand may occur.

$$ss_{px} = \sum_{y}^{rp+y}(Q_{xy} - d_{xy}(\sum_x \sum_y P(d_{xy})) \quad (4)$$

$$P_{sl}(so) = P(\sum Q_{xy} - ss_{px}) < d_{xy})) \quad (5)$$

Inventory on hand depends on sales, order quantities, and safety stock. Also, perished inventory also affects the total inventory level. Based on these assumptions, the inventory level can be calculated in Equation (6).

$$oh_{xy} = \sum_{1}^{y-1}(Q_{xy} - s_{xy}) + ss_{xy} \quad (6)$$

The products that deteriorate will decrease the available quantity. Therefore, the shelf-life of products affects the stock-out probability of a product. Storage costs occur when there is more on-hand inventory than the demand. On-hand inventory is also affected by the quantity of perished items. Based on this assumption, the average on-hand inventory will be as given in Equation (7)

$$P_x(sh) = \begin{cases} \frac{Q_{xy}-(Q_{xy}-ss_{px}-\sum_{y=1}^{p} d_{xy}*P(d_{xy})}{2} \text{ if } Q_{xy} - (Q_{xy} - ss_{px} - \sum_{y=1}^{p} d_{xy} * P(d_{xy}) > 0 \\ 0 \quad otherwise \end{cases} \quad (7)$$

The calculations are crucial for an accurate cost analysis. As seen, the perished quantity affects the stock-out and on-hand inventory levels.

3.2 Model with the Objective of Minimization of Total Cost

An approach to inventory replenishment decision-making is to compute quantities to reduce overall costs. The function that needs to be minimized is the total cost. Ordering costs, storage costs, shortage costs, perishability-related costs, and waste disposal are typical costs that add to the overall cost. Generally, ordering costs are fixed, one-time expenditures associated with placing an order, regardless of quantity. Instead, the costs are predetermined and comprise activities such as order form processing, merchandise receiving, inspection, investigation of unanticipated events, and vendor invoice processing (Silver *et al.*, 1998). Holding costs are the variable expenses associated with ordered quantity and inventory level. Leading costs linked with inventory determinations for perishable commodities are likewise reflected in perishable costs. These charges are accrued when a product's shelf-life is exceeded. Disposal generates additional costs. Waste disposal is governed by stringent legislation in most countries. As a result, disposal is delegated to specialist companies at an additional expense. The disposal actions stated above are associated with the cost of waste disposal.

The remainder of the section proposes a model to accurately determine the overall cost subject to different inventory decisions and their effects on costs such as disposal of perished products. By balancing the cost of inventory and the costs associated with shortages and perishable goods, minimizing the total cost lowers the total cost of inventory, including safety stock. Holding costs are associated with the on-hand inventory. In the proposed model, holding costs cover financial and storage costs. Equation (8) represents the total holding cost. The details of the total holding cost are given afterward.

$$TC_h = TC_f + TC_s \quad (8)$$

In Equation (9), the total cost of finance is proportional to the financial value of inventory. The higher inventory requires higher financial costs due to interest paid for financing. Opportunity costs affect the total finance cost if no additional financing is needed. It is also important to note that in addition to the total cost of finance, the opportunity cost of the money tied to the inventory is also included in the total cost of finance calculation. In traditional models, the financial cost is a part of the inventory holding cost with a linear cost structure. The proposed model employs a detailed explanation of the cost structure. The financial cost is a part of the holding cost relevant to the inventory's financial value.

$$TC_f = \sum_x \sum_y P_x(sh) * h_{xy} * v_x \quad (9)$$

Equation (10) gives the total storage cost associated with the stock quantity. This assumption is important as holding cost is associated with inventory value but not quantity in traditional models. The quantity is used mainly when the cost is variable for external service providers. The typical assumption is that cost is associated with the space employed, and costs are variable. It is represented with a coefficient proportional to on-hand inventory. Based on our assumption, the cost of storage increases incrementally. In literature, holding costs are usually depicted as variable costs associated with the inventory values. Companies that manage their warehouses have incremental cost increases when total inventory quantity surpasses certain thresholds. Due to the increase in warehousing requirements, these costs occur due to the additional workforce needed to manage the inventory and additional machinery, such as forklifts or related investments. Based on these assumptions, Equation (10) gives storage costs.

