International Journal of Industrial Engineering, 32(1), 191-208, 2025

A STUDY OF OPTIMAL ASSIGNMENT MODEL CONSIDERING QUALITY AND WORKER LEVEL IN LIMITED-CYCLE WITH MULTIPLE PERIODS FOR SMART MANUFACTURING

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In recent years, quality irregularities reported by various media have shaken public confidence in companies, impacting both sales and stock prices, even among global leaders in manufacturing. Additionally, the increase in foreign workers and variations in service length have introduced differences in worker skill levels. This study aims to identify optimal worker assignment strategies that balance quality and worker skill levels for efficient production. We propose an optimal assignment model for a smart production line in a limited-cycle, multi-period setting that considers both quality and due dates. Numerical experiments further analyze optimal arrangement strategies and characteristics across different worker skill levels.

Keywords: Limited-Cycle Problem, Multi-Period, Optimal Assignment, Scheduling Problem, Manufacturing Line, Quality Control

(Received on July 12, 2024; Accepted on December 6, 2024)

1. INTRODUCTION

In recent years, incidents of quality fraud, such as falsifying inspection data and conducting inspections by unqualified personnel, have come to light, shaking public confidence in companies. This issue affects even Japan's leading manufacturing companies. There are various causes of such quality irregularities, including the acceptance of irregular practices to favor efficiency and prioritize product delivery schedules to suppliers. Once quality issues are discovered, it is crucial to establish a production plan and sequence that considers both production capacity and quality, as recalling products and enhancing monitoring can be costly.

Additionally, improving productivity and reducing production costs are key priorities in production, directly contributing to corporate profits. By accurately understanding factors such as the production period, worker production speed, and worker quality, a manufacturing company's production control can be significantly enhanced. In today's manufacturing environment, production control has become essential. Efficient organization of the production period, effective use of resources, and economical execution of production activities enable companies to meet expected production targets.

In a line production system, the assembly process—from raw materials to finished products—is divided among workstations, with production progressing as products move down the line. By equalizing the workload allocated to each workstation, smooth production flow can be achieved, a method known as line balancing. Since Bryton first introduced the line balancing problem, it has gained much attention, leading to numerous research publications. However, most of these studies did not consider worker-related factors that contribute to stochastic variability in work time. Worker motivation, health, experience, and skill level can all impact work time variability. Without accounting for these factors, it is challenging

DOI: 10.23055/ijietap.2025.32.1.10181 ISSN 1072-4761

to accurately reflect real-world production dynamics. The past study addresses variations in worker capacity within the line-balancing context. By considering cases where worker task times vary stochastically, they proposed determining optimal worker assignments. Another study presents a new distributed approach to multi-stage job shop scheduling, where a large job shop scheduling problem is divided into multiple smaller problems, each solved by multiple agents. Recently, there has also been substantial research on quality control using digital technology.

Furthermore, while the number of regular employees has seen only moderate growth over the past 40 years, the number of non-regular workers, including part-timers, contract workers, and temporary staff, has increased, as has the number of foreign workers. Therefore, balancing the allocation of regular and non-regular employees across production lines is essential. It is also important to account for the varying skill levels of workers, distinguishing between beginners and skilled workers among regular employees.

From this background, it is evident that an effective worker allocation system is essential in production. Additionally, in a smart factory, both speed and quality need to be considered. However, previous studies have not included quality as a factor. To address this gap, we propose an "optimal assignment model that considers both quality and worker skill level within a limited-cycle, multi-period framework for smart manufacturing." This new model incorporates a quality factor into the multi-period constraint cycle model, analyzing and examining optimal assignment laws. We conducted numerical experiments to derive a specific optimal assignment, to validate the proposed model, and to explore potential new laws for optimal assignment.

2. LITERATURE REVIEW

Many studies have analyzed various objectives for optimization problems in production systems.

As approaches to analyzing manufacturing line design, Nico André *et al* Schmid proposed assembly line balancing and feeding, showing that combined decision-making can reduce costs by up to 20%. Their research highlights the boxed-supply method as a key factor for efficiency in assembly line feeding. Zeynel Abidin Çil *et al*. introduced a new approach that integrates disassembly line balancing with vehicle routing, an essential consideration for adaptable, sustainable production. Using advanced models and algorithms, the study demonstrates effective solutions for complex, large-scale problems in distributed facilities. Hanbo Yang *et al*. proposed a microservices-based cloud-edge CM platform for smart manufacturing to manage large-scale IoT data. This platform combines cloud and edge computing for real-time diagnostics and improved prediction accuracy, achieving 90% diagnostic accuracy and a 50% reduction in prediction error in tests.

As approaches to analyzing optimal worker assignment, Elisa Gebennini *et al*. proposed job assignment optimization for rotating operators, focusing on minimizing walking and ergonomic costs. They introduce a mixed-integer linear programming model to reduce unproductive walking times and ergonomic risks, with a case study in the plastics industry showing that ergonomic improvements can lead to notable cost savings. Feng Liu *et al*. proposed worker assignment in hybrid seru systems by developing a bi-objective model to minimize makespan and balance workloads. For large-scale cases, a K-means-based NSGA-II memetic algorithm provides fast, effective solutions, outperforming other algorithms in terms of speed and performance. Numerical experiments offer insights into effective management.

As approaches to analyzing limited-cycle, multi-period problems, researchers have sought optimal assignments that minimize expected risk or cost within the limited-cycle model. To achieve this, it is necessary to calculate the expected costs for all possible assignments, which requires considerable time. Yamamoto *et al*. defined the problem of efficiently and economically allocating workers to each period, known as the optimal assignment problem, under a multi-period constraint cycle model. Both Yamamoto *et al*. and Kong *et al*. defined the optimal assignment problem in the reset multi-period constraint cycle model, formulated the expected cost of the optimal assignment, and proposed an efficient algorithm to solve it.

Later, Yamamoto *et al*. theoretically derived an optimal assignment rule. As an initial step, they considered a reset multi-period constraint cycle model with two groups of workers who have different work speeds—one group containing a single worker and the other group containing two workers. Under certain conditions, an optimal assignment law was analytically derived. Kong *et al*. similarly examined the case with two groups, where one group had two workers, and the other had one. Song analyzed the scenario with two workers in each of the two groups when the target working time remained constant. Tanizawa studied a model with three groups, each with a constant target time, including a novice, an expert, and a general worker in the group. Zhao investigated the optimal assignment rule when there were groups of one, two, and four workers, while Zhang discussed calculating the optimal assignment when worker capacity varied across periods.

