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CHANNEL COORDINATION OF DUAL-CHANNEL SUPPLY CHAIN WITH C2M MANUFACTURER INTRODUCING THE PRIVATE LABEL

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The thriving development of E-commerce platforms has injected new vitality into the C2M (Customer-to-Manufacturer) business model. However, it has also introduced a new challenger-the private label retailer. Given the escalating influence of private label retailers, the strategic introduction and coordination of the supply chain by C2M manufacturers have emerged as pivotal challenges. We analyze a C2M manufacturer's pricing strategies in three dual-channel supply chain models: centralization, decentralization, and partial centralization. In these models, the C2M manufacturer acts as a Stackelberg leader, while the private label retailer and platform act as followers. Analysis of both symmetric and asymmetric channel scenarios demonstrates that C2M manufacturers favor centralizing the supply chain. Consequently, we explore coordination strategies for Dual-channel supply chains and determine that the C2M manufacturer's contract, which incorporates wholesale and direct channel pricing, effectively coordinates the dual-channel supply, benefiting the retailer and platform but not the C2M manufacturer. We illustrate how such a contract, coupled with a complementary agreement like a two-part tariff or profit-sharing, can effectively coordinate the dual-channel supply chain, creating a win-win-win situation for the C2M manufacturer, private label retailer, and platform.

Keywords: Consumer-to-Manufacturer (C2M) Mode; E-Commerce Platform; Private Label; Dual-Channel; Channel Coordination.

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1. INTRODUCTION

Capitalizing on their advantages in consumer information acquisition and forecasting, e-commerce companies have innovated an effective business model known as Consumer-to-Manufacturer (C2M). SHEIN has leveraged this model to become the leading cross-border e-commerce platform for fast fashion in China, with an estimated market value of $30 billion. (Liu, 2022). C2M encompasses a supply chain approach that directly links consumers and manufacturers through adaptable, tailored production lines to facilitate on-demand manufacturing. This model typically operates via e-commerce platforms acting as virtual intermediaries. Unlike traditional national brands, C2M allows consumers to input demand forecasts, product designs, and assortment preferences (Mak and Max Shen, 2021). As of 2020, the C2M market in China reached $4.73 billion, up from $1.69 billion, comprising 8% of the total e-commerce market and showing significant growth potential (Yumi, 2022). Consumers benefit from purchasing C2M products due to direct links to manufacturers, which streamline production processes and reduce costs. Whether customized or not, C2M manufacturers consistently offer national brands at competitive prices, reflecting the simplicity and efficiency of their supply chain. The launch of Temu by China's PDD Holding in 2022 is significant for the digital commerce sector and possibly heralds the rise of a new generation of e-commerce marketplaces. According to Bloomberg, in May 2023, Temu surpassed SHEIN in terms of user numbers in the United States, with over 70 million people accessing the app. In September 2023, Temu had over 80 million active users in the United States (Bloomberg, 2023).

Who should be worried about the competition from this model? We contend that any company and intermediary selling interchangeable products lacking significant brand value should be concerned. For instance, retailers selling branded products like Bosch, Makita, or Einhell in the tools sector are less vulnerable since the brand serves as a differentiator. However, retailers like Lidl or Kaufland may encounter challenges, mainly when offering unbranded wrenches manufactured in China in their non-food sections. For consumers, there may be no compelling reason to shop with these retailers online since price

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no longer serves as a differentiating factor. Temu, through direct connections with Chinese manufacturers on its platform, can offer significantly lower prices. Thus, many platforms have focused on enhancing supply chain efficiency and sharing infrastructure and technologies with traditional enterprises to usher manufacturers into the digital age. For instance, JD's consumer-to-manufacturer platform has enabled thousands of brands to decrease product demand research time by nearly 80 percent and reduce new product release cycles by nearly 70 percent (Yang, 2023). The rapid renewal of brand-name products has led most retailers to try to grab a slice of the consumer market, too.

In recent decades, private label brands have experienced exponential growth, posing significant competitive threats to national brands (Cuneo *et al*. 2019). This shift also signifies a transition from single-channel to dual-channel strategies. Private label brands, or store brands, are products bearing the retailer's chosen brand name and exclusively owned, controlled, and sold by the retailer (Kumar, 2007). Unlike national brands, retailers assume comprehensive responsibilities for their private label brands, including product positioning, design, sourcing or manufacturing, pricing, shelf placement, and promotions (Morton and Zettelmeyer, 2004).

Many global retailers now prioritize the adoption and expansion of private-label brands as one of their primary objectives (Baltas *et al*., 2007). Globally, private-label brands captured a market share exceeding 16.7% in 2016 (Nielsen, 2018). In Europe, the dominance of the global private label market continues to strengthen, with the overall market share reaching 38.2% in the first quarter of 2024, marking a 0.9% increase compared to the first quarter of 2023 (PLMA, 2024). The development of private labels has driven market diversification, compelling many C2M manufacturers to introduce and sustain their private brands proactively. Furthermore, our coordinated C2M model can create a win-win-win scenario for the C2M manufacturer, platform, and private label retailer: (1) The C2M manufacturer can achieve digital transformation and expand their market reach through online sales, reducing reliance on traditional offline ordering methods. (2) E-commerce platforms leverage downstream traffic and data to enhance upstream manufacturing efficiency, and (3) this support aids manufacturers in developing private label brands for retailers.

Faced with fierce competition from national brands and private label brands, the C2M manufacturer finds it more critical and challenging to make decisions to compete with national brands and choose the appropriate C2M mode. Based on these pricing decisions and practical issues, our research questions arise. (1) In which mode should the C2M manufacturer introduce the private label retailer? (2) Should the manufacturer hold or contract with the private brand retailer? (3) Should the C2M manufacturer integrate with the platform? (4) How do we coordinate the platform and retailer under the decentralization system when the C2M manufacturer is the leader in the supply chain system? (5) Does the partial centralization system have the double marginalization effect? How do we eliminate or even remove it through contracts?

A substantial body of research has examined the dynamics of private-label brands (Chintagunta *et al*., 2002; Hansen and Singh, 2008) and their competitive interactions with national brands (Mills and Strategy, 1995; Sayman and Raju, 2004; Choi and Coughlan, 2006). However, there remains a significant gap in the literature regarding the study of the C2M mode concerning whether manufacturers hold private label retailers and platforms. Various C2M modes—centralization, partial centralization, and decentralization—employed by the manufacturer present an underexplored area concerning their relative effectiveness and limitations in competitive markets. This study aims to address this gap by presenting the following key findings. Firstly, C2M manufacturers display diverse preferences regarding introducing private label retailers. In symmetric channels, the centralization model consistently proves optimal for C2M manufacturers, while partial centralization subjects them to a prisoner's dilemma. When basic demand is high, and brand substitutability is moderate, retaining the private label retailer proves advantageous. Conversely, situations where the platform is held typically under lower basic demand or higher brand substitutability.

Additionally, the decentralization model is consistently the least favorable. Secondly, in asymmetric channels, the centralization model similarly emerges as optimal. For example, in scenarios where retail channel power is dominant and brand substitutability is high, the profit for the C2M manufacturer who holds the platform ranks just below that of the centralization model. On the other hand, the advantages for the C2M manufacturer holding the private label retailer become more pronounced when the direct channel is stronger and brand substitutability is lower. In cases where the direct channel holds more power and brand substitutability is moderate, C2M manufacturers may encounter a prisoner's dilemma under the partial centralization model.

Nevertheless, unlike in symmetric channels, the decentralization model does not consistently represent the least favorable option, especially in extremely low commissions. Thirdly, we designed coordination contracts for scenarios plagued by severe double marginalization effects. We demonstrate that the contract combining wholesale and direct channel pricing effectively coordinates the dual-channel supply chain, benefiting both the retailer and platform. However, it but it does not increase the profits of the C2M manufacturer. Furthermore, through a complementary agreement such as a two-part tariff or profit-sharing arrangement, this contract can successfully achieve a win-win-win situation for the private label retailer, platform, and C2M manufacturer. This results in a mutually beneficial outcome for all parties involved in the dual-channel supply chain.

We organize the rest of the paper as follows: In Section 2, we review the related literature to identify the research gap and position our study. We present the model settings and benchmark case in Section 3. We derive equilibrium outcomes, conduct model analysis in Sections 4-5, and discuss coordination. Finally, in Section 6, we conclude the paper and propose topics for future research. The proofs of all results are presented in the Appendices.

2. LITERATURE REVIEW

Our study is directly related to the literature on the C2M model, which refers to a supply chain model connecting C2M manufacturers with consumers directly through e-commerce platforms as virtual intermediaries. This supply chain eliminates sales, distribution, and inventory, allowing consumers to buy products that are properly matched to their needs. Through the platform, consumers can input demand forecasts and product and assortment designs. Moreover, research on dual-platform competition—where two platforms compete in the same market—suggests that contract choices should be differentiated to mitigate competition (Zhang *et al*., 2019). At the same time, based on an agency contract, the manufacturer sells the product on an online retail platform where a non-neglectable amount of consumer returns occurs due to product misfits (Hao and Kumar, 2023). C2M is related to personalized customization, which has received attention from many scholars (Dewan *et al*.,2003; Gu and Tayi, 2015; Theresa, 2020). However, personalized customization is not always beneficial (Wang *et al*., 2023).

Furthermore, Mak and Max Shen (2021) argue that C2M encompasses not just customization but the entire supply chain. Therefore, some scholars have also noticed the platform-based brand competition brought about by the C2M model (Lyu *et al*., 2023), and we will focus on the connection between C2M manufacturers or retailers with platforms. The rise of large-scale C2M manufacturers and platform development has positioned this model at the forefront for leading Chinese tech companies such as PDD, JD, and Alibaba (Lee, 2018; Mak and Shen, 2021). Yang *et al*. (2024) raise whether manufacturers should leverage platforms to adopt the C2M model based on product differentiation, while He *et al*. (2023) advocate for employing the C2M model to collaborate with e-commerce platforms. Dai *et al*. (2022) further proposed how to solve the problem of profit distribution between C2M manufacturers and platforms to achieve supply chain equilibrium. Fan *et al*. (2022) studied the value of introducing C2M mode by online retailers from the perspective of retailers. Our study from the perspective of C2M manufacturers partially fills this gap by analytically modeling the strategic interactions with retailers and platforms in a competitive situation.

There is also a stream of literature on private labels (store brands) and their encroachment. Private labels can reduce the profits of C2M manufacturers. However, in models where the interaction between manufacturers and retailers depicts this as a retailer-led Stackelberg game, store brands may prove advantageous (Ru *et al*., 2015). Incumbent sellers may benefit more when private-brand is prohibited from being introduced in contract sales(Cheng *et al*., 2023). If the retailer competes by offering high-quality products, the manufacturer should consider reducing wholesale price discounts and improving the quality of the national brand to mitigate conflict pressures (Heese 2010).

In addition to introducing conflict into the supply chain, the quality of private labels also determines the extent of this conflict. By examining the interaction of costs and quality between lower and higher-quality private labels, Li *et al*. (2022) delineate the optimal decisions private label retailers should make regarding encroachment strategies. Retailers are inclined to opt for high-quality private labels due to the influence of producer choices and channel price leadership (Liao *et al*., 2020). Our research also includes the management of supply chain operations concerning branding-related factors (Liao *et al*., 2020; Al-Monawer *et al*., 2021; Yu *et al*., 2022). When the cost of quality investment is sufficiently high, it dictates the circumstances under which contract manufacturers engage in encroachment, resembling the dynamics observed with private labels (Shi, 2019). Chen *et al*. (2019) suggested that a contract manufacturer could introduce its private-label product, competing directly with its original equipment manufacturer, known as factory encroachment. Developing game theoretic models to explore the strategic interactions between manufacturer encroachment and a retailer's introduction of a private label or store brand involves both parties participating in the analysis (Zhang *et al*., 2021). The expansion of private label brands, influenced by retail distribution structures, retailer typologies, and logistical frameworks, is diminishing the market share of manufacturer brands (Cuneo *et al*., 2015). However, surprisingly, Liu *et al*. (2024) indicate that fostering competition tends to be more profitable than significantly differentiating brand positions to mitigate market cannibalization. Li *et al*. (2024) also conclude that introducing new products as a competitive strategy can create a win-win situation for national brand sellers and platforms, exceptionally when executed through a marketplace channel with significant expansion effects. In contrast to existing studies, our analysis explores scenarios where the C2M manufacturer either holds the competing private label retailer or controls the platform, offering them additional strategic flexibility.

Boyaci (2005) and Tsay *et al*. (2004) explored earlier literature on the dual-channel supply chain and coordination mechanisms. Research in this area has been ongoing for a considerable time. Li *et al*. (2023) investigate how a manufacturer should respond to the retailer's stockpiling ability by contracting with the retailer. However, with the rise of the internet, the diversity in the manifestation of dual-channel has expanded significantly. Besides employing contracting with countervailing incentives under asymmetric cost information in a dual-channel supply chain (Huang *et al*.,2023), a manufacturer's contract that sets both a wholesale price and a price for the direct channel can also effectively coordinate the dual-channel supply chain (Chen *et al*., 2012; Zhang *et al*., 2023).