$$TC_S = \sum_x \sum_y (P_x(sh) * h_{xy} + \mathrm{fcs}_x) \tag{10}$$

where

$$i_x = \left\lceil \frac{TS}{fcv_{ix}} \right\rceil$$

$fcs_x = fcv_{ix} * i_x$ $\quad\quad$ i_x=1,2,…j

The total inventory cost includes the anticipated expenses associated with unsold perishable goods during their designated shelf-life. Given that this is a probability value and not a deterministic one, the cost should be computed according to Equation (11) and Equation (12). Waste costs are included in perishable costs. As a simplification, the waste cost is incorporated into the k_{px}.

$$TC_{px} = \sum_x ss_{px} * v_x * (k_{px} + 1) \tag{11}$$

$$TC_p = \sum_x TC_{px} \tag{12}$$

The total cost of a stock-out is given in Equation (13). A customer order must be delivered. An unfilled order creates a financial loss due to lost sales and the long-term effects of disappointment. The higher inability to meet customer demands may cause long-term effects of dissatisfaction. According to research, a disappointed customer will spread this disappointment to 8 different people (Gore, 1996). Based on this data, the study assumes that higher dissatisfaction will quadratically increase costs. We assume that a dissatisfied customer will have increasing costs for each drop-in service level. The assumption is that the dissatisfied customer will likely inform other existing customers that potential customers are alike, affecting the total stock-out cost. The stock-out cost is proportional to the value of goods. Therefore, the cost coefficient is given as k_{so}. The primary assumption of our model assumes that the stock-out costs will increase quadratically when a service level is not achieved. As given in the literature review, stock-out costs increase sharply when a stock-out occurs, as given in the research (Anderson *et al.*, 2006). Based on the assumption and research, the stock-out cost is given in Equation(13) and Equation(14).

$$TC_{so} = \sum_x \sum_y C_{soxy} \tag{13}$$

$$C_{soxy} = k_{so} * v_x * (1 - sl_{rx})^2 \tag{14}$$

Thus, based on the aforementioned costs, the objective is to minimize costs associated with inventory, holding, and stock-outs. The total objective of cost is given in Equation (15)

$$Min\ (TC_h + TC_p + TC_{so}) \tag{15}$$

So, the objective of the model is defined as:

$$Min \sum_{x=1}^{n} \sum_{y=1}^{rp} (oh_{xy} * h_{xy} * v_x) + (oh_{xy} * k_{sx} + fcs_x) + (oh_{xy} * v_x * k_{px}) + (oh_{xy} * v_x) + (P(1 - sl_{rx})^2 * v_x * k_{so})) \tag{16}$$

The objective of the model is to minimize total cost while keeping the service level within acceptable limits. sl_x is service level, as a probability, that should be greater than or equal to zero and less than or equal to 1.

s.t.
$0 \leq sl_x \leq 1$
oh_{xy}, h_{xy}, k_{sx}, fcs_x, kp_x, $kso \geq 0$

The objective function is quadratic because of the asymmetric nature due to stock-out costs. Also, due to possible increases in product range, the problem can be solved within acceptable time limits in real life. The proposed model uses a metaheuristic model to solve the problem within acceptable time limitations. The objective function is quadratic in the model given in Equation (16). The quadratic model is an NP-hard problem (Garey & Johnson, 1979). Also, new orders can be given regularly as the demand or forecasts may change. The time limitation to manage the inventory that represents these changes limits the applicability of exact methods and supports using metaheuristics.

4. REAL-LIFE APPLICATION IN A COMPANY

This section aims to apply the suggested model to a distribution company case study. The proposed model is applied with a real-life dataset in the following sub-section. The data is provided by a distribution company focusing on selling chemical products. The items have different shelf-life and different values per product and selling quantities. In sub-section 4.1, the model is applied, and total cost analysis, actual service level, and perished quantities are calculated to analyze the performance of the proposed model. In sub-section 4.2, the proposed model is compared with the safety stock model based on service level and the safety stock model based on total cost using a traditional approach of linear stock-out costs performed. In the last sub-section, 4.3, a sensitivity analysis is performed by making deviations in the input to analyze the possible outputs of the proposed model. In 4.4, the results based on comparison and sensitivity analysis are elaborated.

4.1 Application of the Proposed Model

The proposed model is performed using data received from a distribution company. The data for 20 items are received. The items are classified as A in terms of ABC analysis. As a result, the products used for this study represent the most important items for the company's inventory management. The products are chemical, and they are subject to chemical decomposition. Based on this chemical decomposition, they have limited shelf lives with different values. The average shelf-life is 2.85 months, with a standard deviation of 0.66. The details of the products are not given in order to keep the information anonymized. All costs are in USD. The study used the following parameters for analysis in Table 1.