3. MODEL EXPLANATION

First, the multi-period constraint cycle model used here is the reset multi-period constraint model. In this model, each period's machining is reset to the target machining time. If processing is delayed, the delay does not impact subsequent periods. Figure 1 illustrates varying worker levels across different plants, and Figure 2 shows the relationship between work time and cost in each plant.

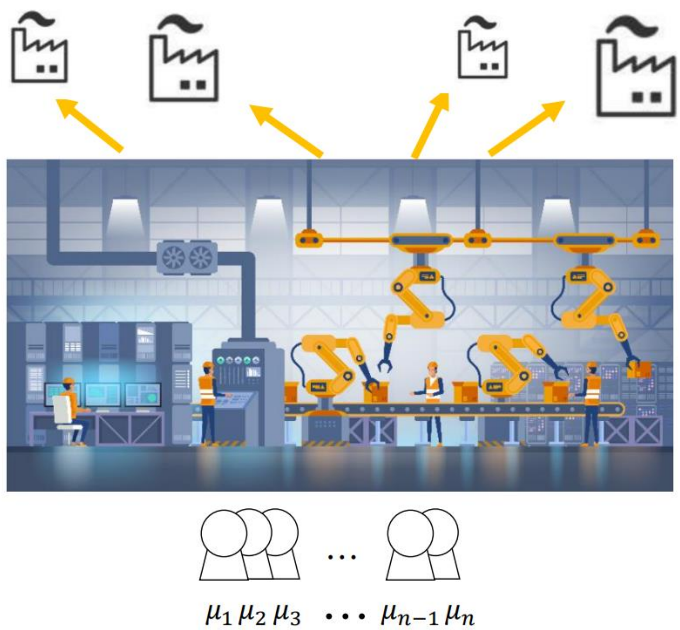


Figure 1. Image of optimal assignment problem considering quality and worker level in limited-cycle with multiple periods

The model is developed based on the following assumptions.

(1) The production line is in series, with n representing the number of periods.
(2) Products progress sequentially through periods 1, 2, ..., n, with each product passing through all n periods.
(3) Z denotes the total cycle time for all processes, also referred to as the target working time. By time Z, all jobs in the current process should be completed and transferred to the next process.
(4) Assuming that operators' production times are independent of each other and that each operator maintains consistent throughput across all periods, the working time of operator l, denoted as T_l, follows a probability distribution with probability density function $f_l(t)$, where t represents time.
  P_l : Probability that worker l is idle
  Q_l : Probability that worker l is late
  TS_l : Expected idle time for worker l
  TL_l : Expected delay time for worker l
(5) Quality assurance times are assumed to be independent, with the quality assurance time for each period I_m following the probability density function $g_m(t)$, where t represents time.
  R_m : Probability that period m is out of quality control
  TR_m: Expected time out of quality control for period m
(6) The processing cost per unit time, $Ct\ (\geq\ 0)$, applies within each process up to the target working time limit.
(7) In each period, if the target working time Z is met or not exceeded, an idle cost $C_s(\geq 0)$ is incurred per unit time.
(8) If production time in a period exceeds Z, a delay cost per unit time is incurred. Consecutive delays incur higher costs, with a delay cost $C_p^{(\alpha)}$ per unit time if delays persist over multiple periods, when α consecutive delay occurs.

The following model descriptions introduce new conditions to previous models:

(9) The quality control state is defined as a condition in which quality is ensured for each period if the time taken does not exceed the guaranteed quality time. Figure 3 illustrates the time variables used in the model. Let S represent

the point in time when the quality characteristic shifts to an out-of-control state. A quality cost per unit time, denoted as $C_q (\geq 0)$, applies in this state.

Here, the quality control state (labeled as "In control" in Figures 3 and 4) is defined as a condition in which the time spent in each period is less than or equal to the quality control time. If the time spent in a period exceeds the in-control time, it is considered out of quality control (labeled as "Out of control" in Figures 3 and 4). Figure 3 visually represents the relationship between target working time, in-control time, and out-of-control time, with the target working time for each period denoted as Z. For example, as shown in Figure 4, when the time in a period exceeds Z (i.e., $Z > I$), the product in that period is out of quality control, resulting in a quality cost incurred in the second process.

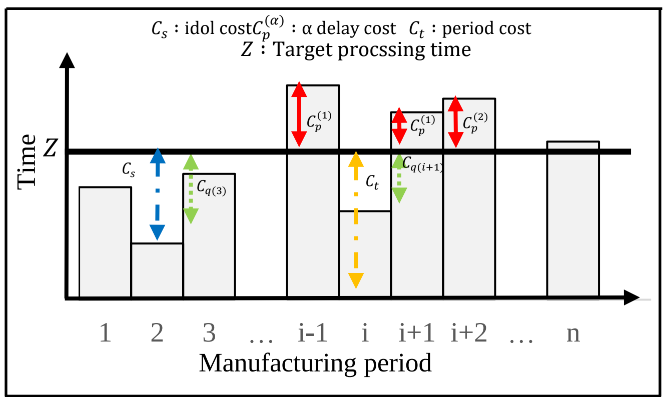
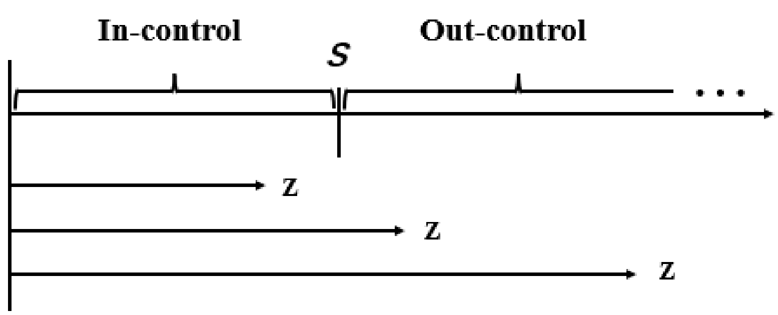