David *et al*. (2015) discovered that traditional contracts are ineffective for dual-channel supply chains. They proposed a linear quantity discount contract tailored for symmetric retailers and provided insights and a methodology to address asymmetric retailer scenarios. Otherwise, the retailer should always bargain the price in a single-channel environment. By contrast, it should not always bargain the price in a dual-channel environment (Matsui, 2022). Analogously, the bargaining model also has (Shen *et al*., 2019).

Chen *et al*. (2021) consider two adoption cases of 3D printing in a dual-channel retail setting. Unlike the literature above, which focuses on establishing contract coordination in dual-channel supply chains, the following explores coordination based on the model structure of a dual-channel supply chain. Developing game-theoretic models to capture both players' pricing decisions and profits, incorporating online consumer reviews under two different channel structures, can enhance the enhancement of dual-channel strategies (Yang *et al*., 2021). For the first time, Jabarzare and Rasti-Barzoki (2020) investigated how packaging companies can influence product quality through packaging and analyzed how different game structures affect optimal pricing, quality decisions, and dual-channel profitability. In addition, some scholars have noticed the dual-channel green supply chain coordination with external intervention. In particular, a unique scenario where the government offers an innovation subsidy. Zhong *et al*. (2023) demonstrate a win–win–win situation for the manufacturer, both selling channels and consumers. When the government establishes a green standard for manufacturers, dual-channel green supply chain management becomes crucial (Gao *et al*., 2021). Based on the manufacturer's corporate social responsibility coefficient, (Li *et al*., 2020) propose designing two contracts to achieve coordination. Xu *et al*. (2018) suggest implementing cap-and-trade regulations by the government to efficiently reduce carbon emissions and promote coordinated development between the economy and the environment. However, few studies have investigated the complete shareholding problem of C2M Manufacturers from a theoretical perspective. To fill this gap, our study investigates the Full shareholding between private-label retailers and platforms from vertical and horizontal perspectives.

We summarize the differences between this paper and the most relevant literature in Table 1.

Table 1. Comparison of this paper with typical literature.

Article	C2M structure	Selling on platforms	agency mode	Brand difference	Channel coordination
Lyu *et al*. (2023)	✓	✓	✓	✓(distance)	
He *et al*. (2023)		✓	✓	✓(distance)	✓
Li *et al*. (2023)					✓
Chen *et al*. (2021)		✓			✓
Cheng *et al*. (2023)		✓	✓	✓(market base)	
Shen *et al*. (2019)		✓			✓
Zhang *et al*. (2019)	✓	✓		✓(quality)	
Li *et al*. (2022)				✓(quality)	
Wang *et al*. (2023)	✓	✓		✓（brand awareness）	
Li *et al*. (2024)	✓	✓	✓	✓(quality and distance)	
This article	✓	✓	✓	✓	✓

3. MODELING SETTING AND BENCHMARK MODEL

This paper considers a supply chain system composed of C2M manufacturers, platforms, and newly entered private label retailers. In this system, manufacturers sell their standard products through platforms and provide products to retailers in an OEM (Original Equipment Manufacturer) manner. Retailers also sell their products through platforms and the platform profits by consigning and charging commissions per item.

3.1 C2M strategies and market segments

As shown in Figure 1, we consider a supply chain composed of three parties: a C2M manufacturer, a private label retailer (She, denoted by subscript R), and a platform (denoted by P). In this supply chain system, we discuss four scenarios: scenarios C, D, MP, and MR. Scenario C exemplifies a centralized supply chain model, where the C2M manufacturer controls both the platform and the private label retailer, setting prices for both the direct channel p_M and the retail channel p_R simultaneously. Conversely, scenario D represents the profit distribution of a supply chain under a decentralization model. The C2M manufacturer determines the direct channel price p_M and the wholesale price w_R, after which the private label retailer determines the retail price p_R. The platform simultaneously charges commissions to both the C2M manufacturer and the private label retailer.

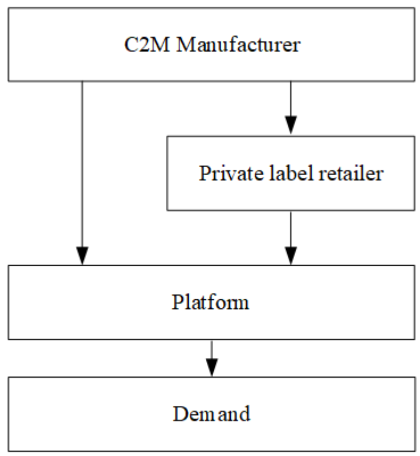

Figure 1. The C2M supply chain structure

The partial centralization model includes scenarios MP and MR. Scenario MP represents a scenario where the C2M manufacturer holds the platform. They decide both the direct price p_M and the wholesale price w_R, after which the private label retailer determines their retail price p_R. Furthermore, due to the C2M manufacturer's ownership of the platform, the retailer also incurs a specific commission payable to the C2M manufacturer. Scenario MR represents a scenario where the C2M manufacturer holds the private label retailer. In this case, the C2M manufacturer determines both the direct channel price p_M and the retail channel price p_R, while the platform charges commissions to the manufacturer through dual channels. Private label brands are products which carry a brand name of a retailer's choice and are fully owned, controlled, and sold exclusively by the retailer. The definition of brand differentiation in this paper comes from Kumar (2007) and Morton and Zettelmeyer (2004) mentioned in the previous introduction. That is, private brand products use the retailer's brand, which is wholly owned and controlled by the retailer and sold exclusively. In contrast, C2M products are fully owned by the manufacturer and use the manufacturer's brand, in which case private label retailers appear as sales channels. For simplicity, we normalize the production cost to zero in this paper (Cai, 2010). Furthermore, it is worth noting that the platform does not participate in decision-making in any scenario discussed in this paper. The symbols used in this paper are summarized in Table 2.

Table 2. Notations for modeling

Notation	Explanation
a_i	Basic demand of Channel $i(i = M, R), (a_i \in (0,1))$
θ	The substitutability of brands in dual-channel $(\theta \in (0,1))$
r_i	The commission charged by the platform to $i(i = M, R), (r_i \in (0,1))$
w_R	The manufacturer's wholesale price
p_i^j	The market price of channel $i(i = M, R)$ under scenario j
D_i^j	The demand of channel $i(i = M, R)$ under scenario j
$\prod_i^j$	The profit of channel $i(i = M, R, P)$ under scenario j
$\prod^j$	The channel-wide profits supply chain under scenario j

3.2 Consumer utility and scenarios

We assume there is only one product for sale. To derive the demand functions under different channel structures, we employ a rational framework established by Ingene and Parry (2004) Chapter 11, Ingene and Parry (2007). We also use similar utility functions for representative consumers, as shown below:

$$U = \sum_{i=M,R} \left(a_i D_i - \frac{D_i^2}{2} \right) - \theta D_M D_R - p_M D_M - p_R D_R. \tag{1}$$

Where θ represents the substitutability between brands, a larger θ indicates higher brand substitutability. The C2M manufacturer introduces a channel with a private label retailer, competing alongside the direct channel. Maximizing utility yields: $D_M = \frac{a_M - \theta a_R - p_M + \theta p_R}{1-\theta^2}$; $D_R = \frac{a_R - \theta a_M - p_R + \theta p_M}{1-\theta^2}$.

During this process, we adopt the Stackelberg game model with the C2M manufacturer as the leader. The profit functions in each scenario are as follows. Using the backward-solving method, we outline the equilibrium solutions for each scenario in Table 3.

Scenario D: $\prod_M^D(p_M, w_R) = (p_M - r_M)D_M + w_R D_R$, $\prod_R^D(p_R) = (p_R - w_R - r_R)D_R$ and $\prod_P^D = r_M D_M + r_R D_R$.
Scenario MP: $\prod_M^{MP}(p_M, w_R) = p_M D_M + (w_R + r_R)D_R$, $\prod_R^{MP}(p_R) = (p_R - w_R - r_R)D_R$.
Scenario MR: $\prod_M^{MR}(p_M, p_R) = (p_M - r_M)D_M + (p_R - r_R)D_R$ and $\prod_P^{MR} = r_M D_M + r_R D_R$.
Scenario C: $\prod_M^C(p_M, p_R) = p_M D_M + p_R D_R$.

Table 3. Equilibrium results in each scenario

	D	MP	MR	C
${w_R}^*$	$\frac{a_R-r_R}{2}$	$\frac{a_R-2r_R}{2}$	/	/
${p_M^i}^*$	$\frac{a_M+r_M}{2}$	$\frac{a_M}{2}$	$\frac{a_M+r_M}{2}$	$\frac{a_M}{2}$
${p_R^i}^*$	$\frac{\theta r_M+r_R-\theta a_M+3a_R}{4}$	$\frac{3a_R-\theta a_M}{4}$	$\frac{a_R+r_R}{2}$	$\frac{a_R}{2}$
${D_M^i}^*$	$\frac{(-2+\theta^2)r_M+\theta r_R-(-2+\theta^2)a_M-\theta a_R}{4(1-\theta^2)}$	$\frac{(2-\theta^2)a_M-\theta a_R}{4(1-\theta^2)}$	$\frac{-r_M+\theta r_R+a_M-\theta a_R}{2(1-\theta^2)}$	$\frac{a_M-\theta a_R}{2(1-\theta^2)}$
${D_R^i}^*$	$\frac{\theta r_M-r_R-\theta a_M+a_R}{4(1-\theta^2)}$	$\frac{a_R-\theta a_M}{4(1-\theta^2)}$	$\frac{\theta r_M-r_R-\theta a_M+a_R}{2(1-\theta^2)}$	$\frac{a_R-\theta a_M}{2(1-\theta^2)}$
${\prod_M^i}^*$	$\frac{(2-\theta^2)r_M^2+r_R^2-(-2+\theta^2)a_M^2-2\theta a_M a_R+a_R^2+2r_M[-\theta r_R+(-2+\theta^2)a_M+\theta a_R]-2r_R(-\theta a_M+a_R)}{8(1-\theta^2)}$	$\frac{(2-\theta^2)a_M^2-2\theta a_M a_R+a_R^2}{8(1-\theta^2)}$	$\frac{r_M^2+r_R^2+2\theta r_R a_M+a_M^2-2(r_R+\theta a_M)a_R+a_R^2-2r_M(\theta r_R+a_M-\theta a_R)}{4(1-\theta^2)}$	$\frac{a_M^2-2\theta a_M a_R+a_R^2}{4(1-\theta^2)}$
${\prod_R^i}^*$	$\frac{(\theta r_M-r_R-\theta a_M+a_R)^2}{16(1-\theta^2)}$	$\frac{(\theta a_M-a_R)^2}{16(1-\theta^2)}$	/	/
${\prod_P^i}^*$	$\frac{(-2+\theta^2)r_M^2-r_M[-2\theta r_R+(-2+\theta^2)a_M+\theta a_R]-r_R(r_R+\theta a_M-a_R)}{4(1-\theta^2)}$	/	$\frac{-r_M^2+r_M(2\theta r_R+a_M-\theta a_R)-r_R(r_R+\theta a_M-a_R)}{2(1-\theta^2)}$	/
$\prod^{i*}$	$\frac{(4-3\theta^2)r_M^2+r_R^2+(-4+\theta^2)\alpha_M^2+2\theta r_M(-r_R+\theta\alpha_M-\alpha_R)+6\theta\alpha_M\alpha_R-3\alpha_R^2+2r_R(-\theta\alpha_M+\alpha_R)}{16(-1+\theta^2)}$	$\frac{(-4+\theta^2)\alpha_M^2+6\theta\alpha_M\alpha_R-3\alpha_R^2}{16(-1+\theta^2)}$	$\frac{r_M^2-2\theta r_M r_R+r_R^2-\alpha_M^2+2\theta\alpha_M\alpha_R-\alpha_R^2}{4(-1+\theta^2)}$	$\frac{a_M^2-2\theta a_M a_R+a_R^2}{4(1-\theta^2)}$

Table 4. Equilibrium under the symmetrical dual-channel

	D	MP	MR	C
${w_R}^*$	$\frac{a-r}{2}$	$\frac{a-2r}{2}$	/	/
${p_M^i}^*$	$\frac{a+r}{2}$	$\frac{a}{2}$	$\frac{a+r}{2}$	$\frac{a}{2}$
${p_R^i}^*$	$\frac{3a+r-a\theta+r\theta}{4}$	$\frac{a(3-\theta)}{4}$	$\frac{a+r}{2}$	$\frac{a}{2}$
${D_M^i}^*$	$\frac{(a-r)(2+\theta)}{4(1+\theta)}$	$\frac{a(2+\theta)}{4(1+\theta)}$	$\frac{a-r}{2(1+\theta)}$	$\frac{a}{2(1+\theta)}$
${D_R^i}^*$	$\frac{a-r}{4(1+\theta)}$	$\frac{a}{4(1+\theta)}$	$\frac{a-r}{2(1+\theta)}$	$\frac{a}{2(1+\theta)}$
${\prod_M^i}^*$	$\frac{(a-r)^2(3+\theta)}{8(1+\theta)}$	$\frac{a^2(3+\theta)}{8(1+\theta)}$	$\frac{(a-r)^2}{2(1+\theta)}$	$\frac{a^2}{2(1+\theta)}$
${\prod_R^i}^*$	$\frac{(a-r)^2(1-\theta)}{16(1+\theta)}$	$\frac{a^2(1-\theta)}{16(1+\theta)}$	/	/
${\prod_P^i}^*$	$\frac{(a-r)r(3+\theta)}{4(1+\theta)}$	/	$\frac{(a-r)r}{1+\theta}$	/
$\prod^{i*}$	$\frac{(a-r)(a(7+\theta)+r(5+3\theta))}{16(1+\theta)}$	$\frac{a^2(7+\theta)}{16(1+\theta)}$	$\frac{a^2-r^2}{2(1+\theta)}$	$\frac{a^2}{2(1+\theta)}$

4. SYMMETRIC DUAL-CHANNEL EQUILIBRIUM ANALYSIS AND COORDINATION

When the C2M manufacturer distributes products through dual-channel, namely direct and retail channels simultaneously via the platform, these channels sell products with the basic demand of a_M and a_R, respectively. Next, we will discuss the profits of each scenario under symmetric dual-channel conditions, where $a = a_M = a_R$, and under symmetric channels, the commission charged by the platform to the C2M manufacturer or private label retailer should also be the same, $r_M = r_{\mathrm{R}} = r$. Amazon (2024), the world's leading e-commerce platform, offers a sales program with a flat commission of \$0.99 per item sold, uniformly applied across all channels. Building on the same basic market demand, this section assumes a consistent commission structure, reinforcing the model's feasibility. We have equilibrium solutions for each scenario under symmetric dual-channel conditions, as shown in Table 4. Next, we will compare these equilibrium results with previous research findings. To ensure non-negativity of wholesale and retail prices, it holds that $a > 2r$.