Table 1. Parameters for the Application

Parameter	Value
$\boldsymbol{h_{xy}}$	0.007
$\boldsymbol{fcs_x}$	715 USD
$\boldsymbol{k_{sx}}$	0.1
$\boldsymbol{k_{px}}$	500 USD
$\boldsymbol{k_{so}}$	3

h_{xy} that represents the holding cost of inventory. The value given is calculated based on the cost of the USD interest rate of the Eurobond for the three years. The monthly interest rate of the Eurobond calculates the value. Our assumption for the fcs_x is that the fixed cost of warehousing changes for the incremental values. The assumption is that the worker can be hired within the limitations, and the additional cost of the new worker is added in case of additional storage. The cost of an additional worker, given as fcs_x, denotes the conversion of the cost of a worker for the employer. On top of the fixed cost, there is also a variable cost associated with warehousing. k_{sx} represents this cost as a percentage of the value of goods. The associated value is calculated based on the feedback from the company. k_{px} that represents the cost of disposal, transportation, cleaning, and getting necessary approvals for perished products. The disposal of chemicals requires certified companies to pick up the goods, transport them to disposal centers, and eliminate goods according to safety regulations. The cost of k_{px} is calculated based on the official disposal cost by a certified company. k_{so} that represents the cost of the stock-out situation. The stock-out cost occurs when an insufficient product supplies the desired quantity or order. The rate of this cost is based on the company's qualitative assumption.

The optimization is performed using the given parameters. The optimization is performed using Matlab 2023a Software running on 16GB RAM with an I7 Processor. The Matlab embedded function for particle swarm optimization is used for optimization. The results of the model are given in Table 2 and shown in Figure 1. Max iterations of the optimization are set for 10000 to save time during optimization, and max stall iteration is set to 1000.

Table 2. Results of the ACSIOM Model

Parameter	Total Cost (USD)
TCf	9,453
TCfc	232,375
TCh	375,284
TCp	594,140
TCso	51,282
TCsv	133,456
TC	1,020,706
$\mu_{sl_{xy}}$	0.70
$\sigma_{sl_{xy}}$	0.13

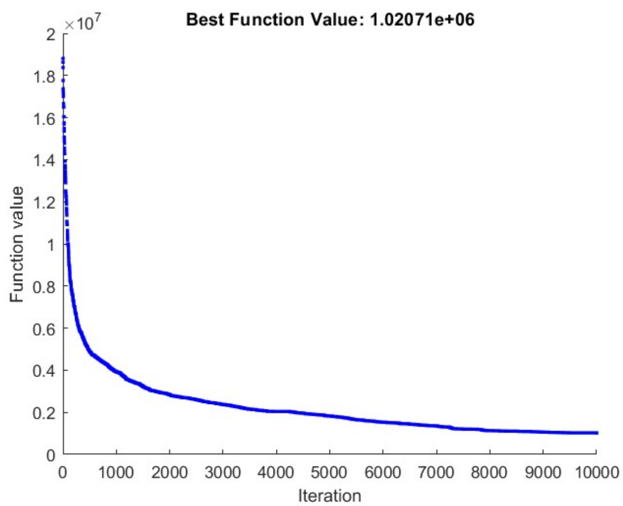

Figure 1. Cost Reduction per Iteration

4.2 Comparison of the Proposed Model with the Traditional Model

There are traditional approaches to inventory replenishment decisions. When there is variation in the inputs, additional safety stocks are calculated to overcome such variations. In order to assess the accuracy of our models, the ACSIOM model (R, S) is compared with an additional safety stock model. The inventory is replenished to an S level in every R period in the (R, S) control system defined as Periodic-Review, Order-Up-to-Level. S level is set to an average demand during R with additional safety stock. In order to overcome variations in demand, safety stock is also integrated into the traditional model. The safety stock formula is given in Equation (3).

As given in Table 3, the costs are lower by 45.33% in the ACSIOM model. The graphical representation is given in Figure 2. The comparison shows that the proposed approach has merit in calculating inventory values for replenishment inventory replenishment. The goal of the proposed ACSIOM model is to minimize the cost. The constraints are in lower numbers to represent the real business scenario where the service level can not be a negative value. As expected, the starting model met the constraints, and PSO-based metaheuristics met the criteria as they are also modeled to cover the mentioned constraints.