Figure 2. Relationship between costs and target work time



Z : Target procssing time S: Point of In control time

Figure 3. Definition of time variables

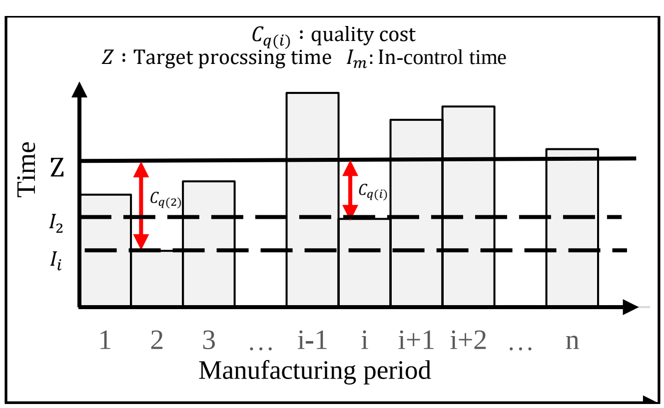


Figure 4. Relationship between quality costs and target work time

4. MATHEMATICAL MODEL AND THEOREM CONSIDERING QUALITY AND WORKER LEVELS

4.1 A mathematical **model considering the quality and two worker levels**

First, we define some notations. For $l = 1,2,\ldots,n$, the assumed production line is illustrated in Figure 5 below.

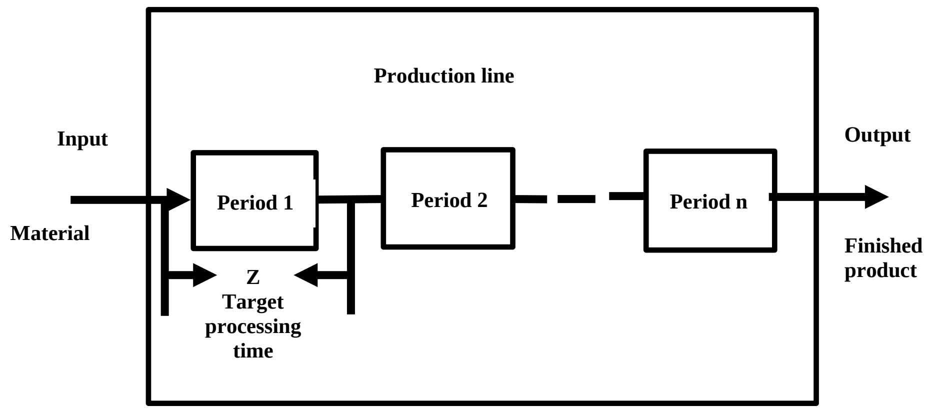


Figure 5. The production line of the model

T_l : The production time of the worker with a mean production rate μ_l.
I_l : The In-control time of the worker with a mean quality control rate λ_l.
$C(\ i\ ;\ T_{(1)},\ T_{(2)},\ \ldots\ ,\ T_{(i)})$: The idle or delay cost incurred in the period i, where the workers with mean production rates $\mu_{\pi(1)}, \mu_{\pi(2)}, \ldots, \mu_{\pi(m)}$ are assigned to periods 1 through i, respectively.
$C(\ i\ ;\ I_{(1)},\ I_{(2)},\ \ldots\ ,\ I_{(i)})$: The idle or delay cost incurred in the period i, where the workers with mean production rates $\mu_{\pi(1)},\ \mu_{\pi(2)},\ \ldots\ ,\ \mu_{\pi(m)}$ are assigned to periods 1 through i, respectively.

Based on assumptions (1)-(9) outlined in Section 2,

$$C(\ i\ ;\ T_{(1)}, T_{(2)}, \ldots, T_{(i)}) = \begin{cases} C_s \cdot (Z - T_{(i)}) & Z > T_{(i)} \\ C_p^{(i-j)} \cdot (T_{(i)} - Z) & Z > T_{(j)}, Z \leq T_{(j+1)}, \ldots, Z \leq T_{(i)} \end{cases} \quad (1)$$

$$C(\ i\ ; I_{(1)}, I_{(2)}, \ldots, I_{(i)}) = \begin{cases} C_q \cdot (Z - I_{(i)}) & Z > I_{(i)} \\ 0 & Z \leq I_{(i)} \end{cases} \quad (2)$$

where the total expected cost is denoted as TC, the expected cost due to idleness and delay is $h\ (i\ ;\ 1, 2, \ldots, n)$, and the quality cost is $q\ (i\ ;\ 1,\ 2,\ \ldots,\ n)$. The cost at the target working time can be expressed as nC_tZ. The expected cost in a production line with *n* periods is given by Equation (1).

$$TC = nC_tZ + h(i; 1,2, \ldots, n) + q(i; 1,2, \ldots, n) \quad (3)$$

However, for $1 \leq i \leq n$ in the case where

$$h(i; 1,2, \ldots, n) = E\left[\sum_{m=1}^{i} C(m; T_{(1)}, T_{(2)}, \ldots, T_{(m)})\right] \quad (4)$$

$$q(i; 1,2, \ldots, n) = E\left[\sum_{m=1}^{i} C(m; I_{(1)}, I_{(2)}, \ldots, I_{(m)})\right] \quad (5)$$

For l = 1,2,..., n ,

P_l : The probability of worker l with production rate μ_l becoming idle, defined as $P_l = P_r\{T_l \leq Z\}$,
Q_l : The probability of the worker l with production rate μ_l experiencing a delay, defined as $Q_l = P_r\{T_l > Z\}$
R_l : The probability that the in-control time with quality control rate λ_l becomes delayed, defined as $R_l = P_r\{I_l \leq Z\}$
TS_l : The expected idle cost for worker l with production rate μ_l, defined as $TS_l = E[(Z - T_l)I(T_l \leq Z)]$,
TL_l : The expected delay cost for worker l with production rate μ_l , defined as $TL_l = E[(T_l - Z)I(T_l > Z)]$.
TR_l : The expected quality cost for worker l with quality control rate λ_l , defined as $TL_l = E[(T_l - Z)I(T_l > Z)]$.

For, 1≤l≤n ,

$$P_l = \int_0^Z \mu_l(t)dt \tag{6}$$

$$Q_l = \int_Z^\infty \mu_l(t)dt \tag{7}$$

$$R_l = \int_0^Z \lambda_l(t)dt \tag{8}$$

$$TS_l = \int_0^Z (Z - t)\mu_l(t)dt \tag{9}$$

$$TL_l = \int_Z^\infty (t - Z)\mu_l(t)dt \tag{10}$$

$$TR_l = \int_0^Z (Z - t)g_l(t)dt \tag{11}$$

4.2 The theorem of the optimal assignment considering quality and two worker levels

The case of one special worker

In this section, we consider the case where the machining times of n -1 general workers follow the same distribution, while the working time distribution of one special worker A is different. Let $\pi(i)$ represent the case where worker A is assigned to period i.