Lemma 1. Stability and Flexibility of Symmetric Dual-Channel System.

(1) In scenario MR, the demand of the direct channel equals the demand of the retail channel; this is also true in scenario C. **(2)** Scenario D, $\frac{\partial {D_M^D}^*}{\partial \theta} < 0$, $\frac{\partial {D_R^D}^*}{\partial \theta} < 0$; $\frac{\partial {D_M^D}^*}{\partial r} < 0$, $\frac{\partial {D_R^D}^*}{\partial r} < 0$; Scenario MP, $\frac{\partial {D_M^{MP}}^*}{\partial \theta} < 0$, $\frac{\partial {D_R^{MP}}^*}{\partial \theta} < 0$; Scenario MR, $\frac{\partial {D_M^{MR}}^*}{\partial r} < 0$, $\frac{\partial {D_R^{MR}}^*}{\partial r} < 0$.

Lemma 1 primarily explores the interactions between manufacturer and private label in various scenarios of symmetric dual-channel, alongside the relationships with commission rates and brand substitutability. **(1)** illustrates that when the manufacturer holds retailer, as in scenarios MR and C, demands in both dual distribution channels are identical. Because of the fixed fee structure in symmetric channels, which does not affect pricing within the channels, equal demands result in these scenarios. In **(2)** scenario D, increased commissions and brand substitutability lead to a significant demand shift from dual-channel to direct channel. Similarly, in scenario MP, increasing brand substitutability shifts demand from the dual channel to the direct channel. Compared to single-channel setups, dual-channel in these scenarios facilitates the transfer of demand and revenue between the two channels. Therefore, the stability and flexibility of dual-channel encourage stakeholders in the system to coordinate supply chains, mitigating dual marginalization effects and alleviating efficiency issues caused by system fragmentation. Additionally, we can observe that in scenarios D and MR, where the commission is applied, the impact of the commission on demand exhibits an inverse relationship, similar to the effect seen with branding. Next, we will analyze the relationships between equilibrium prices and demands in the four scenarios.

Lemma 2. Comparison of the prices and demands results under the symmetric channel.

(1) $w_R^{D^*} > w_R^{MP^*}$;

(2) $p_M^{D^*} = p_M^{MR^*} > p_M^{C^*} = p_M^{MP^*}$;

(3) If $2r < a < 2r/1-\theta$, $p_R^{D^*} > p_R^{MR^*} > p_R^{MP^*} > p_R^{C^*}$; If $2r/1-\theta < a < 1$, $p_R^{D^*} > p_R^{MP^*} > p_R^{MR^*} > p_R^{C^*}$;

(4) If $2r < a < r(2+\theta)/\theta$, $D_M^{MP^*} > D_M^{C^*} > D_M^{D^*} > D_M^{MR^*}$; If $r(2+\theta)/\theta < a < 1$, $D_M^{MP^*} > D_M^{D^*} > D_M^{C^*} > D_M^{MR^*}$;

(5) $D_R^{C^*} > D_R^{MR^*} > D_R^{MP^*} > D_R^{D^*}$.

Our previous table shows that $w_R{}^*$ and $p_M{}^*$ are primarily determined by basic demand and commissions, regardless of product substitutability. This outcome stems from the decisions where brand products from the direct channel are introduced to the market before any retailer competition. The advantage of the direct channel lies in its ability to first enhance brand value and competitive advantage by introducing the private label retailer, thereby increasing brand influence and market share. Secondly, the private label retailer enables better control over product quality and pricing, thereby enhancing profit margins. Additionally, they facilitate stronger customer relationships, fostering higher customer loyalty and repeat purchases. We also observe that wholesale prices are directly proportional to basic demand. This occurs because, in scenarios where manufacturers do not hold retailers, manufacturers often increase wholesale prices to boost demand and profits in their direct channels, responding to high market demand. The relationship between wholesale prices and commission is straightforward. Higher wholesale prices reduce retailer profit margins. To protect retailer profits, the commission must remain at lower levels; otherwise, retailer partners may withdraw from the supply chain, which is detrimental to the manufacturer and platform.

In **(1)**, we observe that wholesale prices are higher in scenario D. In scenario MP, the manufacturer holding the platform can profit from both commission and wholesale prices. Therefore, manufacturers reduce wholesale prices to mitigate the dual marginalization effects on private-label retailers. In **(2)**, we find that for $p_M{}^*$, in scenarios D and MR, where the manufacturer does not hold the platform, the outcomes are higher compared to scenarios C and MP, where the manufacturer does hold the platform. This is due to the commission charged by the platform when the manufacturer does not own it. Interestingly, in scenarios D, MR, and C, MP, the difference in retail prices in the direct channel is $r/2$. This indicates that when manufacturers do not hold platforms, in the decentralization model, they increase the retail price and direct channel price by $r/2$ to offset the profit loss caused by platform commission. Similarly, in the partial centralization model under scenario MR, the manufacturer increases the price by $r/2$ in the direct channel when they do not hold the platform. Additionally, **(3)** reveals that the price $p_R{}^*$ of retail is $r/2$ higher than in the centralization model under scenario C, again aiming to mitigate the impact of platform commission.

For **(4)** and **(5)**, the comparison is straightforward, revealing the variations in demands across channels in different scenarios. The decentralization model exhibits inefficiencies for two primary reasons. Firstly, the wholesale prices in the retail channel lead to double marginalization, reducing their demands. Secondly, as higher channel prices result in fewer orders, reduced platform earnings prompt higher commissions, exacerbating the double marginalization effect.

Proposition 1. Under constraint, the profit variation of each participant,

(1) If $2r < a < \frac{2[2r+\sqrt{r^2(3+\theta)}]}{1-\theta}$, $\prod_M^{C^*} > \prod_M^{MP^*} > \prod_M^{MR^*} > \prod_M^{D^*}$, Region II;

$\frac{2[2r+\sqrt{r^2(3+\theta)}]}{1-\theta} < a < 1$, $\prod_M^{C^*} > \prod_M^{MR^*} > \prod_M^{MP^*} > \prod_M^{D^*}$, Region I in Figure 2.

(2) If $a > 2r$, $\prod_R^{MP^*} > \prod_R^{D^*}$ constantly.

(3) If $a > 2r$, $\prod_P^{MR^*} > \prod_P^{D^*}$ constantly.

(4) If $2r < a < \frac{2\sqrt{2}r}{\sqrt{1-\theta}}$, $\prod^{C^*} > \prod^{MP^*} > \prod^{MR^*} > \prod^{D^*}$, Region II;

$\frac{2\sqrt{2}r}{\sqrt{1-\theta}} < a < 1$, $\prod^{C^*} > \prod^{MR^*} > \prod^{MP^*} > \prod^{D^*}$, Region I in Figure 3.

Proposition 1 emphasizes that in the decentralization model, scenario D leads to the lowest profits for both the manufacturer and the entire supply chain. From figure 2, we observe that: In Region I, characterized by moderate market demand and moderate brand substitutability ($a \in (0.2239,1), \theta \in (0,0.7636)$), the manufacturer achieves higher profits in the centralization model compared to scenarios with only holding the retailer or the platform, and significantly more than in the decentralization model. In Region II, where the basic demand is lower and the brand substitutability is higher, the advantages become more pronounced. Here, the manufacturer attains maximum profits in the centralization model, followed by scenarios hold platform or hold retailer, with the lowest profits under the decentralization model. This observation is straightforward: the C2M manufacturer benefits more from the centralization model, while profits are lowest under the decentralization model. Interesting insights emerge regarding the profit variations in scenarios MP and MR: In healthier markets, specifically in Region I, the profit under the partial centralization model exceeds that under the decentralization model ($\prod_M^{MR^*} > \prod_M^{MP^*}$). This indicates that holding only the retailer is a preferable decision for manufacturers in scenarios with moderate to high basic demand, where holding the retailer can effectively capture the market share. Moreover, with brand substitutability within normal ranges, holding retailers proves profitable. However, within Region II, characterized by increasing basic demand and higher brand substitutability, market distortions quickly arise. Here, either basic demand is very low or brand substitutability is extremely high, making it challenging for late-entering retailers to gain traction. Consequently, in such cases, the profit for the C2M manufacturer holding the platform surpasses that of the retailer.

From Figure 3, it is evident that: Region I holds a dominant position when comparing the overall profit of the supply chain, where $a \in (0.0848528,1)$. Combining insights from both figures, we conclude that in a partial centralization model, with basic demand held constant, the manufacturer should prioritize scenarios MR followed by MP as brand substitutability increases. Conversely, with brand substitutability fixed, the manufacturer should prioritize scenario MP as market demand increases, followed by scenario MR.

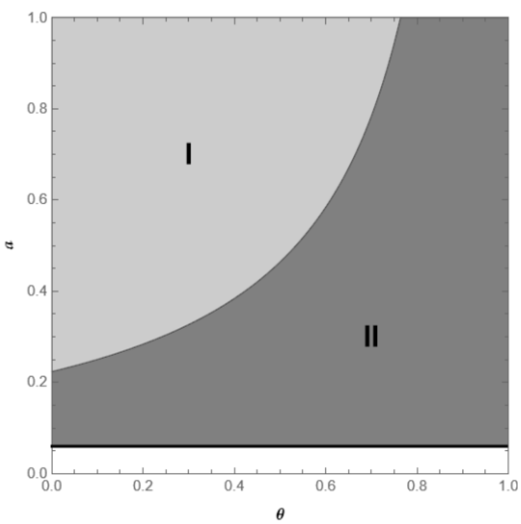

Figure 1. Profit comparison of C2M manufacturers in four scenarios (r = 0.03)

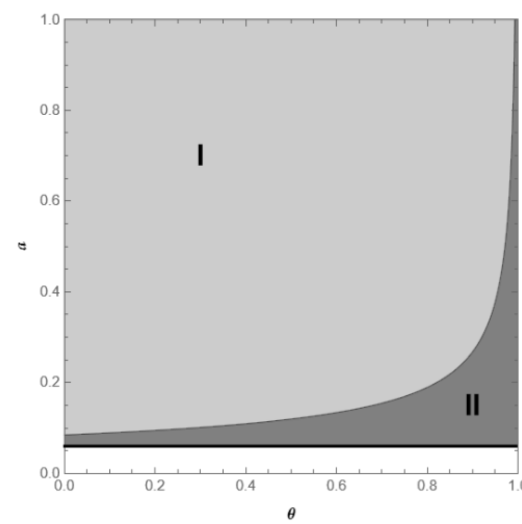

Figure 2. Profit comparison of channel-wide in four scenarios (r = 0.03)

Corollary 1. Scenario C always represents the optimal solution. However, the optimal choices made by the C2M manufacturer consistently benefit the entire supply chain in Regions I and II. In Region III, the C2M manufacturer and the entire supply chain face a prisoner's dilemma when facing the partial centralization scenario. Figure 4.

