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STRATEGIC CHOICES OF ONLINE RETAILERS IN LIVE-STREAMING E-COMMERCE

Tao Jiang*, Yuying Sun, and Youwei Guo

College of Economics and Management
Shandong University of Science and Technology
Shandong, China
*Corresponding author Email: jiangt@sdust.edu.cn

Some online retailers, based on traditional channels, have launched live-streaming channels. To study the optimal live-streaming choices of online retailers, this paper models the supply chain composed of a manufacturer and an online retailer by establishing three modes: no live-streaming mode, influencer live-streaming mode, and retailer self-live-streaming mode. The online retailer's optimal choice of live-streaming mode is derived by analyzing the equilibrium solutions. The results suggest that when the sales ability of the employee-streamer and the live-streaming consumer purchase rate are high, it's always advantageous to opt for retailer self-live-streaming mode. Affected by the influencer's commission rate and fixed participation fee, the influencer live-streaming mode is harmful to the online retailer. However, if the influencer chooses a profit-sharing mechanism, the online retailer and influencer can achieve a Pareto profit improvement, and online retailers will open the influencer live-streaming mode.

Keywords: Live-streaming e-commerce; The streamer's sales ability; Channel selection; Game theory

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1. INTRODUCTION

Digital technology has opened the path for live-streaming e-commerce, fundamentally transforming the consumption habits of consumers. Through real-time interactions with consumers, live-streaming sales go beyond the traditional one-way shopping experience and enhance consumers' trust, as pointed out by Li *et al*. (2021). Moreover, streamers utilize live-streaming channels to reveal their appearance, functionality, and usage of the products to consumers in real time. Taking advantage of the strengths of e-commerce sales, retailers such as Walmart and Amazon have initiated live-streaming channels to seize market share rapidly. During the 'Double Eleven' period of 2024, the cumulative sales of comprehensive e-commerce platforms and live-streaming e-commerce platforms amounted to 1,441.8 billion yuan, registering a year-on-year growth of 26.6%. The sales of live-streaming e-commerce platforms reached 332.5 billion yuan, with a year-on-year increase of 54.6%. Although live-streaming e-commerce sales have offered numerous development opportunities to online retailers, they are not without challenges. The selection of streamers and potential channel conflicts can pose risks to online retailers, potentially resulting in financial losses.

Unlike traditional sales, live-streaming sales channels typically incorporate streamers as key players. These streamers assist online retailers in selling products, introduce consumers to various offerings, and engage in interactive sessions with them. Consequently, consumers gain a deeper understanding of product details, enhancing their willingness to purchase. The involvement of streamers introduces distinct differences between traditional sales and live-streaming sales. Zhang and Tang (2023) pointed out that the involvement of live streamers makes a significant difference between traditional sales and live-streaming sales. Firstly, the presentation method is different. Traditional sales mainly show products to consumers through pictures, text, and videos. However, the live-streaming channel can show more details of products to consumers through live streamers. Secondly, the sales effect of products is different. The live-streaming channel is more likely to stimulate consumers' purchasing desire. Therefore, the live-streaming channel is attracting the attention of online retailers and consumers.

In practice, some retailers opt to collaborate with streamers (He *et al.*, 2022; Zhen *et al.*, 2024; Cheng *et al.*, 2024). M.A.C often conducts live events on social media platforms, where its professional makeup artists (i.e., employee-streamers) demonstrate the latest makeup techniques online and answer questions from consumers. This format not only increases the brand's transparency but also enhances consumers' understanding and trust in the products. When retailers choose to collaborate with influencers, they frequently sign contracts with them to clarify the fixed participation fee and commission rate. Additionally, some retailers will select the retailer self-live-streaming mode (Xin *et al*., 2023; Jiang and Guo, 2024).

After launching the live-streaming channel, Walmart has attracted more potential consumers and promoted its products to a greater number of viewers, thereby expanding its consumer base and enhancing brand awareness.