[Theorem 1]

1) If $C_p^{(i)}$ is non-decreasing with respect to $Q_B < Q_A$ and $\frac{TL_B}{Q_B} < \frac{TL_A}{Q_A}$, then the assignment π (1) is the optimal assignment.
2) If $C_p^{(i)}$ is non-decreasing with respect to $Q_B \geq Q_A$ and $\frac{TL_B}{Q_B} \geq \frac{TL_A}{Q_A}$,then the optimal arrangement $\pi(i)$ exists for $\left(i \geq \frac{n}{2}\right)$.

[Proof]
See Appendix 1.

The case of two special workers

In this section, we consider the case where the machining times of n−2 general workers follow the same distribution, while the working time distributions of two special workers A. Let $\pi(i,j)$ represent the case where worker A is assigned to period i,j.

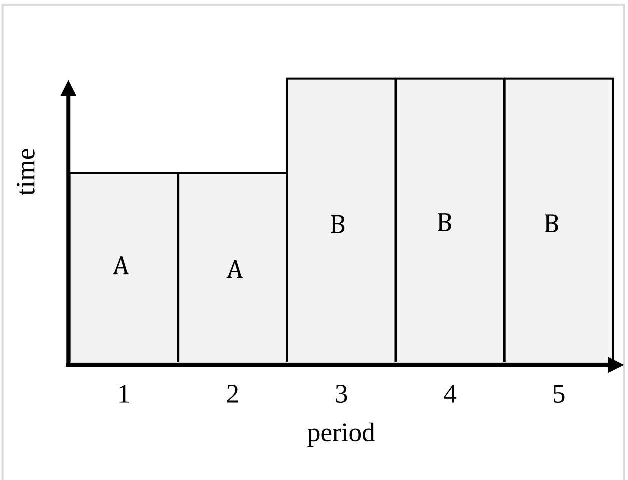


Figure 6. With two slow workers

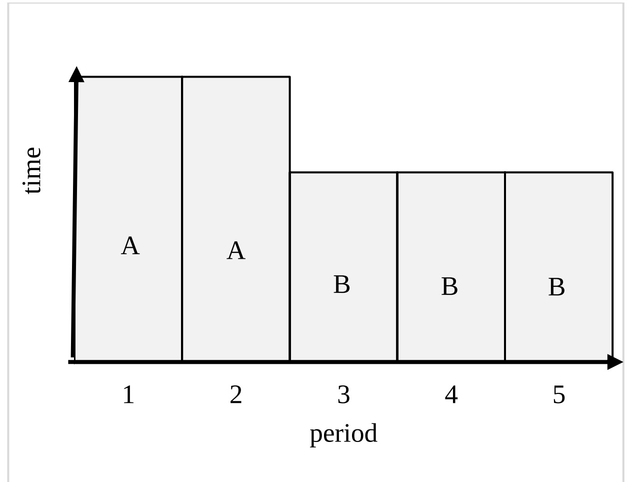


Figure 7. With two fast workers

<u>The case of two slow workers</u>

[Theorem 2]

$$M = \frac{\frac{TL_A}{Q_A}}{\left(\frac{TL_A}{Q_A} - \frac{TL_B}{Q_B}\right)} \cdot \left(\frac{1}{Q_B} - \frac{1}{Q_A}\right) \tag{12}$$

Also, for o satisfying $1 \leq o \leq u$

$$L_o = \frac{C_p^{(j_o+1)} - C_p^{(j_o)}}{C_p^{(j_o)} - C_p^{(j_o-1)}} \tag{13}$$

However, if $j_o = \sum_{k=1}^{o} K_k$, then $C_p^{(i)}$ is non-decreasing with respect to i, $Q_A > Q_B$ and $\frac{TL_A}{Q_A} > \frac{TL_B}{Q_B}$,

1-1) If max $L_o < M$, then $\pi(1, n)$ is the optimal arrangement.
1-2) If max $L_o > M$, then $\pi(1, 2)$ is the optimal arrangement, and the following is a summary of these results.

[Proof]
See Appendix 2

<u>The case of two fast workers</u>

[Theorem 3]

If $C_p^{(i)}$ is non-decreasing with respect to $Q_A < Q_B$ and $\frac{TL_A}{Q_A} < \frac{TL_B}{Q_B}$, then the optimal arrangement $\pi(i, j)$ is given by $\left\{\pi(i, j) | i \geq \frac{j-1}{2}, j \geq \frac{n}{2}\right\}$.

[Proof]
See Appendix 3

5. NUMERICAL CONSIDERATION

This chapter presents the results of numerical experiments and discussion of the optimum arrangement. The following conditions are used: $T = 2$, $C_t = 10$, $C_s = 20$, $C_p^{(n)} = 30 + 10n$, and $C_q = 50$. Let μ_A represent the production speed of worker A, and let λ be the rate of change of quality assurance time. A smaller rate of change in quality assurance time indicates a longer quality assurance time.

Total expected cost and Optimal assignment with one special worker

The case of one slow worker

Let A represent the worker who works slowly, and B represents the other workers. The values are set as follows:
$\lambda = 0.4,0.5,0.6$、$\mu_A = 0.4$、$\mu_B = 0.5,0.6,0.7$

Table 1 below shows the total expected cost in the case of optimal assignment.

Table 1. Behavior of total expected cost at optimal assignment with changes in quality and worker level (with one slow worker)

λ	μ_A	μ_B	Total expected cost (9 processes)	Total expected cost (10 processes)
0.4	0.4	0.5	833.68	925.91
0.4	0.4	0.6	778.23	863.33
0.4	0.4	0.7	748.8	830.16
0.5	0.4	0.5	884.27	982.13
0.5	0.4	0.6	828.83	919.55
0.5	0.4	0.7	799.39	886.38
0.6	0.4	0.5	929.08	1031.91
0.6	0.4	0.6	873.63	969.33
0.6	0.4	0.7	844.2	936.16

The total expected cost increased as the in-control time λ increased. This suggests that the shorter the in-control time, the higher the risk and associated costs. Additionally, as the speed of worker B increased, the cost decreased. This is likely because an increase in worker speed allowed tasks to be completed ahead of the target time, reducing overall costs. The total expected cost also increased with the number of processes, likely due to the increased risk and the cumulative cost of each task as the number of processes grew.