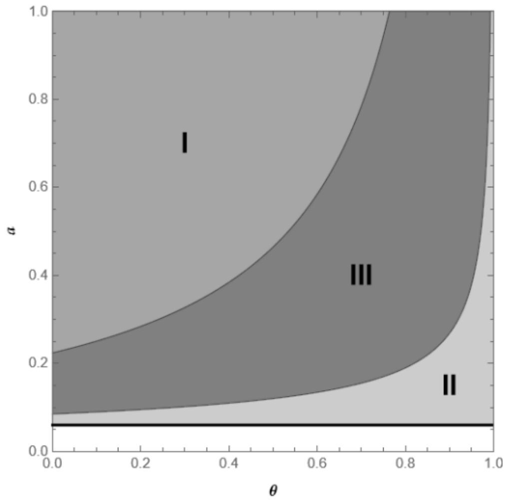

Figure 3. Comprehensive comparison

Corollary 1 asserts that under a commission of 0.03, scenario C consistently emerges as the optimal choice, while scenario D yields the lowest profit. This is rooted in the ability of all supply chain members to mitigate double marginalization effects through cooperative gameplay in the centralization model.

However, profit dynamics in a partial centralization model are not static. In Regions I and II, where basic demand is significant and brand substitutability varies, manufacturers' decisions not only maximize their profits but also optimize overall supply chain returns. Region I typically features substantial basic demand and moderate brand substitutability, whereas Region II is characterized by lower demand and higher brand substitutability. Here, holding the platform generates higher profits than holding the retailer. In Region III, the manufacturer and the supply chain face a prisoner's dilemma, where individual decisions may maximize personal profits but collectively reduce supply chain profitability. In such scenarios, centralization management may not be favorable given the adverse market conditions and brand substitutability, necessitating alternative strategies to coordinate and enhance supply chain efficiency. Figure 4 explicitly illustrates how manufacturer should select channel structures to achieve optimal returns under varying market conditions. In Region I, priority should be given to holding retail channels (scenario MR), followed by platform ownership (scenario MP). In Region II, as basic demand increases and brand substitutability rises, it is recommended to hold the platform (MP) rather than the retailer (MR).

The above analysis explored the equilibrium solutions for different scenarios under symmetric dual-channel setups, revealing that the centralized model in scenario C consistently remains optimal. Meanwhile, it also revealed the predicament under the partial centralization model. Consequently, our next step will examine how C2M manufacturers should design contracts with private label retailers to incentivize optimal profit optimization within their respective scenarios.

4.1 Symmetrical dual-channel supply chain coordination mechanism

We analyze the equilibrium solution to discuss how C2M manufacturers and private label retailers in symmetric dual-channel settings can formulate contracts to eliminate dual marginalization effects. In scenarios C and MR under consideration, manufacturers hold retailer, and there is a constant $\prod_M^{C^*} > \prod_M^{MP^*}, \prod^{C^*} > \prod^{MP^*}$; and $\prod_M^{MR^*} > \prod_M^{D^*}, \prod^{MR^*} > \prod^{D^*}$. We adopt a cooperative game approach to establish contract agreements between manufacturers and retailers in scenarios MP and D. Our initial focus will be on addressing coordination issues, specifically in scenario MP.

4.1.1 Coordination Mechanism and Numerical Analysis of Scenario D

In the decentralization supply chain scenario D, for a given wholesale price (w_R) and direct channel prices ($p_M^{D^*}$), p_R^D is concave in $\prod_R^D$. From $\frac{d\prod_R^D}{dp_R^D} = 0$, we derive that.

$$p_R^D = \frac{1}{2}(\theta p_M + r + w_R - \theta a + a). \tag{2}$$

According to **Lemma 2** item **(2)**, we discover $p_M^{D^*} = p_M^{MR^*}$. This implies that the manufacturer's optimal price in the direct channel is the same in the partial centralization dual- channel (MR) and the decentralization dual-channel supply chain. Let w_{R1} denote the wholesale price specified in a contract $(w_{R1}, p_M^{MR^*})$ with the wholesale price and the direct channel price offered by the manufacturer. If the contract $(w_{R1}, p_M^{MR^*})$ can induce the retailer to order $D_R^{MR^*}$, the manufacturer should have the retailer set the self-interested retail price p_R^D in **(7)** equal to $p_R^{MR^*}$. Thus, the retailer will order $D_R^{MR^*}$ and the order quantity in the direct channel is $D_M^{MR^*}$. Given $p_M^{MR^*}$ and w_{R1}, the retail price p_R^D in **(7)** becomes

$$p_R^D(w_{R1}) = \frac{1}{2}\left(r + a + \frac{1}{2}(r-a)\theta + w_{R1}\right). \tag{3}$$

By setting **(3)** equal to $p_R^{MR^*}$, we have

$$w_{R1} = \frac{1}{2}(a\theta - r\theta). \tag{4}$$

We substitute various equations to calculate the profits of each part of the supply chain in the coordinated scenario D$(w_{R1}, p_M^{MR^*})$

$$\prod\nolimits_M^D(w_{R1}) = \left(p_M^{MR^*} - r\right)D_M^{MR^*} + w_{R1}D_R^{MR^*} = \frac{(r-a)^2}{4}. \tag{5}$$

$$\prod\nolimits_R^D(w_{R1}) = (p_R(w_{R1}) - w_{R1} - r)D_R^{MR^*} = \frac{(r-a)^2(1-\theta)}{4(1+\theta)}. \tag{6}$$

$$\prod\nolimits_P^D(w_{R1}) = rD_M^{MR^*} + rD_R^{MR^*} = \frac{r(a-r)}{1+\theta}. \tag{7}$$

$$\prod\nolimits^D(w_{R1}) = \frac{a^2 - r^2}{2(1+\theta)}. \tag{8}$$

Proposition 2. When using contract $(w_{R1}, p_M^{MR^*})$ to coordinate Scenario D, $\prod^D(w_{R1}) = \prod^{MR^*}$, it indicates that the contract offered by the C2M manufacturer can coordinate Scenario D.

Based on the above analysis, the contract $(w_{R1}, p_M^{MR^*})$ for the manufacturer can coordinate the supply chain, resulting in positive profits. Comparing Formulas **(5)** to **(8)** with the profits of the manufacturer, the platform, and the overall supply chain in Table 4 of Scenario D, we observe that this contract increases the profits of the platform and the entire supply chain but does not increase the profits of the manufacturer. Next, we will implement a complementary two-part tariff agreement alongside the $(w_{R1}, p_M^{MR^*})$ contract to coordinate Scenario D, ensuring a win-win-win outcome for the C2M manufacturer, private label retailer, and the platform.

The two-part tariff agreement has been extensively studied in the literature (Moorthy 1987) (Coughlan and Wernerfelt 1989) for a single retail channel supply chain, where the manufacturer sets the wholesale price equal to the unit production cost and charges a lump sum fee to the retailer. In this paper, we find that with a two-part tariff agreement, the manufacturer can charge a lump sum fee (F) when it offers a contract $(w_{R1}, p_M^{MR^*})$, so that the contract $(w_{R1}, p_M^{MR^*}, F)$ can coordinate the dual-channel supply chain and enable the retailer, platform and manufacturer to be a win-win-win.

When a lump sum fee F satisfies $\prod_M^D(w_{R1}) + \mathrm{F} > \prod_M^D$, the manufacturer will prefer to offer a $(w_{R1}, p_M^{MR^*}, F)$ contract, which yields

$$F > \frac{(r-a)^2(1-\theta)}{8(1+\theta)} = F_1, \tag{9}$$

When a lump sum fee F satisfies $\prod_R^D(w_{R1}) - F > \prod_R^D$, the retailer will accept a $(w_{R1}, p_M^{MR^*}, F)$ contract, which yields

$$F < \frac{3(r-a)^2(1-\theta)}{16(1+\theta)} = F_2, \quad (10)$$

Considering **(9)** and **(10)**, we see that the range of the lump sum fee (F) in which both the retailer and the manufacturer benefit is (F_1, F_2). We summarize the results in **Proposition 3**.

Proposition 3. If the private label retailer pays a specified cost F to the C2M manufacturer, where F belongs to (F_1, F_2), then the contract$(w_{R1}, p_M^{MR^*}, F)$ can coordinate decentralization Scenario D. This ensures a win-win-win for the C2M manufacturer, the platform, and the private label retailer. Where w_{R1}, F_1 and F_2 are given by **(4), (9), and (10),** respectively.

Our numerical experiments demonstrate how this contract coordinates. Scenario D ensures that the manufacturer, retailer, and platform achieve increased profits. We set $\theta = 0.5$, $r = 0.1$, and maintain $a > r$. Table 5 illustrates the outcomes for different values of a in Scenario D and Scenario D optimized with contract $(w_{R1}, p_M^{MR^*})$. In Scenario D, as a increases, both manufacturer and retailer should increase their wholesale and retail prices, primarily due to self-interest in the decentralization setting. However, through comparison, it becomes evident that contract $(w_{R1}, p_M^{MR^*})$ can coordinate Scenario D, enhancing profits for the retailer and the platform and improving channel-wide supply chain efficiency while not increasing profits for the manufacturer. This reaffirms **Proposition 2**.

Figure 5 illustrates how the upper and lower limits of the one-time fee F vary with changes in a under the contract $(w_{R1}, p_M^{MR^*}, F)$. As a increases, both F_1 and F_2 increase. This indicates that with enhanced basic demand, the one-time fee F increases, expanding the fee space and negotiation potential.

Table 5. Results for scenario D and coordination under contract $(w_{R1}, p_M^{MR^*})$ without a two-part tariff agreement, where $\theta = 0.5$, $r = 0.1$

a	Scenario D							Coordinated scenario D under a contract $(w_{R1}, p_M^{MR^*})$						
	$w_R^{D^*}$	$p_M^{D^*}$	$p_R^{D^*}$	$\prod_M^{D^*}$	$\prod_R^{D^*}$	$\prod_P^{D^*}$	$\prod^{D^*}$	w_{R1}	$p_M^{MR^*}$	$p_R^{D^*}(w_{R1})$	$\prod_M^{D^*}(w_{R1})$	$\prod_R^{D^*}(w_{R1})$	$\prod_P^{D^*}(w_{R1})$	$\prod^{D^*}(w_{R1})$
0.4	0.12	0.25	0.2875	0.0262	0.0018	0.0175	0.0456	0.075	0.25	0.25	0.0225	0.0075	0.02	0.05
0.5	0.2	0.3	0.35	0.0466	0.0033	0.0233	0.0733	0.1	0.3	0.3	0.04	0.0133	0.0266	0.08
0.6	0.25	0.35	0.4125	0.0729	0.0052	0.0291	0.1072	0.125	0.35	0.35	0.0625	0.0208	0.0333	0.1166
0.7	0.3	0.4	0.475	0.105	0.0075	0.035	0.1475	0.15	0.4	0.4	0.09	0.03	0.04	0.16
0.8	0.35	0.45	0.5375	0.1429	0.0102	0.0408	0.1939	0.175	0.45	0.45	0.1225	0.0408	0.0466	0.21
0.9	0.4	0.5	0.6	0.1866	0.0133	0.0466	0.2466	0.2	0.5	0.5	0.16	0.0533	0.0533	0.2666

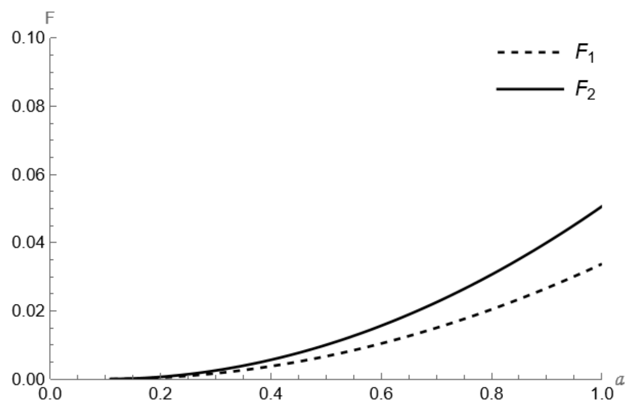

Figure 4. Range of F for contract $(w_{R1}, p_M^{MR^*}, F)$

4.1.2 Coordination Mechanism and Numerical Analysis of Scenario MP

We calculate it in a similar way as in the previous section. From $\frac{d\prod_R^{MP}}{dp_M^{MP}} = 0$, we derive that

$$p_R^{MP} = \frac{1}{2}(\theta p_M + r + w_R - \theta a + a). \tag{11}$$

and

$$w_{R2} = \frac{1}{2}(a\theta - 2r). \tag{12}$$

$$\prod_M^{MP}(w_{R2}) = p_M^{C^*} D_M^{C^*} + (w_{R2} + r) D_R^{C^*} = \frac{a^2}{4}. \tag{13}$$

$$\prod_R^{MP}(w_{R2}) = (p_R(w_{R2}) - w_{R2} - r) D_R^{C^*} = \frac{a^2(1-\theta)}{4(1+\theta)}. \tag{14}$$

$$\prod^{MP}(w_{R2}) = \frac{a^2}{2(1+\theta)}. \tag{15}$$

Proposition 4. When using contract $(w_{R2}, p_M^{C^*})$ to coordinate Scenario MP, $\prod^{MP}(w_{R2}) = \prod^{C^*}$, this indicates that the contract offered by the C2M manufacturer can coordinate Scenario MP.