Nonetheless, if online retailers adopt the live-streaming e-commerce sales approach, many difficulties may emerge. Firstly, introducing a live-streaming channel might cause intense conflicts between the two channels for homogeneous products (Ma and Yang, 2024), resulting in a profit decline. Secondly, if online retailers introduce live-streaming sales, their operating costs may increase. Cooperation with influencers means exchanging a high fixed participation fee and commission rate for the significant personal influence value of influencers. For instance, Luo Yonghao, as an influencer for the online education company Zebra AI in 2020, received a relatively high fixed participation fee (1.2 million yuan). However, his total sales were only 529,200 yuan, far from his fixed participation fee and below the company's expectations (Chen *et al*., 2023).

Furthermore, live-streaming channel subscribers will undoubtedly face certain inconvenience costs. Live streaming is usually conducted by streamers within a specific period, so consumers can only enjoy the benefits of the live-streaming channel during that time frame (Pan *et al*., 2023). Given these challenges, it is uncertain whether online retailers should add a live-streaming channel based on the traditional channel. If they do add live-streaming channels, retailers should add which type of live-streaming channels.

Based on the above evidence, this paper focuses on the following questions: (1) Is live-streaming channels worthy of being introduced to online retailers? (2) If online retailers opt for live-streaming channels, which option will they introduce, influencer live-streaming mode or retailer self-live-streaming mode? (3) How does the live-streaming channel influence product pricing?

We consider a manufacturer and an online retailer in a supply chain and examine three modes, namely, mode N - no live-streaming mode, mode S - retailer self-live-streaming mode, and mode I - influencer live-streaming mode. Theoretical models for each mode are established and solved. Subsequently, by comparing the profits of the retailer in the three modes, we derived the specific conditions under which retailers choose to open live-streaming mode (influencer live-streaming mode or retailer self-live-streaming mode). Finally, the mode is expanded, that is non-zero production cost and the price cross-sensitivity of live-streaming and traditional channels.

Our findings are presented as follows. Firstly, we concentrate on analyzing the sales ability of the streamer and the live-streaming consumer purchase rate regarding whether the online retailer initiates the live-streaming channel. Secondly, we develop two live-streaming modes: influencer live-streaming mode and retailer self-live-streaming mode. When the live-streaming consumer purchase rate is high, the online retailer can generate the highest profit in retailer self-live-streaming mode, and thus, the retailer will select it. When the sales ability of the influencer and the live-streaming consumer purchase rate are high, the profit of the online retailer will always be lower than that under no live-streaming mode. In other words, the online retailer will not choose the influencer live-streaming mode under any circumstances. Finally, we analyze how the influencer utilizes the profit-sharing mechanism to achieve Pareto improvement in the profits of the influencer and the online retailer, providing theoretical support for the online retailer.

The remainder of the paper is structured as follows. The literature review concerning live-streaming e-commerce is presented in Section 2. Section 3 formulates three modes, namely, no live-streaming mode, influencer live-streaming mode, and retailer self-live-streaming mode. Section 4 conducts a comparison of the equilibrium profits. Section 5 is extended to incorporate the production cost of the manufacturer and the price cross-sensitivity analysis of the two channels. Section 6 summarizes the conclusions.

2. LITERATURE REVIEW

2.1. Live-streaming e-commerce

The rise of online retail has significantly impacted consumers through live-streaming channels. First, we analyze how the live-streaming channel influences consumer purchasing behaviors. Live-streaming sales provide real-time product experiences through authentic interactions, enhancing brand awareness and impression, bridging the gap between consumers and brands, and increasing engagement and trust (Li *et al.*, 2022; Qiong and Jin, 2022). In live-streaming e-commerce, streamers use their professional knowledge and social functions to interact with the audience in real time, attracting more consumers to make purchases (Chen and Yang, 2023). They establish emotional connections that enhance consumers' emotional identification with the products, influencing purchasing decisions (Tao *et al.*, 2024; Meng and Lin, 2023). Live-streaming sales can satisfy viewers' information needs through multi-party real-time interactions, mobilize users' emotions, and actively purchase products (Zhang *et al*., 2023). Real-time interaction between streamers and consumers in live-streaming e-commerce can facilitate consumers to promote product understanding, reduce information asymmetry, and increase purchase intention (Xu *et al*.,2022). It can be seen that in live-streaming e-commerce sales, consumer behavior is influenced by the streamer, and the influence of different streamers (i.e., what kinds of live-streaming modes the decision maker chooses) is particularly important for live streaming.