Table 2 below shows the optimal assignment. Here, "Number of periods" refers to the order in which materials are processed, with $\lambda = 0.4$, $\mu_A = 0.4$, $\mu_B = 0.5$.

Table 2. Optimal assignment with one slow worker

	Number of periods									
n	1	2	3	4	5	6	7	8	9	10
9	**A**	B	B	B	B	B	B	B	B	
10	**A**	B	B	B	B	B	B	B	B	B

The case of one fast worker

Let A be the fastest worker and B the other workers. The values are set as follows:
$\lambda = 0.4, 0.5, 0.6$、$\mu_A = 0.6$、$\mu_B = 0.3, 0.4, 0.5$

Table 3 below shows the total expected cost in the case of optimal assignment.

Table 3. Behavior of total expected cost at optimal assignment with changes in quality and worker level (with one fast worker)

λ	μ_A	μ_B	Total Expected Cost (9 processes)	Total Expected Cost (10 processes)
0.4	0.6	0.5	815.8	908.01
0.4	0.6	0.4	917.42	1023.74
0.4	0.6	0.3	1127.94	1264.95
0.5	0.6	0.5	866.39	964.23
0.5	0.6	0.4	968.02	1079.96
0.5	0.6	0.3	1178.54	1321.16
0.6	0.6	0.5	911.2	1014.01
0.6	0.6	0.4	1012.82	1129.75
0.6	0.6	0.3	1223.34	1370.95

The total expected cost increased as the in-control time λ increased. This suggests that a shorter in-control time results in higher risks and increased costs. Additionally, the cost increased as the speed of worker *B* decreased. These likely raised costs because a slower worker *B* increased the proportion of work time that exceeded the target, thus raising costs. The total expected cost also increased with the number of processes, likely due to the added risk and cumulative cost associated with each additional task.

Table 4 below shows the optimal assignment under the conditions $\lambda = 0.4,\ \mu_A = 0.6,$ and $\mu_B = 0.5$.

Table 4. Optimal assignment with one fast worker

	Number of periods									
n	1	2	3	4	5	6	7	8	9	10
9	B	B	B	B	B	**A**	B	B	B	
10	B	B	B	B	**A**	B	B	B	B	B

<u>Total expected cost and Optimal assignment with two special workers</u>

<u>The case of two slow workers</u>

Let A be the worker who works slowly and B the other workers. The values are set as follows:

$\lambda = 0.4, 0.5, 0.6$, $\mu_A = 0.4$、$\mu_B = 0.5, 0.6, 0.7$

Table 5 below shows the total expected cost in the case of optimal assignment.

Table 5. Behavior of total expected cost at optimal assignment with changes in quality and worker level (with two slow workers)

λ	μ_A	μ_B	Total Expected Cost (9 processes)	Total Expected Cost (10 processes)
0.4	0.4	0.5	845.31	937.55
0.4	0.4	0.6	795.31	880.41
0.4	0.4	0.7	768.44	849.81
0.5	0.4	0.5	895.91	993.76
0.5	0.4	0.6	845.91	936.62
0.5	0.4	0.7	819.04	906.02
0.6	0.4	0.5	940.71	1043.55
0.6	0.4	0.6	890.71	986.41
0.6	0.4	0.7	863.85	955.81

The total expected cost increased as the in-control time λ increased. This suggests that a shorter in-control time results in higher risks and increased costs. Additionally, as the speed of worker *B* increased, the cost decreased. This likely occurred because the increase in worker speed allowed tasks to be completed earlier than the target time, thereby reducing costs. The total expected cost also increased with the number of processes, likely due to the added risk and cumulative cost associated with each additional task.

Table 6 below shows the optimal assignment under the conditions $\lambda = 0.4,\ \mu_A = 0.4,$ and $\mu_B = 0.5$.

Table 6. Optimal assignment with two slow workers

	Number of periods									
n	1	2	3	4	5	6	7	8	9	10
9	**A**	B	B	B	B	B	B	B	**A**	
10	**A**	B	B	B	B	B	B	B	B	**A**

<u>The case of two fast workers</u>

Let *A* represent the faster worker, and *B* represent the other workers. The values are set as follows:
$\lambda = 0.4, 0.5, 0.6, \mu_A = 0.6$、$\mu_B = 0.3, 0.4, 0.5$

Table 7 below shows the total expected cost for the optimal assignment.

Table 7. Behavior of total expected cost at optimal assignment with changes in quality and worker level (with two fast workers)

λ	μ_A	μ_B	Total Expected Cost (9 periods)	Total Expected Cost (10 periods)
0.4	0.6	0.5	808.42	900.57
0.4	0.6	0.4	894.33	1000.18
0.4	0.6	0.3	1068.14	1203.03
0.5	0.6	0.5	859.02	956.79
0.5	0.6	0.4	944.93	1056.4
0.5	0.6	0.3	1118.73	1259.25
0.6	0.6	0.5	903.82	1006.57
0.6	0.6	0.4	989.73	1106.18
0.6	0.6	0.3	1163.54	1309.03

The total expected cost increased as the in-control time λ increased, suggesting that shorter in-control times lead to greater risk and higher costs. The cost also increased as the speed of worker *B* decreased, likely because the proportion of work time exceeding the target increased with slower worker speeds. Additionally, the total expected cost rose with the number of processes, likely due to increased risk and the cumulative cost of each task.

Table 8 below shows the optimal assignment under the following conditions:

- $\lambda = 0.4,\ \mu_A = 0.6,$ and $\mu_B = 0.5$ when the number periods are 9 and 10,
- $\lambda = 0.5,\ \mu_A = 0.6,$ and $\mu_B = 0.5$ when the number periods are 9 and 10.

Table 8. Optimal assignment with fast workers($\mu_A = 0.6,\ \mu_B = 0.5$)

λ	*n*	1	2	3	4	5	6	7	8	9	10
0.4	9	B	B	B	**A**	B	B	**A**	B	B	
0.5	9	B	B	B	**A**	B	**A**	B	B	B	
0.4	10	B	B	B	**A**	B	B	**A**	B	B	B
0.5	10	B	B	B	**A**	B	B	**A**	B	B	B

This example illustrates that the optimal assignment laws from the preceding three chapters are consistently satisfied.