Similar to the previous section, when a lump sum fee F satisfies $\prod_M^{MP}(w_{R2}) + F > \prod_M^{MP}$, the manufacturer will prefer to offer a $(w_{R2}, p_M^{C^*}, F)$ contract, which yields

$$F > \frac{a^2(1-\theta)}{8(1+\theta)} = F_3, \tag{16}$$

When a lump sum fee F satisfies $\prod_R^{MP}(w_{R2}) - F > \prod_R^{MP}$, the retailer will accept a $(w_{R2}, p_M^{C^*}, F)$ contract, which yields

$$F < \frac{3a^2(1-\theta)}{16(1+\theta)} = F_4, \tag{17}$$

Considering **(16)** and **(17)**, we see that the range of the lump sum fee (F) in which both the retailer and the manufacturer benefit is (F_3, F_4). We summarize the results in **Proposition 5.**

Proposition 5. If the private label retailer pays a specified cost F to the C2M manufacturer, where F belongs to (F_3, F_4), then the contract $(w_{R2}, p_M^{C^*}, F)$ can coordinate partial centralization Scenario MP. This ensures a win-win outcome for the C2M manufacturer and the private label retailer, where w_{R2}, F_3 and F_4 are given by **(12), (16), (17)** respectively.

Our numerical experiments illustrate how this contract coordinates Scenario MP to ensure increased profits for both the manufacturer and the retailer. We set $\theta = 0.5$, $r = 0.1$, and maintain $a > 2r$. Table 6 shows the results for different values of a in Scenario MP and optimized Scenario MP with the contract $(w_{R2}, p_M^{C^*})$. In both Scenario MP and optimized Scenario MP with the contract $(w_{R2}, p_M^{C^*})$, as a increases, the manufacturer should decrease wholesale prices to increase profits for the retailer and the channel-wide supply chain. Similarly, the retailer should decrease their retail prices to increase their profits. This reaffirms **Proposition 4**. We also find that the contract $(w_{R2}, p_M^{C^*}, F)$ can coordinate Scenario MP, increasing profits for the retailer and enhancing channel-wide supply chain efficiency, but it does not increase profits for the manufacturer.

Table. 6. Results for scenario MP and coordination under contract $(w_{R2}, p_M^{C^*})$ without a two-part tariff agreement, where $\theta = 0.5$, $r = 0.1$

a	Scenario MP						Coordinated scenario MP under a contract $(w_{R2}, p_M^{C^*})$					
	$w_R^{MP^*}$	$p_M^{MP^*}$	$p_R^{MP^*}$	$\prod_M^{MP^*}$	$\prod_R^{MP^*}$	$\prod^{MP^*}$	w_{R2}	$p_M^{C^*}$	$p_R^{MP^*}(w_{R2})$	$\prod_M^{MP^*}(w_{R2})$	$\prod_R^{MP^*}(w_{R2})$	$\prod^{MP^*}(w_{R2})$
0.4	0.1	0.2	0.25	0.0466	0.0033	0.05	0	0.2	0.2	0.04	0.0133	0.0533
0.5	0.15	0.25	0.3125	0.0729	0.0052	0.0781	0.025	0.25	0.25	0.0625	0.0208	0.0833

0.6	0.2	0.3	0.375	0.105	0.0075	0.1125	0.05	0.3	0.3	0.09	0.03	0.12
0.7	0.25	0.35	0.4375	0.1429	0.0102	0.1531	0.075	0.35	0.35	0.1255	0.0408	0.1633
0.8	0.3	0.4	0.5	0.1866	0.0133	0.2	0.1	0.4	0.4	0.16	0.0533	0.2133
0.9	0.35	0.45	0.5625	0.2362	0.0168	0.2531	0.125	0.45	0.45	0.2025	0.0675	0.27

Figure 6 illustrates how the upper and lower limits of the one-time fee F vary with changes in a under the contract $(w_{R2}, p_M^{C^*}, F)$. Here, the range between F_3F_4 is larger than the range between F_1F_2 mentioned earlier, but the overall trend remains consistent, as a increases, the negotiation space available to manufacturer and retailers expands

Contracts $(w_{R1}, p_M^{MR^*}, F)$ and $(w_{R2}, p_M^{C^*}, F)$ effectively coordinate scenarios D and MP, respectively. The parameter F operates within specified ranges, ensuring increased profitability for both the retailer and the platform, while also elevating the manufacturer's profits. Higher values of F favor the C2M manufacturer, whereas lower values benefit the retailer. The exact value of F significantly hinges on bargaining capabilities within the supply chain. Coughlan and Wernerfelt (1989) highlight the effectiveness of two-part tariff contracts in single-channel retail supply chains, where manufacturers lead and can theoretically set profit margins arbitrarily. In contrast, executing such contracts $(w_{R1}, p_M^{MR^*}, F)$ and $(w_{R2}, p_M^{C^*}, F)$ in dual-channel supply chains is comparatively straightforward. Manufacturer set wholesale prices w_{R1} and w_{R2} above their production costs but below traditional wholesale prices (w_R), benefiting retailers who face lower upfront costs before lengthy sales seasons. Moreover, manufacturers commit to and inform retailers about direct channel pricing. Given a specified range of one-time total fees (F) that manufacturers charge, retailers accept these contracts as they realize they can achieve higher profits compared to decentralization dual-channel supply chains. Thus, the contracts $(w_{R1}, p_M^{MR^*}, F)$ and $(w_{R2}, p_M^{C^*}, F)$ are easily implementable solutions.

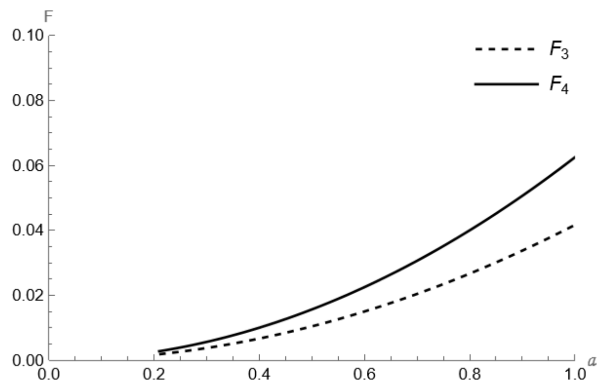

Figure 5. Range of F for contract $(w_{R2}, p_M^{C^*}, F)$

5. ASYMMETRIC DUAL-CHANNEL EQUILIBRIUM ANALYSIS AND COORDINATION

We define $\Omega = a_M/a_R$ as the relative channel status of Channel M to Channel R in asymmetric dual-channel environments. If $\Omega > 1$, Channel M initially excels in basic demand compared to Channel R; conversely, if $\Omega < 1$, Channel R holds this advantage. $\Omega = 1$ denotes a symmetric channel system, previously discussed in earlier chapters. We explore the combined effects of brand substitutability and channel asymmetry across different scenarios. By comparing efficiency-profit ratios among different scenarios with $a_R = 1$, the results are summarized in the table below Table 7.

To ensure that prices and demands are all non-negative, we have the following conditions：

$$r_R < \frac{1}{2} \text{ and } max\,\{\theta, r_M + \theta(1 - r_R)\} < \Omega < min\,\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\}.$$

Based on the results in Table 7, we can derive the relationships between equilibrium prices and demands in the following four scenarios.

Table 7. Equilibrium under the asymmetric dual-channel

	D	MP	MR	C
${w_R}^*$	$\frac{1-r_R}{2}$	$\frac{1-2r_R}{2}$	/	/
${p_M^i}^*$	$\frac{\Omega+r_M}{2}$	$\frac{\Omega}{2}$	$\frac{\Omega+r_M}{2}$	$\frac{\Omega}{2}$
${p_R^i}^*$	$\frac{3-\theta\Omega+\theta r_M+r_R}{4}$	$\frac{3-\theta\Omega}{4}$	$\frac{1+r_R}{2}$	$\frac{1}{2}$
${D_M^i}^*$	$\frac{2\Omega-\theta(1+\theta\Omega)+(-2+\theta^2)r_M+\theta r_R}{4(1-\theta^2)}$	$\frac{2\Omega-\theta(1+\theta\Omega)}{4(1-\theta^2)}$	$\frac{\Omega-\theta-r_M+\theta r_R}{2(1-\theta^2)}$	$\frac{\Omega-\theta}{2(1-\theta^2)}$
${D_R^i}^*$	$\frac{1-\theta\Omega+\theta r_M-r_R}{4(1-\theta^2)}$	$\frac{1-\theta\Omega}{4(1-\theta^2)}$	$\frac{1-\theta\Omega+\theta r_M-r_R}{2(1-\theta^2)}$	$\frac{1-\theta\Omega}{2(1-\theta^2)}$
${\prod_M^i}^*$	$\frac{1-\Omega\left(2\theta+\left(-2+\theta^2\right)\Omega\right)-\left(-2+\theta^2\right)r_M^2+r_R(-2+2\theta\Omega+r_R)+2r_M(\theta+(-2+\theta^2)\Omega-\theta r_R)}{8(1-\theta^2)}$	$\frac{1-\Omega(2\theta+(-2+\theta^2)\Omega)}{8(1-\theta^2)}$	$\frac{1+\Omega^2+r_M^2+2\theta\Omega r_R+r_R^2-2(\theta\Omega+r_R)-2r_M(-\theta+\Omega+\theta r_R)}{4(1-\theta^2)}$	$\frac{1-2\theta\Omega+\Omega^2}{4(1-\theta^2)}$
${\prod_R^i}^*$	$\frac{(1-\theta\Omega+\theta r_M-r_R)^2}{16(1-\theta^2)}$	$\frac{(1-\theta\Omega)^2}{16(1-\theta^2)}$	/	/
${\prod_P^i}^*$	$\frac{-r_M(\theta+(-2+\theta^2)\Omega+(-2+\theta^2)r_M)-r_R+\theta(\Omega+2r_M)r_R+r_R^2}{4(1-\theta^2)}$	/	$\frac{-r_M^2-r_R(-1+\theta\Omega+r_R)+r_M(-\theta+\Omega+2\theta r_R)}{2(1-\theta^2)}$	/
$\prod^{i^*}$	$\frac{3-\Omega\left(6\theta+\left(-4+\theta^2\right)\Omega\right)+\left(-4+3\theta^2\right)r_M^2+(-2+2\theta\Omega-r_R)r_R+2\theta r_M(1-\theta\Omega+r_R)}{16(1-\theta^2)}$	$\frac{3-\Omega(6\theta+(-4+\theta^2)\Omega)}{16(1-\theta^2)}$	$\frac{1-2\theta\Omega+\Omega^2-r_M^2+2\theta r_M r_R-r_R^2}{4(1-\theta^2)}$	$\frac{1-2\theta\Omega+\Omega^2}{4(1-\theta^2)}$

Lemma 3. Comparison of the prices and demands results under the asymmetric channel.

(1) $w_R^{D^*}>w_R^{MP^*}$;

(2) ${p_M^D}^*={p_M^{MR}}^*>{p_M^C}^*={p_M^{MP}}^*$;

(3) If $max\,\{\theta, r_M+\theta(1-r_R)\}<\Omega<\frac{1-2r_R}{\theta}$, $p_R^{D^*}>p_R^{MP^*}>p_R^{MR^*}>p_R^{C^*}$;

If $\frac{1-2r_R}{\theta}<\Omega<min\,\{\frac{1}{\theta},\frac{1+\theta r_M-r_R}{\theta}\}$, $p_R^{D^*}>p_R^{MR^*}>p_R^{MP^*}>p_R^{C^*}$;

(4) If $r_M+\theta(1-r_R)<\Omega<\frac{1+\theta r_M-r_R}{\theta}$, ${D_M^{MR}}^*>{D_M^D}^*$;

If $0<\Omega<\frac{1}{\theta}$, $D_M^{MP^*}>{D_M^C}^*$;

(5) If $r_M+\theta(1-r_R)<\Omega<\frac{1+\theta r_M-r_R}{\theta}$, $D_R^{MR^*}>D_R^{D^*}$;

If $\theta<\Omega<\frac{1}{\theta}$, $D_R^{C^*}>D_R^{MP^*}$.

Lemma 3 primarily compares scenarios in asymmetric dual-channel settings. **(1)** and **(2)** points are like results found in symmetric dual-channel scenarios. Despite changes in wholesale and direct channel prices in asymmetric dual-channel setups, their relative sizes in corresponding scenarios remain unaffected; hence, the comparison results remain unchanged. **(3)** is akin to findings from symmetric dual-channel scenarios as well. In centralization scenario C, retail price is consistently the lowest, while in decentralization scenario D, retail prices are the highest. In partial centralization scenarios MP and MR exhibit non-uniform retail prices; when Ω is smaller (indicating lower demand in the direct channel market), $p_R^{MP^*}>p_R^{MR^*}$; conversely, when Ω is larger (indicating higher demand in the direct channel market), $p_R^{MR^*}>p_R^{MP^*}$. Demand in direct and retail channels is influenced by the presence of commissions, leading us to compare the relationships between scenarios D and MR, and scenarios C and MP. This comparison will aid us in future supply chain coordination efforts.