Decision-makers should consider streamer-related factors and costs if they choose to introduce a live-streaming channel. Wang and Wang (2023) examined three models of live streaming in competitive environments, i.e., no live-streaming mode, third-party live-streaming mode, and self-owned live-streaming mode. The study shows that when the hassle cost for consumers to purchase in the live-streaming channel is low, live-streaming sales will always result in price increases for the online retailer. Hao and Yang (2022) modeled live-streaming sales in both resale and agency sales formats, exploring which mode of pricing is more favorable to vendors and platforms. For example, the resale format may become a better choice for platforms and vendors when considering the impact of consumer returns. In this paper, based on previous literature, we explore the choice of different live-streaming modes, namely no live-streaming mode, retailer self-live-streaming mode, or influencer live-streaming mode.

2.2. Channel selection

With the development of e-commerce, many traditional retailers have chosen to transform, not only selling products through traditional channels but also establishing live-streaming channels for sales, as exemplified by Bonobos and Warby Parker (Wang *et al*., 2016). For manufacturers, multi-channel sales are beneficial to their development (Xin *et al*., 2023). For consumers, multi-channel sales offer the option to purchase products through their preferred channels (Liu *et al*., 2022). Building upon their existing sales channels, manufacturers are willing to expand their sales presence on competitive retail platforms such as JD.com and Tmall (Dai *et al*., 2022). Based on certain commission rates and mismatch costs, competing retailers are willing to open up more sales channels (Huang *et al*., 2024; Zhang *et al*., 2022). A single channel does not necessarily maximize manufacturers' profits, and a combination of multiple channels is the most preferred model by both manufacturers and consumers (Zhang *et al*., 2023). Wang *et al*. (2024) found that when the basic compensation coefficient for streamers and the traditional promotion cost coefficient increase. Manufacturers tend to choose a dual-channel strategy combining traditional promotion and live-streaming channels by the influencers. Ji *et al*. (2023) studied whether decision-makers should introduce new channels under dynamic pricing schemes and found that opting for a live-streaming channel will increase the prices of the traditional channel. Wang and Guo (2023) investigated the issue of live-streaming sales in the supply chain and who manufacturers or retailers should partner with for this purpose. By establishing a game theory model, they discussed three possible live-streaming sales models and studied their impacts on consumers and supply chain members. The results indicated that retailers should conduct live-streaming sales, while manufacturers can only benefit from this model when the streaming media's sales capability is high.

In most literature, the decision of whether manufacturers choose a live-streaming channel is predominantly analyzed from the perspectives of the sales ability of live streamers (Pan *et al.,* 2022; Chen *et al.,* 2024), the fixed participation fee of live streamers (Zhang and Tang, 2022), commission rate (Zhou *et al*., 2024), and the proportion of consumers in the live-streaming channel (Du *et al*., 2023). These studies focus on manufacturers' live-streaming strategies, with relatively fewer examining retailers' streaming strategies. Lower commission rates for live streamers can increase product demand and profits (Zhang *et al*., 2023; Liu *et al*., 2024). Ma and Yang (2024) explored manufacturers' streaming strategies and found that the live-streaming channel can contribute to a win-win situation for both manufacturers and e-commerce platforms. Based on this, this paper examines retailers' streaming strategies from their perspective, taking into account all the aforementioned influencing factors.

Most of the aforementioned literature focuses on whether manufacturers should open live-streaming channels by analyzing their relationship with consumers. In contrast, this paper discusses the same issue from a retailer's perspective, constructing a game model involving a manufacturer, an online retailer, and streamers, and examining factors such as the streamers' sales ability, commission rate, fixed participation fee, and the live-streaming consumer purchase rate.