6. CONCLUSION

Quality fraud issues are currently causing significant losses for companies. Additionally, with globalization, it is essential for workers of varying skill levels to collaborate in manufacturing. Therefore, this paper proposed a quality-considering model

based on the concept of multi-period constraint cycles to achieve efficient production in a series-type production line while ensuring product quality. We derived a theorem on the optimal assignment rule for cases involving one or two special workers and confirmed the optimal assignment through numerical experiments. Furthermore, changing the target working hours affected the optimal assignment, suggesting that the setting of target working hours influences the optimal arrangement.
This research is expected to contribute to smart production systems, as the optimal allocation of personnel can enhance production speed. Future work includes deriving optimal assignment rules for three worker-level groups (skilled, novice, and standard workers) and exploring optimal assignment rules for different worker levels. Additionally, since this model is abstract, it will be important to apply it to concrete production lines. Another avenue for further research involves examining the regularity of optimal placements when the number of processes is increased beyond current levels to accommodate more complex production requirements.

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APPENDIX

[Appendix 1]
Proof in the same way as [Appendix 2].

[Appendix 2]
First, two auxiliary theorems are noted here.
The simple flow of the proof is to prove that process i costs less than process $i+1$, and then prove that process 1 costs less than process i. This means that the optimal allocation is to place the special worker in process 1.

① For o satisfying $1 \le o \le u$,

$$h(n,\pi(i,j+1)) - h(n,\pi(i,j)) = \left\{ \begin{matrix} \left(\begin{matrix} (Q_A - Q_B) \cdot TL_B \cdot A(i,j) + (Q_A - Q_B) \cdot \left(C_p^{(j-i+1)} - C_p^{(j-1)} \right) \cdot Q_B^{j-i-1} \cdot \left(\frac{Q_A}{Q_B} \cdot TL_B - TL_A \right) \\ + \left(C_p^{(j+1)} - C_p^{(j)} \right) \cdot Q_A^2 Q_B^{j-1} \cdot \left(\frac{TL_A}{Q_A} - \frac{TL_B}{Q_B} \right) \end{matrix} \right) \\ i = i_o, j = j_o \\ (Q_A - Q_B) \cdot TL_B \cdot A(i,j) \\ i_o + 1 \le i_{o+1} - 1, j_o + 1 \le j \le j_{o+1} - 1 \end{matrix} \right\} \tag{14}$$

However,

$$A(i,j) = \sum_{\alpha=1}^{n-j-1} \left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)} \right) \cdot Q_B^{\alpha-1} \right) - \sum_{\alpha=1}^{j-i-1} \left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)} \right) \cdot Q_B^{\alpha-1} \right) - \sum_{\alpha=j-i}^{j-1} \left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)} \right) \cdot Q_B^{\alpha-1} \right) \cdot \frac{Q_A}{Q_B} \tag{15}$$

$$Q_A = \int_Z^\infty f_A(t)dt \tag{16}$$

$$Q_B = \int_Z^\infty f_B(t)dt \tag{17}$$

$$R_l = \int_Z^\infty g_l(t)dt \tag{18}$$

$$TL_A = \int_Z^\infty (t-Z) f_A(t)dt \tag{19}$$

$$TL_B = \int_Z^\infty (t-Z) f_B(t)dt \tag{20}$$

$$TR_l = \int_0^Z (Z-t) g_l(t)dt \tag{21}$$

② For o satisfying $1 \le o \le u$,

$$h(n,\pi(i+1,j)) - h(n,\pi(i,j)) = \left\{ \begin{matrix} \left(\begin{matrix} (Q_A - Q_B) \cdot TL_B \cdot B(i,j) + (Q_A - Q_B) \cdot \left(C_p^{(j-i)} - C_p^{(j-i-1)} \right) \cdot Q_B^{j-i-2} \cdot TL_A \\ + \left(C_p^{(i+1)} - C_p^{(i)} \right) \cdot Q_B^i \cdot Q_A \cdot \left(\frac{TL_A}{Q_A} - \frac{TL_B}{Q_B} \right) \end{matrix} \right) \\ i = i_o, j = j_o \\ (Q_A - Q_B) \cdot TL_B \cdot B(i,j) \\ i_o + 1 \le i_{o+1} - 1, j_o + 1 \le j \le j_{o+1} - 1 \end{matrix} \right\} \tag{22}$$

However,

$$B(i,j) = \sum_{\alpha=1}^{j-i-2}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right)\cdot Q_B^{\alpha-1}\right) + \sum_{\alpha=j-i}^{n-i-1}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right)\cdot Q_B^{\alpha-1}\right)\cdot\frac{Q_A}{Q_B} - \sum_{\alpha=1}^{i-1}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right)\cdot Q_B^{\alpha-1}\right) \tag{23}$$

For i and j satisfying $1 \le i < j \le n$,

$$f\left(n,\pi(i,j)\right) - \left(n,\pi(1,j)\right) \ge 0 \tag{24}$$

For i and j satisfying $2 \le j \le n$,

$$h\left(n,\pi(i,j)\right) - h\left(n,\pi(1,j)\right) \le 0 \tag{25}$$

If we prove above relationship, we can say that $\pi(1,n)$ is the optimal assignment.
We will prove it in the following two steps, a) and b).

For i and j satisfying $1 \le i < \frac{j-1}{2} \le n$

$$h\left(n,\pi(i,j)\right) - h\left(n,\pi(1,j)\right) \ge 0 \tag{26}$$

We show that this equation holds.

For i and j satisfying $1 \le i < j \le n$, from the auxiliary theorem ①,

$$h\left(n,\pi(i+1,j)\right) - h\left(n,\pi(i,j)\right) = (Q_A - Q_B)\cdot TL_B\cdot\left(\sum_{\alpha=1}^{j-i-2} g(\alpha) - \sum_{\alpha=1}^{i-1} g(\alpha)\right) + S1_{i.j} \tag{27}$$

then,

$$S1_{i.j} = (Q_A - Q_B)\cdot\sum_{\alpha=j-i}^{n-j-1}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right)\cdot Q_B^{\alpha-1}\right)\cdot TL_B\cdot\frac{Q_A}{Q_B} + (Q_A - Q_B)\cdot\left(C_p^{(j-i)} - C_p^{(j-i-1)}\right)\cdot Q_B^{j-i-2}\cdot TL_A \quad + \left(C_p^{(i+1)} - C_p^{(i)}\right)\cdot Q_B^{i}\cdot Q_A\cdot\left(\frac{TL_A}{Q_A} - \frac{TL_B}{Q_B}\right) \tag{28}$$