Table 3. Comparison of ACSIOM with the Traditional Approach

Parameter	ACSIOM (USD)	Traditional model (USD)	Comparison (%)
TCf	9,453	23,563	59.88%
TCfc	232,375	392,535	40.80%
TCh	375,284	748,747	49.88%
TCp	594,140	1,106,615	46.31%
TCso	51,282	11,501	-345.88%
TCsv	133,456	332,649	59.88%
TC	1,020,706	1,866,863	45.33%
$\mu_{sl_{xy}}$	0.70	0.60	-17.21%
$\sigma_{sl_{xy}}$	0.13	0.13	3.25%

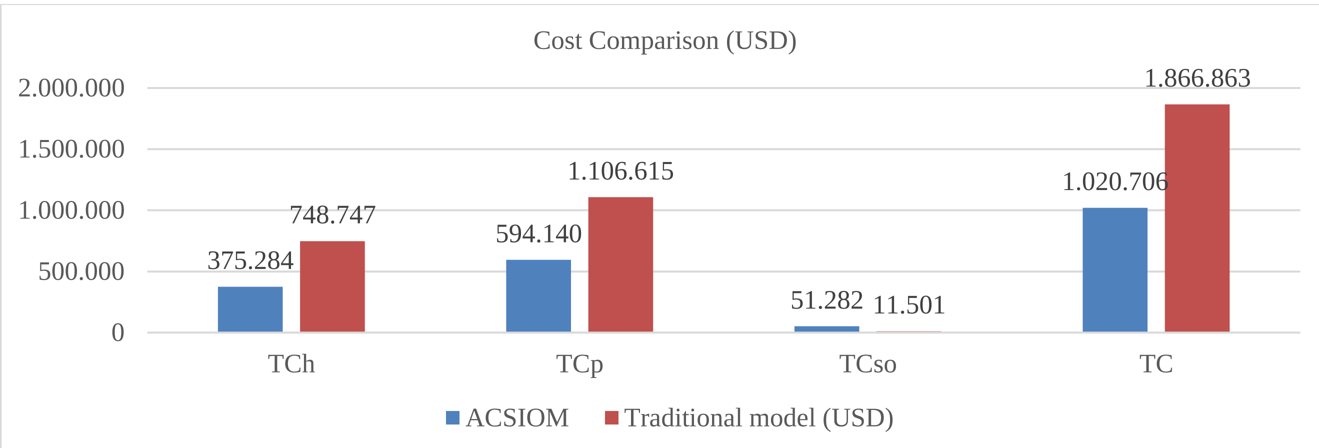

Figure 2. Cost Comparison of ACSIOM vs (R, S) Model

4.3 Sensitivity Analysis

A sensitivity analysis is performed to assess the robustness and behavior of the proposed model. As given in Table 4, the inputs are manipulated to assess the outcomes of the ACSIOM Model under changing parameters. The sensitivity analysis is performed on 4 scenarios and compared with the ACSIOM model.

The primary assumption is that shelf life is pivotal in the inventory model's effectiveness. Its importance increases when a short shelf-life increases the risk of perishability. Scenario 1 aims to analyze this assumption. Shelf life is reduced by 50% to increase the possibility of perishability. As shown below, the total cost increases dramatically in such a reduction. As given in Table 4, the cost increases by 84% vs. the ACSIOM results. The decrease in shelf-life increased the quantity of perished products. The costs increased as a result. These changes indicate that the model can accommodate higher waste costs without substantial cost escalation, providing some buffer for unexpected waste increases.

In Scenario 2, the cost of waste is increased by 100% without changing any other parameters. As expected, although not significantly, the cost increased by 4%. The model employs the changes in parameters as expected in Scenario 2. The increase in waste cost per MT increases the total cost as expected.

In Scenario 3, the stock-out cost is increased by 100%; the results are slightly different than Scenario 2. When the stock-out increases, due to the nature of asymmetric cost increases, the model aims to lower the cost by focusing more on increasing service level. As a result, the cost is similar, but the service level is increased. This outcome suggests that the model prioritizes service levels when stock-out costs rise, effectively balancing cost and service level to minimize the more severe penalties associated with stock-outs.