Table 1 shows the difference between our study and the main related literature.

Table 1 Difference between our study and the main related literature

Literature	Dual channel sales	Online retailer	Consumer segmentation	Influence factors of the live-streaming sales
Xin *et al.* (2023)	−	−	−	√
Du *et al.* (2023)	√	−	√	√
Zhang *et al.* (2023)	√	−	−	√
Wang *et al.* (2016)	−	√	√	−
Dai *et al.* (2022)	√	−	−	−
Pan *et al.* (2022)	√	−	−	−

Literature	Dual channel sales	Online retailer	Consumer segmentation	Influence factors of the live-streaming sales
Wang and Guo (2023)	√	√	–	–
Hao and Yang (2023)	√	√	–	–
Cheng *et al.* (2024)	√	–	––	√
This study	√	√	√	√

3. MODEL FRAMEWORK AND EQUILIBRIUM SOLUTION

3.1. Problem description

In this paper, we consider a supply chain consisting of a manufacturer and an online retailer (hereafter referred to as the retailer). The retailer has two types of live-streaming modes: influencer live-streaming mode and retailer self-live-streaming mode. In retailer self-live-streaming mode, the streamer is typically an employee of the retailer. During this process, first, the streamer decides the sales effort level. Second, the manufacturer determines the wholesale price. Finally, the retailer sets the retail prices for both the live-streaming channel and the traditional channel. In both live-streaming modes, according to (Chintagunta, 1992; He *et al.*, 2020; Liu and Liu, 2021), the live-streaming cost is denoted by $\frac{e_i^2}{2}$. Table 2 shows the symbols and definitions, where $i = N$ means no live-streaming mode. The retailer only sells products through traditional sales channel (i.e., traditional channel). $i = S$ means that the retailer self-live-streaming mode; $i = SN$ means consumers purchase through traditional channel in retailer self-live-streaming mode. $i = I$ means that influencer live-streaming mode, $i = IN$ means consumers purchase through traditional channel in influencer live-streaming mode. The streamer is the employee-streamer in retailer self-live-streaming mode, and the streamer is usually the influencer in influencer live-streaming mode. The supply chain structure is shown in Figure 1. The game sequence is shown in Figure 2.