Proposition 6. Under constraint, the profit variation of C2M manufacturer, If $\frac{\theta+2r_M-2\theta r_R-\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2} < \Omega < \frac{\theta+2r_M-2\theta r_R+\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2}$, $\prod_M^{C\,*} > \prod_M^{MP*} > \prod_M^{MR*} > \prod_M^{D\,*}$, Region I; $\Omega < \frac{\theta+2r_M-2\theta r_R-\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2}$, $\prod_M^{C\,*} > \prod_M^{MR*} > \prod_M^{MP*} > \prod_M^{D\,*}$, Region II. As r_M increases until $r_{M1} = \Omega - \theta(1-r_R)$ and as r_R increases until $r_{R1} = \frac{1}{2}(2 - 2\theta\Omega + 2\theta r_M - \sqrt{2}\sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 + 4\Omega r_M - 4\theta^2\Omega r_M - 2r_M{}^2 + 2\theta^2 r_M{}^2})$, $\prod_M^{MP*}$ also increases until it completely dominates $\prod_M^{MR*}$.

Proposition 6 is relatively intuitive. Like the dual channel with symmetric relative channel status, $\prod_M^{C\,*}$ is always optimal whether their relative channel power is stronger or weaker. Meanwhile, consumers always benefit from scenario C because centralization virtually eliminates the double marginalization effect in this supply chain, allowing them to get a lower price and, consequently, higher demand. It is not surprising that scenario C is consistently the best among the four scenarios. We also study the impact of commissions in dual channels and provide the thresholds for each commission. It is easier to understand that the higher the commission, the friendlier it is for the scenarios that hold the platform, namely scenario MP and scenario C.

An interesting question is whether asymmetric channel power affects the equilibrium for all members, including the entire supply chain. We can see from Figure 7 that an increase in Ω is more beneficial to the scenario MP holding the platform. At the same time, the decrease in Ω is more beneficial to the scenario MR holding the retailer. In scenario MP, the enhanced power of the direct selling channel will increase the manufacturer's profit. In addition, we also see that high brand substitutability is almost destructive to the manufacturers of scenario MR because undifferentiated brand substitutability will make consumers' choices almost depend only on the relative channel strength.

Corollary 2. Consistently $\prod_R^{MP*} > \prod_R^{D*}$;

When $0 < \theta < \frac{r_R}{r_M}$ and $r_M + \theta(1-r_R) < \Omega < \frac{1+\theta r_M - r_R}{\theta}$, $\prod_P^{MR*} > \prod_P^{D*}$;

When $\frac{r_R}{r_M} < \theta < 1$ and $r_M + \theta(1-r_R) < \Omega < \frac{1+\theta r_M - r_R}{\theta}$, $\prod_P^{D*} > \prod_P^{MR*}$.

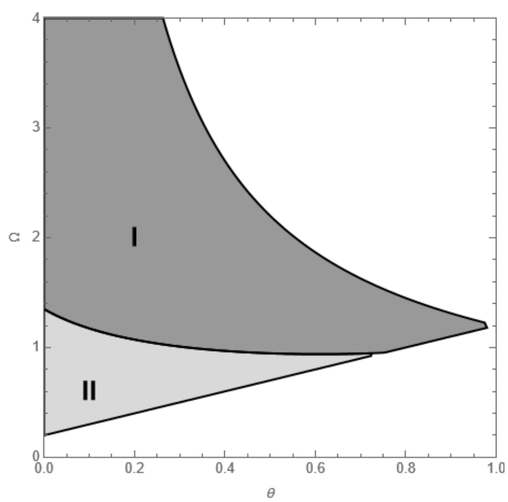

Figure 6. Profit comparison of C2M manufacturers in four scenarios ($r_M = 0.2, r_R = 0.1$)

Corollary 2 primarily compares the profit circumstances between the retailer and the platform. It can be observed that in the absence of a retailer, holding a platform is more advantageous for the retailer. In the situation without a platform, when the value of r_R/r_M is greater, the platform profit in the decentralization scenario D will be higher. This is evident as the increase in commission leads to an increase in the platform's profit. Conversely, when the value of r_R/r_M is smaller, the advantage of scenario MR becomes more prominent.

Proposition 7. Under the constraint, the profit variation of channel-wide, If $\frac{\theta-2\sqrt{\theta^2({r_M}^2-2\theta r_M r_R+{r_R}^2)}}{\theta^2} < \Omega < \frac{\theta+2\sqrt{\theta^2({r_M}^2-2\theta r_M r_R+{r_R}^2)}}{\theta^2}$, $\prod^{C^*} > \prod^{MP^*} > \prod^{MR^*} > \prod^{D^*}$, Region I; $\Omega < \frac{\theta-2\sqrt{\theta^2({r_M}^2-2\theta r_M r_R+{r_R}^2)}}{\theta^2}$, $\prod^{C^*} > \prod^{MR^*} > \prod^{MP^*} > \prod^{D^*}$, Region II. As r_M increases until $r_{M2} = \frac{1}{2}(2\theta r_R + \sqrt{1-2\theta\Omega+\theta^2\Omega^2-4{r_R}^2+4\theta^2{r_R}^2})$ and as r_R increases until $r_{R2} = \frac{1}{2}(2\theta r_M - \sqrt{1-2\theta\Omega+\theta^2\Omega^2-4{r_M}^2+4\theta^2{r_M}^2})$, $\prod^{MP^*}$ also increases until it completely dominates $\prod_M^{MR^*}$. To see Figure 8.

Corollary 3. Scenario C always represents the optimal solution. However, the optimal choices made by the C2M manufacturer consistently benefit the entire supply chain in Regions I and II. In Region III, the C2M manufacturer and the entire supply chain face a prisoner's dilemma when facing the partial centralization scenario. To see Figure 9.

Like the symmetric dual-channel analysis, in **Proposition 7** and **Corollary 3**, we compare the size of the channel-wide profit, the commission threshold, and the situation under which the manufacturer and the whole supply chain will face the prisoner's dilemma. Distinctively, herein, we emphasize how brand substitutability and the relative channel status of asymmetric channels impact the selection of the manufacturer. As Scenario C is centralization, it is consistently favored by the manufacturer. It is worth noting that lower Ω and brand substitutability are more beneficial for scenario MR, and conversely, larger Ω and brand substitutability are more beneficial for scenario MP. However, when Ω is relatively widespread and brand substitution is moderate, C2M manufacturers will face a prisoner's dilemma in the face of partial centralization scenarios.

In addition, we note that exceptional cases occur when both r_M and r_R are at the minimum. We summarize it as the following corollary.

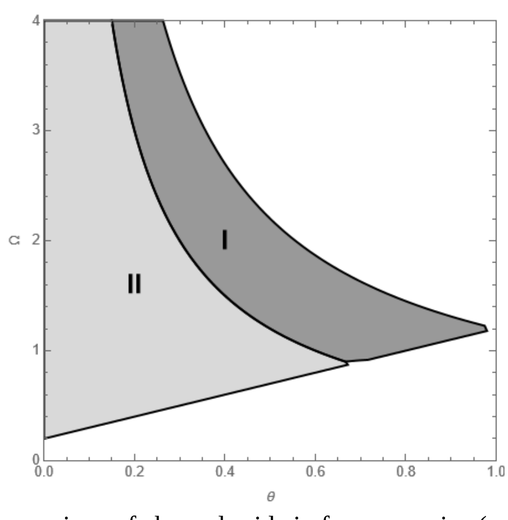

Figure 7. Profit comparison of channel-wide in four scenarios ($r_M = 0.2, r_R = 0.1$)

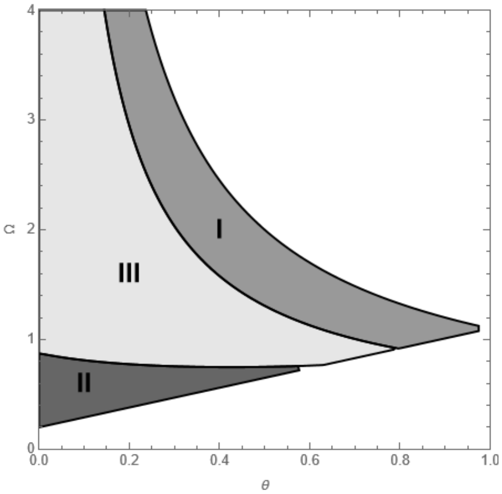

Figure 8. Comprehensive comparison

Corollary 4. When r_M and r_R are at the minimum. If $\frac{1-3\theta r_M+3r_R}{\theta} < \Omega < \frac{(-4+3\theta^2){r_M}^2+2\theta r_M(1+r_R)-r_R(2+r_R)}{2\theta(\theta r_M-r_R)}$, $\prod^{D^*}$ is better than $\prod^{MR^*}$and $\prod^{MP^*}$, respectively.

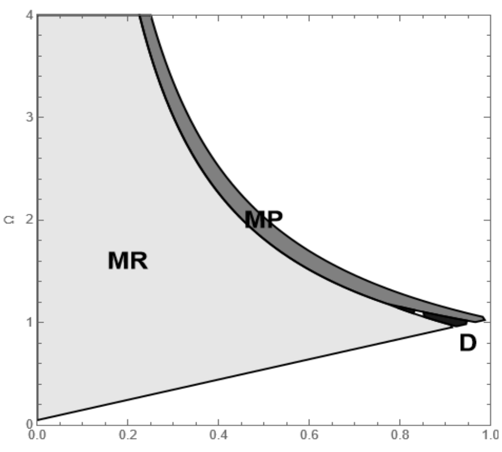

Figure 9. Minimal commission ($r_M = 0.05, r_R = 0.01$)

Corollary 4 shows the extreme case where decentralization outperforms the partial centralization scenario when commissions are at their minimum. Moreover, Figure 10 illustrates that this region emerges solely when the relative channel powers are nearly equal, and the degree of brand substitutability is high. The feasible region has been maintained at a minimal level. This is because the very small commission will reduce the marginalization effect brought by the platform, and the close channel power and the high brand substitutability will weaken the marginalization effect of the unowned retailers. We will explore this further.

5.1 Asymmetric dual-channel supply chain coordination mechanism

Above, we have discussed the optimal strategy choices of C2M manufacturers in the face of four scenarios. Next, we will discuss the coping strategies of C2M and supply chain coordination when faced with the invasion of private label retailers. We will separately explore and compare Scenario MR and Scenario D, as well as Scenario MP and Scenario C.

When coordinating Scenario D, we find that although the manufacturer's profit is always $\prod_M^{MR^*} > \prod_M^{D^*}$, the channel-wide profits of the supply chain $\prod^{MR^*}$ and $\prod^{\boldsymbol{D}^*}$ do not present a single-size relationship. Through Table 7, we have $r_M + \theta(1-r_R) < \Omega < \frac{1+\theta r_M - r_R}{\theta}$. In this case, we find that:

Proposition 8. When $0 < r_R < r_M < 1$, If $r_M + \theta(1-r_R) < \Omega < \frac{1-3\theta r_M + 3r_R}{\theta}$, Scenario MR is optimal; If $max\,\{\theta, r_M + \theta(1-r_R)\} < \Omega < \frac{1+\theta r_M - r_R}{\theta}$, Scenario D is optimal. When $0 < r_M < r_R < 1$, Scenario MR is optimal.

Proposition 8 clarifies the manufacturer's strategies in scenarios D and MR, shedding light on how commissions and the relative positions of channels influence the overall supply chain profitability. When $r_R > r_M$, scenario MR consistently emerges as the optimal strategy for the manufacturer. Conversely, when $r_M > r_R$, scenario D gradually becomes prevalent, indicating that increasing r_M diminishes the dominance of scenario MR. Figure 11 illustrates that with a fixed relative channel position Ω, as θ increases, the manufacturer should sequentially choose scenario MR followed by scenario D. When θ is low, a higher Ω suggests scenario MR is preferable. For moderate θ, increasing Ω suggests choosing between MR and D, and for high θ, increasing Ω favors scenario D.

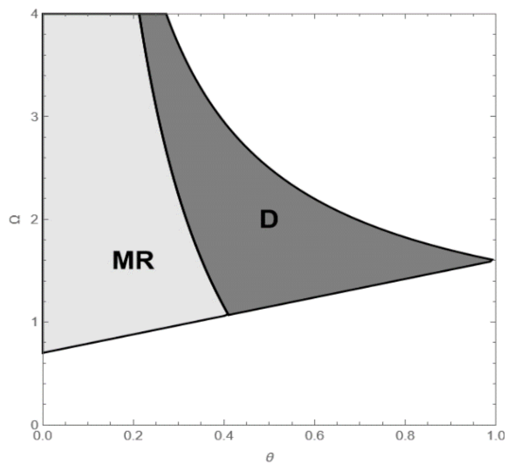

Figure 10. Profit comparison of channel-wide in scenario MR and D ($r_M = 0.7, r_R = 0.1$)

Therefore, there is no coordination condition between scenario MR and scenario D. Subsequently, we will focus on coordinating the MP supply chain.

5.1.1 Coordination Mechanism and Numerical Analysis of Scenario MP

According to **Proposition 6 and 7**, $\prod_M^{C^*} > \prod_M^{MP^*}$, $\prod^{C^*} > \prod^{MP^*}$, indicate that scenario C consistently outperforms scenario MP due to its dual marginalization effects. Thus, our focus now shifts to coordinating scenario MP. It is noteworthy that this strategy applies under the conditions $r_R < \frac{1}{2}$ and $\Omega > \theta$.