Also,

$$g(\alpha) = \left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right)\cdot Q_B^{\alpha-1} \tag{29}$$

If $j_o = \sum_{k=1}^{o} K_k$, then $C_p^{(i)}$ is nondecreasing with respect to i, $Q_A > Q_B$ and $\frac{TL_A}{Q_A} > \frac{TL_B}{Q_B}$, then from equation (19)

$$S1_{i.j} \ge 0 \tag{30}$$

Also, from equation (20), for i and j satisfying $1 \le i < (j-1)/2 \le n$,

$$\sum_{\alpha=1}^{j-i-2} g(\alpha) - \sum_{\alpha=1}^{i-1} g(\alpha) = \sum_{\alpha=i}^{j-i-2} g(\alpha) \ge 0 \tag{31}$$

From equations (41) and (42), and from equation (39), for i and j satisfying $1 \le i < (j-1)/2 \le n$,

$$h\left(n,\pi(i+1,j)\right) - h\left(n,\pi(i,j)\right) \ge 0 \tag{32}$$

From the above, (38) is proven.

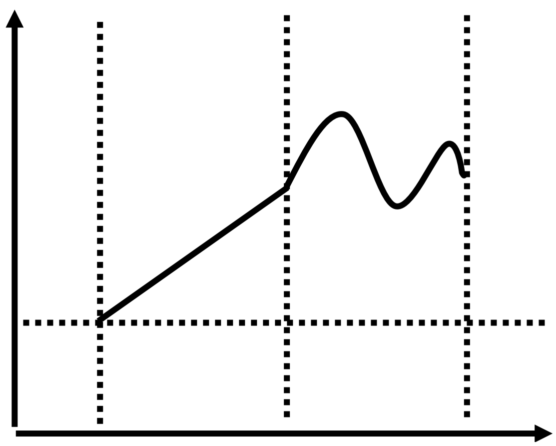

Figure 6. Variation of $f(n,\pi(i,j))$ with respect to i in Theorem 1-1)

Next, for *i* and *j* satisfying $1 \le i < j \le n$,

$$h(n;\pi(i,\ j)) - h(n;\pi(1,\ j)) \ge 0 \tag{33}$$

The above equation is shown to hold.

Now, for *i* and *j* satisfying $1 \le i < j \le n$,

$$h(n;\pi(i,j)) - h(n;\pi(1,j)) = \sum_{\beta=1}^{i-1} h(n;\pi(\beta+1,j)) - h(n,\pi(\beta,j)) \ = (Q_A - Q_B)\cdot TL_B \cdot \sum_{\beta=1}^{i-1}\left(\sum_{\alpha=1}^{j-\beta-2} g(\alpha) - \sum_{\alpha=1}^{\beta-1} g(\alpha)\right) + \sum_{\beta=1}^{i-1} S1_{\beta.j} \tag{34}$$

For *i* and *j* satisfying $1 \le i < j \le n$,

$$\sum_{\beta=1}^{i-1}\left(\sum_{\alpha=1}^{j-\beta-2} g(\alpha) - \sum_{\alpha=1}^{\beta-1} g(\alpha)\right) = \begin{cases} \sum_{\beta=1}^{i}\sum_{\alpha=1}^{j-\beta-2} g(\alpha) & i < \frac{j-1}{2} \\ \sum_{\beta=1}^{j-i-2}\sum_{\alpha=\beta}^{j-\beta-2} g(\alpha) & j-2 > i \ge \frac{j-1}{2} \\ 0 & i = j-2 \end{cases} \tag{35}$$

If $C_p^{(\alpha)}$ is nondecreasing with respect to α and $Q_A > Q_B$, then from equations (42) and (47), from equation (46), for i and *j* satisfying $1 \le i < j \le n$,

$$h(n,\pi(i,j)) - h(n,\pi(1,j)) \ge 0 \tag{36}$$

The above equation holds.
Equations (38) and (45) are satisfied, which proves equation (35).

Next,

- $\{j_o + 1 \le j \le j_{o+1} - 1 | 2 \le j_o + 1, j_{o+1} - 1 \le n (1 \le o \le u)\}$

- $\{j = j_o | 2 \leq j_o \leq n(1 \leq o \leq u)\}$

We show below that equation (37) holds in the two steps of c) and d), separately for j satisfying these two equations. For convenience of proof under the condition that $C_p^{(i)}$ is nondecreasing with respect to i, let $j_o = \sum_{K=1}^{o} K_k$ for o satisfying $1 \leq o \leq u$.

For j satisfying $\{j_o + 1 \leq j \leq j_{o+1} - 1 | 2 \leq j_o + 1, j_{o+1} - 1 \leq n(1 \leq o \leq u)\}$

$$h(n, \pi(i,j)) - h(n, \pi(1,j)) \leq 0 \quad (37)$$

We show that the above equation holds.
Now, for j satisfying $\{j_o + 1 \leq j \leq j_{o+1} - 1 | 2 \leq j_o + 1, j_{o+1} - 1 \leq n(1 \leq o \leq u)\}$, from auxiliary theorem ①,

$$h(n, \pi(1, j+1)) - h(n, \pi(1,j)) = (Q_A - Q_B) \cdot TL_B \cdot \left(\sum_{\alpha=1}^{n-j-1}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right) \cdot Q_B^{\alpha-1}\right) - \sum_{\alpha=1}^{j-2}\left(\left(C_p^{(\alpha+1)} - C_p^{(\alpha)}\right) \cdot Q_B^{\alpha-1}\right)\right) \quad (38)$$

If $C_p^{(\alpha)}$ is nondecreasing with respect to α, then from equation (41), for j satisfying $\left\{j_o + 1 \leq j \leq j_{o+1} - 1 | \frac{n+1}{2} \leq j_o + 1, j_{o+1} - 1 \leq n(1 \leq o \leq u)\right\}$,

$$\sum_{\alpha=1}^{n-j-1} g(\alpha) - \sum_{\alpha=1}^{j-2} g(\alpha) = -\sum_{\alpha=i}^{j-i-2} g(\alpha) \leq 0 \quad (39)$$