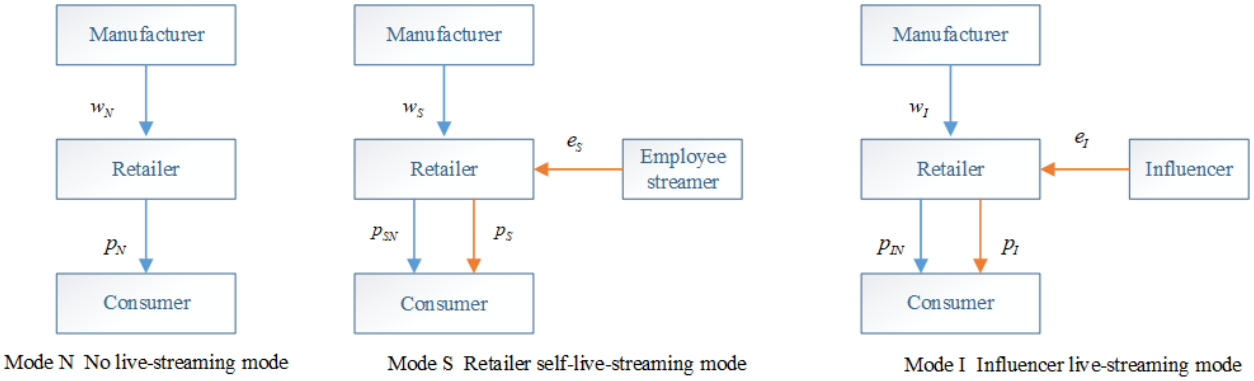


Figure 1. Dual-channel supply chain structure

Table 2. Summary of symbols

symbol	definition	symbol	definition
p_i	Retail price of the product $(i = N, S, SN, I, IN)$	θ	Live-streaming consumer purchase rate
w_i	Wholesale price of the product$(i = N, S, I)$	r	The commission rate of the influencer$(r > 0)$
D_i	Demand$(i = N, S, SN, I, IN)$	a	Market size$(a > 0)$
π_M^i	The profit of the manufacturer$(i = N, S, I)$	b	Fixed participation fee of the influencer
π_R^i	The profit of the retailer$(i = N, S, I)$	β	The influencer's sales ability
π_I^i	The profit of the influencer$(i = N, S, I)$	λ	The employee-streamer's sales ability
e_i	Sales effort level of the streamer$(i = S, I)$	c	Manufacturer's unit cost of production
π^{i*}	The profit of total supply chain $(i = S、N、I)$	μ	Price cross sensitivity

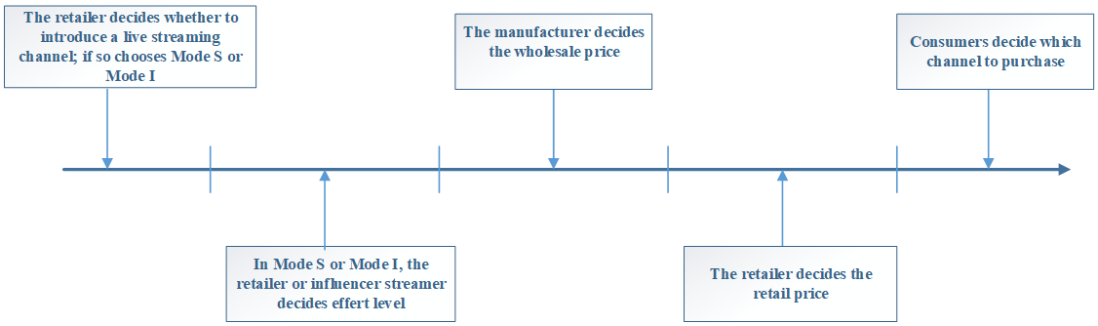


Figure 2. Game sequence

The retailer bears the live-streaming cost in mode S, and the influencer bears the live-streaming cost in mode I. In this paper, consumers can only purchase products through one channel, either the traditional channel or the live-streaming channel (Zhang and Tang, 2023). To simplify the analysis, in our main text, we do not take into account the manufacturer's production costs, i.e., $c = 0$ (Anderson, 2022; Niu *et al*., 2019) and the impact of consumer cross-price sensitivity when opening a live-streaming channel. In Section 5, we relax the assumption. The impact of consumer cross-price sensitivity when opening a live-streaming channel is not taken into account, and the commission rate of streamers$r$is exogenous (Xin *et al*., 2023; Zhang and Tang, 2023).

To ensure that operational decisions in each channel are positive, in mode S, the live-streaming consumer purchase rate θ is in the range of $\left\{\frac{3}{8}, \frac{12-3\lambda^2}{16-3\lambda^2}\right\}$, and the sales effort level of employee-streamer$\lambda$is in the range of $\left\{0, \sqrt{\frac{16}{5}}\right\}$. Additionally, in mode I, the live-streaming consumer purchase rate θ is in the range of $\left\{\frac{1}{4}, \frac{12-3\beta^2 r}{16-3\beta^2 r}\right\}$, and the sales effort level of the influencer$\beta$is in the range of $\left\{0, \sqrt{\frac{32}{9r}}\right\}$.

3.2. No live-streaming mode (Mode N)

In no live-streaming mode, consumers can only buy products from the traditional distribution channel. $D_N = a - p_N$is used to represent the demand function (Zhang *et al.,* 2019; Du *et al.,* 2023; Liu *et al.,* 2024), where$a$indicates the market size and $p_N$indicates the retail price.