In the partial centralization supply chain scenario MP, for a given wholesale price (w_R) and direct channel prices $(p_M^{MP^*})$, p_M^{MP} is concave in $\prod_R^{MP}$. From $\frac{d\prod_R^{MP}}{dp_M^{MP}} = 0$, we derive that.

$$p_R^{MP} = \frac{1}{2}(\theta p_M + r_R + w_R - \theta\Omega + 1). \tag{18}$$

According to **Lemma 3** item **(2)**, we discover $p_M^{C^*} = p_M^{MP^*}$. We calculate it in a similar way as in the previous section

$$w_{R3} = \frac{1}{2}(\theta\Omega - 2r_R). \tag{19}$$

$$\prod_M^{MP}(w_{R3}) = p_M^{C^*} D_M^{C^*} + (w_{R3} + r_R)D_R^{C^*} = \frac{\Omega^2}{4}. \tag{20}$$

$$\prod_R^{MP}(w_{R3}) = (p_R(w_{R3}) - w_{R3} - r_R)D_R^{C^*} = \frac{(-1+\theta\Omega)^2}{4(1-\theta^2)}. \tag{21}$$

$$\prod^{MP}(w_{R3}) = \frac{1 - 2\theta\Omega + \Omega^2}{4(1-\theta^2)}. \tag{22}$$

Proposition 9. When using the contract $(w_{R3}, p_M^{C^*})$ to coordinate Scenario MP, $\prod^{MP}(w_{R3}) = \prod^{MR^*}$, it indicates that the contract offered by the C2M manufacturer can coordinate Scenario MP.

Comparing **(20)** with $\prod_M^{MP^*}$, we found that this contract did not bring additional profit to the C2M manufacturer. Therefore, we adopt a complementary contract profit-sharing agreement to coordinate the supply chain, similar to others used in practice.

In the complementary profit-sharing contract, the manufacturer and the retailer should negotiate a split of the channel-wide profit. We denote the manufacturer's profit share as ρ and the retailer's profit share as $(1-\rho)$, where $\rho \in (0.1)$. The contract $(w_{R3}, p_M^{C^*}, \rho)$ is preferred by the manufacturer if $\rho\prod^{MP}(w_{R3}) - \prod_M^{MP^*} > 0$, which requires

$$\rho > \frac{-1 + 2\theta\Omega - 2\Omega^2 + \theta^2\Omega^2}{2(-1 + 2\theta\Omega - \Omega^2)} = \rho_1, \tag{23}$$

The contract $(w_{R3}, p_M^{C^*}, \rho)$ is preferred by the retailer if $(1-\rho)\prod^{MP}(w_{R3}) - \prod_R^{MP^*} > 0$, which requires

$$\rho < \frac{-3 + 6\theta\Omega - 4\Omega^2 + \theta^2\Omega^2}{4(-1 + 2\theta\Omega - \Omega^2)} = \rho_2, \tag{24}$$

We summarize the results in the following proposition.

Proposition 10. When ρ falls within range (ρ_1, ρ_2), contract $(w_{R3}, p_M^{C^*}, \rho)$ can coordinate the supply chain of asymmetric dual-channel scenario MP, ensuring a win-win situation for both the C2M manufacturer and the private label retailer. Where w_{R3}, ρ_1, ρ_2 are given by **(19), (23),** and **(24)** respectively.

Formulas **(23)** and **(24)** indicate that a lower ρ value favors the retailer, while a slightly higher ρ value benefits the manufacturer more. The specific ρ value largely depends on the bargaining power of the manufacturer and the retailer within the supply chain. **Proposition 10** outlines a reasonable range for ρ, within which our proposed complementary contract $(w_{R3}, p_M^{C^*}, \rho)$ benefits both the manufacturer and the retailer. Implementing this contract results in higher profits for both parties compared to not implementing it, achieving a win-win situation and optimizing channel-wide profit in scenario MP.

Our numerical experiments demonstrate how the contract coordinates scenario MP to ensure greater profits for both the manufacturer and the retailer. With parameters set at $\theta = 0.4$ and $r_R = 0.1$, Table 8 illustrates the outcomes under different Ω values after implementing the contract $(w_{R3}, p_M^{C^*})$. Our findings reveal that as Ω increases, the retailer should lower their retail prices, while the manufacturer faces the dilemma of reduced profits due to declining wholesale prices. The contract $(w_{R3}, p_M^{C^*})$ effectively coordinates scenario MP, enhancing profits for the retailer and channel-wide supply chain efficiency, albeit with limited profit gains for the manufacturer. This reinforces our **Proposition 9.**

Table 8. Results for scenario MP and coordination under contract $(w_{R3}, p_M^{C^*})$ without a profit-sharing agreement, where $\theta = 0.4,\ r_R = 0.1$

Ω	Scenario MP						Coordinated scenario MP under a contract $(w_{R3}, p_M^{C^*})$					
	$w_R^{MP^*}$	$p_M^{MP^*}$	$p_R^{MP^*}$	$\prod_M^{MP^*}$	$\prod_R^{MP^*}$	$\prod^{MP^*}$	w_{R3}	$p_M^{C^*}$	$p_R^{MP^*}(w_{R3})$	$\prod_M^{MP^*}(w_{R3})$	$\prod_R^{MP^*}(w_{R3})$	$\prod^{MP^*}(w_{R3})$
0.5	0.4	0.25	0.7	0.1577	0.0476	0.2053	0	0.25	0.5	0.0625	0.1904	0.2529
0.7	0.4	0.35	0.68	0.1996	0.0385	0.2382	0.04	0.35	0.5	0.1225	0.1542	0.2767
0.9	0.4	0.45	0.66	0.2634	0.0304	0.2939	0.08	0.45	0.5	0.2025	0.1219	0.3244
1.1	0.4	0.55	0.64	0.3491	0.0233	0.3725	0.12	0.55	0.5	0.3025	0.0933	0.3958
1.3	0.4	0.65	0.62	0.4567	0.0171	0.4739	0.16	0.65	0.5	0.4225	0.0685	0.491
1.5	0.4	0.75	0.6	0.5863	0.0119	0.5982	0.2	0.75	0.5	0.5625	0.0476	0.6101

Figure 12 illustrates how the upper and lower limits of manufacturers' shares vary with changes in the contract $(w_{R3}, p_M^{C^*}, \rho)$. We observe that with increasing Ω, both ρ_1 and ρ_2 show an upward trend, leading to increased profit shares for manufacturers. It is noticeable that ρ_1 increases faster than ρ_2, indicating that as direct channels strengthen, the negotiating space gradually diminishes

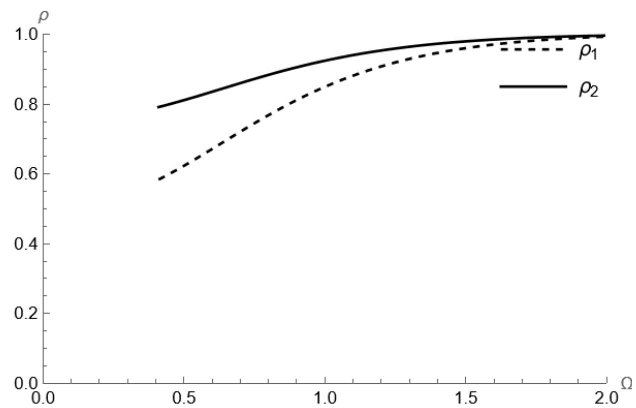

Figure 11. Range of ρ for contract $(w_{R3}, p_M^{C^*}, \rho)$

6. MANAGEMENT INSIGHTS AND CONCLUSIONS

Our study investigates optimal strategies for the C2M manufacturer concerning the introduction of the private label retailer. It examines how basic demand, relative channel strength, and brand substitutability influence interactions within the supply chain. Additionally, we analyze scenarios involving double marginalization and propose coordinated approaches. The research centers on a supply chain consisting of a C2M manufacturer, a platform, and the potential introduction of a private-label retailer. The primary decision for the C2M manufacturer revolves around introducing a private-label retailer.

We analyze and coordinate dual-channel supply chains under symmetric and asymmetric conditions. In symmetric dual-channel scenarios, centralization in Scenario C consistently yields superior performance, whereas decentralization in Scenario D ranks the lowest. Furthermore, under negotiated commissions, the partial centralization scenario MR outperforms the MP scenario when basic demand and brand substitutability are high. Conversely, Scenario MP becomes favorable under minimal basic demand and high brand substitutability, presenting a prisoner's dilemma for the C2M manufacturer. In asymmetric dual-channel scenarios, Scenario C remains optimal, while Scenario D does not always perform the worst.

Following analysis, we coordinate Scenario D based on Scenario MR and Scenario MP based on Scenario C. We find that the contract featuring wholesale and direct channel prices set by the C2M manufacturer can effectively coordinate the dual-channel supply chain. Furthermore, we propose complementary agreements--specifically, a two-part tariff agreement and a profit-sharing agreement--to demonstrate that these contracts can foster coordination within the dual-channel supply chain. Specifically, when the lump sum fee paid by the private label retailer to the C2M manufacturer falls within a specified range, and the negotiated private label retailer's profit share also falls within a specified range, these agreements facilitate a win-win-win scenario among the private label retailer, platform, and C2M manufacturer. This contrasts with the outcomes observed in the decentralization and partial centralization dual-channel supply chain scenarios.

Our findings enrich managerial insights. Firstly, blindly holding a private label retailer for dual-channel expansion may not always be optimal. High brand substitutability prompts C2M manufacturers to strategically hold a private label retailer, like Fast Retailing Company and Inditex Group, the parent company of Zara, to manage brand substitutability and enhance market share amidst competition. In contrast, lower brand substitutability favors the holding platform. Tesla is an excellent example of this model. Its business strategy of holding the platform not only maintains its dominant position among similar brands but also uses the C2M model to make pricing decisions quickly. Furthermore, centralization typically proves optimal. Companies like SHEIN, Temu, and JD.com exemplify successful implementations of centralization models. It is worth noting that Shenzhou International, a major contract manufacturer for Nike, Uniqlo, Adidas, and other brands, attempted transformation into a C2M manufacturer but failed. The primary reason was the high substitution rate between its products and those it produced as an OEM. This also underscores that decentralization is often not viable for C2M manufacturers.

Lastly, our proposed complementary coordination contracts enhance the profitability of supply chain members and C2M manufacturers in the supply chain scenarios where the double marginalization effect exists. Actually, most enterprises fail to achieve centralization. Hence, the implementation of the complementary contracts we proposed can improve the efficiency of the supply chain. For instance, P&G's adoption of two-part pricing and profit-sharing contracts with retailers ensures the incentives for large-volume purchases while maintaining the retailers' profitability.

Looking forward, future research could explore commission rates based on price and revenue in various scenarios, as well as strategies under different power structures. The growing influence of e-commerce platforms and the emergence of high-quality private label retailers as leaders in Stackelberg games also warrant investigation. Moreover, considering the quality factors of national brands versus private label brands could yield diverse outcomes in future studies.

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APPENDIX

Proof of Table 3:

Under scenario D: $\frac{d^2\prod_R^D}{dp_R^2}=\frac{2}{-1+\theta^2}<0$, $\prod_R^D$ is a concave function of p_R. Solving $\frac{d\prod_R^D}{dp_R}=\frac{\theta p_M-2p_R+r_R+w_R-\theta a_M+a_R}{1-\theta^2}=0$, the reaction function of the retailer can be derived as $p_R(p_M,w_R)=\frac{1}{2}(\theta p_M+r_R+w_R-\theta a_M+a_R)$. Inserting $p_R(p_M,w_R)$ into the manufacturer profit function, $\prod_M^D(p_M,w_R)=\frac{1}{2(1-\theta^2)}((p_M-r_M)((-2+\theta^2)p_M+2a_M+\theta(r_R+w_R-\theta a_M-a_R))-w_R(-\theta p_M+r_R+w_R+\theta a_M-a_R))$ is obtained. $\frac{\partial^2\prod_M^D}{\partial p_M{}^2}=\frac{2-\theta^2}{-1+\theta^2}<0$, $\frac{\partial^2\prod_M^D}{\partial p_M{}^2}\cdot\frac{\partial^2\prod_M^D}{\partial w_R{}^2}-(\frac{\partial^2\prod_M^D}{\partial p_M w_R})^2=\frac{2}{1-\theta^2}>0$ since $0<\theta<1$, $\prod_M^D$ is a concave function over (p_M,w_R). Solving the first-order condition $\frac{\partial\prod_M^D(p_M,w_R)}{\partial p_M}=0$ and $\frac{\partial\prod_M^D(p_M,w_R)}{\partial w_R}=0$, the optimal $p_M^{D^*}=\frac{1}{2}(r_M+a_M)$ and $w_R{}^*=\frac{1}{2}(-r_R+a_R)$ for manufacturer are obtained. Inserting $p_M^{D^*}$ and $w_R{}^*$ into $p_R(p_M,w_R)$, the optimal $p_R^{D^*}$ for retailer is derived as $p_R^{D^*}=\frac{1}{4}(\theta r_M+r_R-\theta a_M+3a_R)$. . The equilibrium profit of the manufacturer, retailer and the platform are $\prod_M^{D^*}=\frac{(-2+\theta^2)r_M^2-r_R^2+(-2+\theta^2)a_M^2+2\theta a_M a_R-a_R^2+2r_R(-\theta a_M+a_R)-2r_M(-\theta r_R+(-2+\theta^2)a_M+\theta a_R)}{8(-1+\theta^2)}$, $\prod_R^{D^*}=\frac{(\theta r_M-r_R-\theta a_M+a_R)^2}{16(1-\theta^2)}$, $\prod_P^{D^*}=\frac{-((-2+\theta^2)r_M^2)+r_R(r_R+\theta a_M-a_R)+r_M(-2\theta r_R+(-2+\theta^2)a_M+\theta a_R)}{4(-1+\theta^2)}$, $\prod^{D^*}=\frac{(4-3\theta^2)r_M^2+r_R^2+(-4+\theta^2)a_M^2+2\theta r_M(-r_R+\theta a_M-a_R)+6\theta a_M a_R-3a_R^2+2r_R(-\theta a_M+a_R)}{16(-1+\theta^2)}$, respectively.