Also, since $Q_A > Q_B$ and from equation (51), it follows from equation (50)
for j satisfying $\left\{j_o + 1 \leq j \leq j_{o+1} - 1 | \frac{n+1}{2} \leq j_o + 1,\ j_{o+1} - 1 \leq n(1 \leq o \leq u)\right\}$,

$$h(n,\ \pi(1,\ j+1)) - h(n,\ \pi(1,\ j)) \leq 0 \quad (40)$$

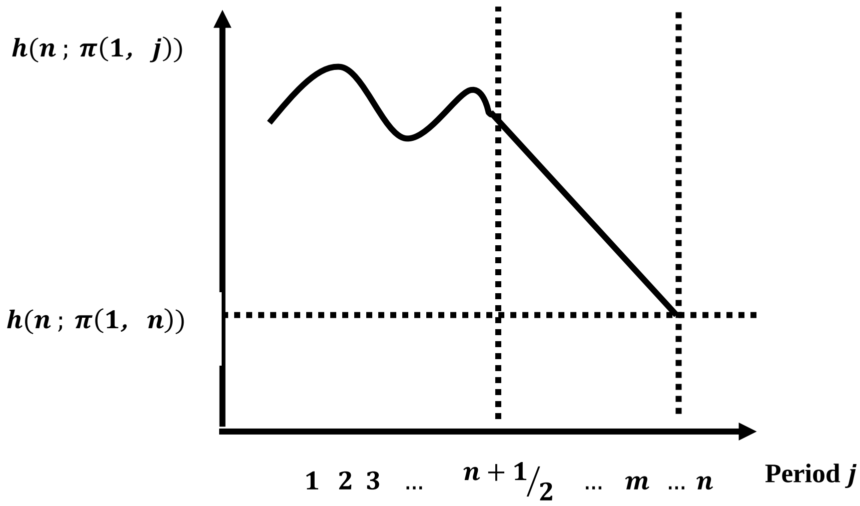

Figure 7. Variation of $f(n,\pi(i,j))$ with respect to j in Theorem 1-1)

Also, for j satisfying $\left\{j_o+1\le j\le j_{o+1}-1 | 2\le j_o+1,\ \ j_{o+1}-1\le \frac{n+1}{2}(1\le o\le u)\right\}$

$$h(n,\ \pi(1,\ n))-h(n,\ \pi(1,\ j))\le 0 \tag{41}$$

From equations (52) and (53), equation (49) is proved.

d) for j satisfying $\{j=j_o|2\le j_o\le n(1\le o\le u)\}$

$$h(n,\ \pi(1,\ n))-h(n,\ \pi(1,\ j))\le 0 \tag{42}$$

We show that the above equation holds.

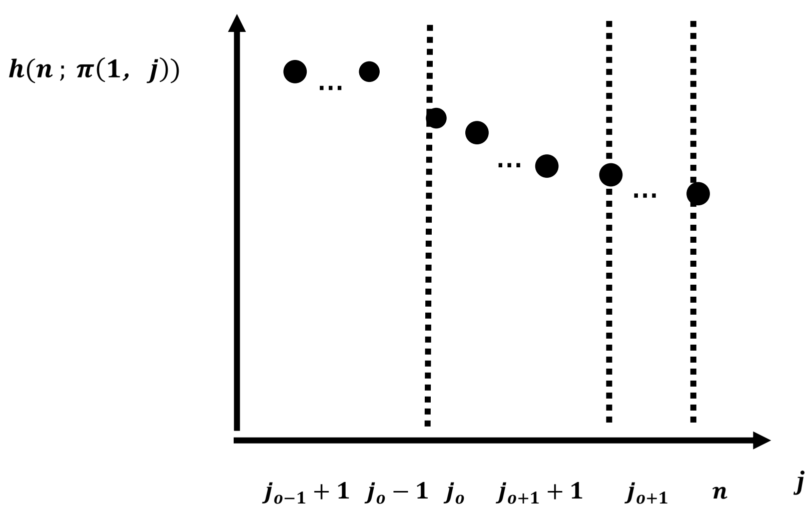

Fig8. the image of the change of h(n;π(1, j)) in $\{j_o+1\le j\le j_{o+1}-1|2\le j_o+1,\ \ j_{o+1}-1\le n(1\le o\le u)\}$ and $\{j=j_o|2\le j_o\le n(1\le o\le u)\}$

$$S2_{i.\ \ j} = -(Q_A - Q_B) \cdot Q_B^{j-1} \cdot D(j) \tag{43}$$

However,

$$D(j) = \left(\left(C_p^{(j+1)} - C_p^{(j)} \right) \cdot \left(\frac{Q_A^2 \left(\frac{TL_A}{Q_A} - \frac{TL_B}{Q_B} \right)}{-(Q_A - Q_B)} \right) + \left(C_p^{(j)} - C_p^{(j-1)} \right) \cdot \left(\frac{TL_A}{Q_B} \right) \right) \tag{44}$$

From auxiliary theorem ② and equation (41), for j satisfying $\{j = j_o | 2 \leq j_o \leq n(1 \leq o \leq u)\}$

$$h\big(n,\ \ \pi(1,\ \ j+1)\big) - h\big(n,\ \ \pi(1,\ \ j)\big) \ = (Q_A - Q_B) \cdot TL_B \cdot \left(\sum_{\alpha=1}^{n-j-1} g(\alpha) - \sum_{\alpha=1}^{j-2} g(\alpha) \right) + S2_{i.\ \ j} \tag{45}$$

If $C_p^{(\alpha)}$ is nondecreasing with respect to α, then $maxL_o < M$,

$$D(j) \geq 0 \tag{46}$$

Since $Q_A > Q_B$, from equation (55)

$$S2_{i.\ \ j} \leq 0 \tag{47}$$

Since $Q_A > Q_B$ and from equation (59), from equation (57), for j satisfying $\left\{ j = j_o | \frac{n+1}{2} \leq j_o \leq n(1 \leq o \leq u) \right\}$

$$h\big(n,\ \ \pi(1,\ \ j+1)\big) - h\big(n,\ \ \pi(1,\ \ j)\big) \leq 0 \tag{48}$$

Also, for j satisfying $\left\{ j = j_o | 2 < j_o < \frac{n+1}{2} (1 \leq o \leq u) \right\}$,

$$h\big(n,\ \ \pi(1,\ \ n)\big) - h\big(n,\ \ \pi(1,\ \ j)\big) \leq 0 \tag{49}$$

Equation (54) is proved by the fact that equations (60) and (61) hold.
Equation (37) is proved by the fact that equations (49) and (54) hold.
Theorem 1-1) is proved by the fact that equations (36) and (37) hold.

1-2)
Proof in the same way as 1-1).

[Appendix 3]
Proof in the same way as [Appendix 2].