In mode N, the following formulas give the profit function of the manufacturer and retailer, where w_N indicates the wholesale price.

$$\pi_R^N = D_N w_N \tag{1}$$

$$\pi_M^N = (p_N - w_N) D_N \tag{2}$$

Using backward induction, we solve (1) and (2) to obtain Lemma 1.
Proof. See the Appendix.
Lemma 1. The equilibrium solutions are shown in Table 3 in mode N.

Table 3. Equilibrium solutions in no live-streaming mode

	w_N^*	p_N^*	D_N^*	π_M^{N*}	π_R^{N*}
Equilibrium solution	$\frac{a}{2}$	$\frac{3a}{4}$	$\frac{a}{4}$	$\frac{a^2}{8}$	$\frac{a^2}{16}$

Lemma 1 states that the equilibrium solutions depend only on the market size (a). With the expansion of the market size, the equilibrium solutions will increase.

3.3. Retailer self-live-streaming mode (Mode S)

Consumers can purchase products from both the live-streaming and traditional channels when the retailer chooses self-live-streaming mode. We useθrespectively to represent the live-streaming consumer purchase rate. We use$D_S = \theta a - p_S + \lambda e_S$ to express the live-streaming channel's demand function (Liu and Liu, 2022; Xin *et al.,* 2023), where $e_S$represents the employee-streamer's sales effort level, $p_S$shows the live-streaming channel's retail price andλrepresents the employee-streamer's sales ability. The traditional channel's demand function is $D_{SN} = (1-\theta)a - p_{SN}$, where$p_{SN}$represents the traditional channel's retail price.

In Mode *S*, the following formulas respectively give the profit functions.

$$\pi_M^S = w_S(D_{SN} + D_S) \tag{3}$$

$$\pi_R^S = (p_{SN} - w_S)D_{SN} + (p_S - w_S)D_S - \frac{e_S^2}{2} \tag{4}$$

Since the employee-streamer belongs to the retailer, the live-streaming cost is borne by the retailer. Using backward induction, we solve (3) and (4) to obtain Lemma 2.

Lemma 2. In mode *S*, Table 4 shows the equilibrium solutions.

Table 4. The equilibrium solutions under the retailer self-live-streaming mode

Equilibrium solution			Equilibrium solution
e_S^*	$\frac{a\lambda(8\theta-3)}{16-5\lambda^2}$	D_{SN}^*	$\frac{a(12-16\theta+3\lambda^2\theta-3\lambda^2)}{2(16-5\lambda^2)}$
w_S^*	$\frac{2a(\lambda^2\theta-\lambda^2+2)}{16-5\lambda^2}$	D_S^*	$\frac{a(16\theta+\lambda^2\theta-\lambda^2-4)}{2(16-5\lambda^2)}$
p_S^*	$\frac{a(16\theta+5\lambda^2\theta-5\lambda^2+4)}{2(16-5\lambda^2)}$	π_M^{S*}	$\frac{4a^2(\lambda^2\theta-\lambda^2+2)^2}{(5\lambda^2-16)^2}$
p_{SN}^*	$\frac{a(20-16\theta+7\lambda^2\theta-7\lambda^2)}{2(16-5\lambda^2)}$	π_R^{S*}	$\frac{a^2(5-\lambda^2\theta^2+2\lambda^2\theta-\lambda^2+16\theta^2-16\theta)}{2(16-5\lambda^2)}$

Lemma 2 shows that the equilibrium solutions are mainly affected by the market size (a), the live-streaming consumer purchase rate (θ) and the employee-streamer's sales ability (λ).

Corollary 1. The impact of key parameters on optimal results in retailer self-live-streaming mode are given in Table 5.

Table 5 The impact of key parameters on optimal results in retailer self-live-streaming mode

Parameter	e_S^*	w_S^*	p_{SN}^*	p_S^*	D_{SN}^*	D_S^*	π_M^{S*}	π_R^{S*}
a	+	+	+	+	+	+	+	+
λ	+	+	+	+	−	+	+	+
θ	+	+	−	+	−	+	+	$-\frac{3}{8} < \theta < \widetilde{\theta_1}$ $+\ \widetilde{\theta_1} < \theta < \theta_1$

+: increase; −: decrease. $\theta_1 = \frac{12-3\lambda^2}{16-3\lambda^2}$, $\widetilde{\theta_1} = \frac{8-\lambda^2}{16-\lambda^2}$.