Also, in Scenario 4, another assumption is tested by changing the parameters. The review period is increased by 100%. The assumption is that an increased review period would also increase the possibility of perishing. As expected, the total cost increased due to increased perishing. This cost increase reinforces the need for regular review intervals to maintain optimal inventory levels and minimize perishing. The sensitivity analysis confirms that the ACSIOM model performs as expected under various scenarios, demonstrating robustness and adaptability to different parameter changes. These insights validate the model's practical applicability and highlight critical areas that significantly influence inventory costs, such as shelf-life and stock-out costs. The model works as expected under different parameters based on these sensitivity analyses and related comparisons. After conducting the sensitivity analysis, the proposed model performs as expected.

Table 4. ACSIOM Sensitivity Analysis Results

Model	Amendment	Total Cost (USD)	Difference
ACSIOM		1,020,706	
Scenario-1	P_{xy} / 2	1,873,457	0.84
Scenario-2	k_w * 2	1,058,883	0.04
Scenario-3	k_{so} * 2	945,286	-0.07
Scenario-4	rv_x * 2	1,040,685	0.02

5. CONCLUSION

The study aims to optimize inventory replenishment decisions under an asymmetric cost structure. This assumption is an essential aspect, as stock-out leads to customer dissatisfaction. Customer dissatisfaction may lead to other potential or existing customers. This dissatisfaction would increase quadratically and cause asymmetric stock-out costs. Another assumption is that the holding costs should reflect some costs that change for specific thresholds. The assumption is based on the fact that certain costs occur when some investments should be made to cover increased workloads. The model is constructed to integrate the perishability of products, stock-outs, and costs as a part of holding costs. The model aims to minimize cost, a widely used approach for inventory replenishment decisions. The proposed model is applied in a real-life case with 20 products with data covering 48 months. Based on the application, the results are encouraging. The total cost is 45.33% lower than the traditional (R, S) model with safety stock. The result indicates the applicability of the proposed model.

Sensitivity analysis is performed to analyze the robustness of the proposed ACSIOM Model and the model's behavior under changing conditions. As expected, the total cost increases dramatically with a lower shelf-life. The perishability of the products dramatically increases the lower shelf-life. The costs also increased accordingly by 84%. Similar results are also obtained after different scenarios as a part of sensitivity analysis. Based on the sensitivity analysis results, we can conclude that the model performs as expected.

The theoretical implications of the study are essential. Traditional models are complex and cannot be applied to products with lower shelf-life. Also, increased stock-out costs with lower service levels may undermine the performance of the traditional approaches. The proposed model integrates these asymmetric costs and shelf-life into the model for an inventory replenishment decision. Practically, the proposed model performs better than the traditional approach, allowing the decision-makers to make more accurate decisions. The model's applicability with different parameters allows the decision-makers to change the starting assumptions and eventual results. GA is applied to optimize the proposed model. The time required for exact methods limits their applications. Inventory replenishment decisions can be made regularly, sometimes multiple times per day. In order to meet the realities of business, time-sensitive solutions are vital. In order to meet these requirements, GA, a widely used metaheuristic, is applied.

The study has limitations. The study uses a modeling approach for inventory replenishment. Such application is optimized using the particle swarm optimization method as a metaheuristic. Although the application is performed in Matlab Software, the real-life application of the model would need a Graphical User Interface (GUI). Such an application is vital for users to manage the inventory replenishment process effectively. So, due to the cumbersome nature of the programming, the use of the proposed model may be limited. The product details are taken from a real-life case. Although 20 products are chosen for the model, the limitation of the number of products may undermine the performance of the proposed model. An extended dataset may alter the results. Another limitation is the application of the proposed model. The limited number of constraints is also a limiting factor for the applicability of the proposed model. Although the applied model can be optimized for a single product, the proposed model uses multiple products. This change is used since real-life cases may have constraints that must be combined with multiple decision variables. These constraints are not employed in the model to reflect the problem. Some examples of such constraints are total waste, total storage, total purchasing budget, and order quantity constraints. When the model is employed using multiple products, these constraints can be added when required at the expense of additional computational complexity.

Future studies aim to overcome the limitations and use the results for further research. The proposed model is applied to Matlab for analysis. The proposed model will use a decision support system (DSS) and alternative inventory replenishment approaches. The application of DSS would help the decision-makers to perform inventory replenishment optimization. Also, the proposed study will be performed using an extended dataset to analyze the results of the proposed model. An extension of the dataset may help analyze different products and the performance of the proposed model. Also, a real-life case may contribute to the applicability of the proposed model.

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