Similar to the above calculation method, we can obtain the equilibrium solutions for scenarios C, MP and MR.

Tables 4 and 7 can be easily obtained based on the results in Table 3, so omitted here

Proof of Lemma 1: The result can be easily verified, thus omitted here.

The symmetric dual-channel condition $a>2r$ can be easily obtained.

Proof of Lemma 3: Under $r_R<\frac{1}{2}$ and $max\,\{\theta,r_M+\theta(1-r_R)\}<\Omega<\frac{1+\theta r_M-r_R}{\theta}$.

(1), (2) can be easily verified, thus omitted here.

(3) $p_R^{MP^*}-p_R^{MR^*}=\frac{1}{4}(1-2r_R-\theta\Omega)$, when $max\,\{\theta,r_M+\theta(1-r_R)\}<\Omega<\frac{1-2r_R}{\theta}$, $p_R^{MP^*}>p_R^{MR^*}$. $p_R^{D^*}-p_R^{MP^*}=$

$\frac{1}{4}(\theta r_M + r_R) > 0$. $p_R^{D^*} > p_R^{MP^*}$. $p_R^{MR^*} - p_R^{C^*} = \frac{r_R}{2} > 0$, $p_R^{MR^*} > p_R^{C^*}$. Thus $p_R^{D^*} > p_R^{MP^*} > p_R^{MR^*} > p_R^{C^*}$.

When $\frac{1-2r_R}{\theta} < \Omega < \frac{1+\theta r_M - r_R}{\theta}$, $p_R^{MR^*} > p_R^{MP^*}$. Thus $p_R^{D^*} > p_R^{MR^*} > p_R^{MP^*} > p_R^{C^*}$.

(4) Scenario D and MR follow the restriction $r_M + \theta(1 - r_R) < \Omega < \frac{1+\theta r_M - r_R}{\theta}$.

$D_M^{D^*} - D_M^{MR^*} = \frac{\theta(-1+\theta\Omega-\theta r_M+r_R)}{4(-1+\theta^2)}$, since $\Omega < \frac{1+\theta r_M - r_R}{\theta}$, $D_M^{D^*} - D_M^{MR^*} < 0$, $D_M^{D^*} < D_M^{MR^*}$.

Scenario C and MP follow the restriction $\theta < \Omega < \frac{1}{\theta}$ and $r_R < \frac{1}{2}$.

$D_M^{C^*} - D_M^{MP^*} = \frac{\theta-\theta^2\Omega}{4(-1+\theta^2)}$, since $\Omega < \frac{1}{\theta}$, $D_M^{C^*} - D_M^{MP^*} < 0$, $D_M^{C^*} < D_M^{MP^*}$.

(5) Scenario D and MR follow the restriction $r_M + \theta(1 - r_R) < \Omega < \frac{1+\theta r_M - r_R}{\theta}$.

$D_R^{MR^*} - D_R^{D^*} = \frac{-1+\theta\Omega-\theta r_M+r_R}{4(-1+\theta^2)}$, since $\Omega < \frac{1+\theta r_M - r_R}{\theta}$, $D_R^{MR^*} - D_R^{D^*} > 0$, $D_R^{MR^*} > D_R^{D^*}$.

Scenario C and MP follow the restriction $\theta < \Omega < \frac{1}{\theta}$ and $r_R < \frac{1}{2}$.

$D_R^{C^*} - D_R^{MP^*} = \frac{1-\theta\Omega}{4-4\theta^2}$, since $\Omega < \frac{1}{\theta}$, $D_R^{C^*} - D_R^{MP^*} > 0$, $D_R^{C^*} > D_R^{MP^*}$.

Proof of Proposition 6:

To compare $\prod_M^{MP^*}$ and $\prod_M^{MR^*}$, we temporarily define $\Delta\prod M2$ as the Manufacturer's profits in MP minus the one in MR. Solving $\Delta\prod M2 = 0$ yields two roots:

$$\Omega_1 = \frac{\theta+2r_M-2\theta r_R-\sqrt{-2(-2+\theta^2)r_M^2+2\theta^2 r_R^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2},$$

$$\Omega_2 = \frac{\theta+2r_M-2\theta r_R+\sqrt{-2(-2+\theta^2)r_M^2+2\theta^2 r_R^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2}.$$

Given the non-negative condition of price and demand ($r_R < \frac{1}{2}$ and $max\,\{\theta, r_M + \theta(1 - r_R)\} < \Omega < min\,\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\}$), due to complexity, we omit this condition from display. However, in the scope of the following discussion, we will expand under this condition and take their intersection under the following conditions. Thus, we can get: When $\frac{\theta+2r_M-2\theta r_R-\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2} < \Omega < \frac{\theta+2r_M-2\theta r_R+\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2}$,

$\prod_M^{MP^*} > \prod_M^{MR^*}$. $\prod_M^{C^*} - \prod_M^{MP^*} = \frac{(-1+\theta\Omega)^2}{8(1-\theta^2)} > 0$, $\prod_M^{C^*} > \prod_M^{MP^*}$. $\prod_M^{MR^*} - \prod_M^{D^*} = \frac{(-1+\theta\Omega-\theta r_M+r_R)^2}{8(1-\theta^2)} > 0$, $\prod_M^{MR^*} > \prod_M^{D^*}$. Thus $\prod_M^{C^*} > \prod_M^{MP^*} > \prod_M^{MR^*} > \prod_M^{D^*}$. When $\Omega < \frac{\theta+2r_M-2\theta r_R-\sqrt{-2(-2+\theta^2)r_M{}^2+2\theta^2 r_R{}^2+4r_M(\theta-\theta^3+\theta(-2+\theta^2)r_R)}}{\theta^2}$, $\prod_M^{MR^*} > \prod_M^{MP^*}$, Thus $\prod_M^{C^*} > \prod_M^{MR^*} > \prod_M^{MP^*} > \prod_M^{D^*}$.

Depending on how the image changes. Since $\Omega > r_M + \theta(1 - r_R)$, We can solve for the $r_{M1} = \Omega - \theta(1 - r_R)$ is the threshold. Since $\Omega_1 = \frac{\theta + 2r_M - 2\theta r_R - \sqrt{-2(-2+\theta^2)r_M{}^2 + 2\theta^2 r_R{}^2 + 4r_M(\theta - \theta^3 + \theta(-2+\theta^2)r_R)}}{\theta^2}$, We can yields two roots: $r_{R1} = \frac{1}{2}(2 - 2\theta\Omega + 2\theta r_M - \sqrt{2}\sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 + 4\Omega r_M - 4\theta^2\Omega r_M - 2r_M{}^2 + 2\theta^2 r_M{}^2})$ and $\widehat{r_{R1}} = \frac{1}{2}(2 - 2\theta\Omega + 2\theta r_M + \sqrt{2}\sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 + 4\Omega r_M - 4\theta^2\Omega r_M - 2r_M{}^2 + 2\theta^2 r_M{}^2})$ since $0 < r_R < \frac{1}{2}$, r_{R1} is valid. Thus r_{R1} is the threshold of r_R.

Proof of Corollary 2:

$\prod_R^{D^*} - \prod_R^{MP^*} = -\frac{(-1+\theta\Omega)^2(-1+\theta\Omega-\theta r_M+r_R)^2}{256(-1+\theta^2)^2} < 0$, Thus $\prod_R^{D^*} < \prod_R^{MP^*}$.

$\prod_P^{D^*} - \prod_P^{MR^*} = \frac{(\theta r_M - r_R)(1-\theta\Omega+\theta r_M-r_R)}{4(1-\theta^2)}$, When $\theta < \frac{r_R}{r_M}$ and $\Omega < \frac{1+\theta r_M - r_R}{\theta}$, $\prod_P^{D^*} - \prod_P^{MR^*} < 0$. Thus $\prod_P^{MR^*} > \prod_P^{D^*}$. When $\theta > \frac{r_R}{r_M}$ and $\Omega < \frac{1+\theta r_M - r_R}{\theta}$, $\prod_P^{D^*} - \prod_P^{MR^*} > 0$. Thus $\prod_P^{D^*} > \prod_P^{MR^*}$.

Proof of Proposition 7:

To compare $\prod^{MP^*}$ and $\prod^{MR^*}$, we temporarily define $\Delta\prod 2$ as the channel-wide profits in MP minus the one in MR. Solving $\Delta\prod 2 = 0$ yields two roots: $\Omega_3 = \frac{\theta - 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$, $\Omega_4 = \frac{\theta + 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$. Given the non-negative condition of price and demand ($r_R < \frac{1}{2}$ and $max\,\{\theta, r_M + \theta(1 - r_R)\} < \Omega < min\,\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\}$), due to complexity, we omit this condition from display. However, in the scope of the following discussion, we will expand under this condition and take their intersection under the following conditions. Thus, we can get: When $\frac{\theta - 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2} < \Omega < \frac{\theta + 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$, $\prod^{MP^*} > \prod^{MR^*}$. $\prod^{C^*} - \prod^{MP^*} = -\frac{(-1+\theta\Omega)^2}{16(-1+\theta^2)} < 0$, $\prod^{C^*} < \prod^{MP^*}$.

$\prod^{MR^*} - \prod^{D^*} = \frac{(-1+\theta\Omega+3\theta r_M - 3r_R)(-1+\theta\Omega-\theta r_M + r_R)}{16(1-\theta^2)}$, in this section, we only discuss higher r_M and r_R in which $max\,\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\} < \frac{1-3\theta r_M + 3r_R}{\theta}$, result $\prod^{MR^*} - \prod^{D^*} > 0$, thus $\prod^{MR^*} > \prod^{D^*}$, $\prod^{C^*} > \prod^{MP^*} > \prod^{MR^*} > \prod^{D^*}$. When $\Omega < \frac{\theta + 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$, $\prod^{MP^*} < \prod^{MR^*}$. Thus $\prod^{C^*} > \prod^{MR^*} > \prod^{MP^*} > \prod^{D^*}$.

Depending on how the image changes. Since $\Omega > \frac{\theta - 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$, We can yields two roots: $r_{M2} = \frac{1}{2}\left(2\theta r_R + \sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 - 4r_R{}^2 + 4\theta^2 r_R{}^2}\right)$ and $\widehat{r_{M2}} = \frac{1}{2}\left(2\theta r_R + \sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 - 4r_R{}^2 + 4\theta^2 r_R{}^2}\right)$. Since $0 < r_M < 1$, r_{M2} is valid. Thus r_{M2} is the threshold of r_M. Since $\Omega > \frac{\theta - 2\sqrt{\theta^2(r_M{}^2 - 2\theta r_M r_R + r_R{}^2)}}{\theta^2}$, We can yields two roots: $r_{R2} = \frac{1}{2}(2\theta r_M - \sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 - 4r_M{}^2 + 4\theta^2 r_M{}^2})$ and $\widehat{r_{R2}} = \frac{1}{2}(2\theta r_M + \sqrt{1 - 2\theta\Omega + \theta^2\Omega^2 - 4r_M{}^2 + 4\theta^2 r_M{}^2})$ since $0 <$

$r_R < \frac{1}{2}$, r_{R2} is valid. Thus r_{R2} is the threshold of r_R.

Proof of Corollary 4:

We will discuss the case of $min\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\} > \frac{1-3\theta r_M + 3r_R}{\theta}$, which is discussed in Proposition 7. In this section, we only discuss the minimum r_M and r_R in which $min\{\frac{1}{\theta}, \frac{1+\theta r_M - r_R}{\theta}\} < \frac{1-3\theta r_M + 3r_R}{\theta}$, result $\prod^{MR^*} < \prod^{D^*}$. To find an upper bound on scenario D, we make $\prod^{D^*} - \prod^{MR^*} = 0$ solve for Ω, result

$\Omega < \frac{(-4+3\theta^2){r_M}^2 + 2\theta r_M(1+r_R) - r_R(2+r_R)}{2\theta(\theta r_M - r_R)}$.

Proposition 8 can be easily obtained in combination with **Proposition 7 and Corollary 4,** so it is omitted