Table 5 reveals that the profits of the retailer and the manufacturer will rise along with the growth of the market size (a). Thus, the retailer ought to adopt certain measures to augment the market size to obtain an increase in profits. For instance, the retailer can urge the influencer to enhance the sales effort level, thereby expanding the size of their market and consequently increasing profits.

Table 5 represents the sales ability of the employee-streamer (λ) at each equilibrium solution. The equilibrium wholesale price and the sales effort level of the employee-streamer are positively correlated with the sales ability of the employee-streamer. In our paper, consumers are classified into two groups, namely, consumers on the live-streaming channel and consumers on the traditional channel. As the sales ability of the employee-streamer increases, it will inevitably result in an increase in the number of consumers in the live-streaming channel and, consequently, a decrease in the number of consumers

in the traditional channel. Therefore, the retailer should provide the employee-streamer with more training to enhance sales skills and attract more consumers, thereby increasing profits.

Table 5 indicates the impact of the live-streaming consumer purchase rate (θ) on each equilibrium solution. The market is certain, so when the live-streaming consumer purchase rate increases, the sales demand of the live-streaming channel will increase, which will inevitably lead to a decrease in the traditional channel. The retailer can stimulate the growth of demand by lowering the retail price of the traditional channel. The retailer's profit is influenced by the live-streaming consumer purchase rate, which is positively associated with the retailer's profit when the live-streaming purchase rate is relatively high. This is because, under this mode, the cost of live streaming is borne by the retailer, and if fewer consumers make purchases through the live-streaming channel, the profit generated by live streaming is insufficient to cover the cost of live streaming.

3.4. Influencer live-streaming mode (Mode I)

Similar to the retailer's self-live-streaming mode, products can be bought in two channels when the retailer chooses the influencer live-streaming mode. Therefore, $D_I = \theta a - p_I + \beta e_I$ is used to illustrate the live-streaming channel's demand function, where e_I represents the influencer's sales effort level, p_I represents the live-streaming channel's retail price and β represents the influencer's sales ability. $D_{IN} = (1-\theta)a - p_{IN}$ is used to show the traditional channel's demand function, where p_{IN} represents the traditional channel's retail price.

The following formulas give the profit functions in mode I.

$$\pi_M^I = w_I(D_{IN} + D_I) \tag{5}$$

$$\pi_R^I = (p_{IN} - w_I)D_{IN} + (1-r)(p_I - w_I)D_I - b \tag{6}$$

$$\pi_L^I = r(p_I - w_I)D_I + b - \frac{e_I^2}{2} \tag{7}$$

Where r represents the unit product commission rate and b represents the fixed participation fee of the influencer. Using backward induction, we solve (5) - (7) to obtain Lemma 3.

Lemma 3. Table 6 shows the equilibrium solution under Model I.

Table 6. The equilibrium solutions under influencer live-streaming mode

	Equilibrium solution		**Equilibrium solution**
e_I^*	$\frac{3a\beta r(4\theta-1)}{32-9\beta^2 r}$	D_{IN}^*	$\frac{a(12-16\theta-3\beta^2 r+3\beta^2 r\theta)}{32-9\beta^2 r}$
w_I^*	$\frac{a(3\beta^2 r\theta-3\beta^2 r+8)}{32-9\beta^2 r}$	D_I^*	$\frac{4a(4\theta-1)}{32-9\beta^2 r}$
p_I^*	$\frac{a(16\theta-3\beta^2 r+3\beta^2 r\theta+4)}{32-9\beta^2 r}$	π_M^{I*}	$\frac{a^2(3\beta^2 r\theta-3\beta^2 r+8)^2}{(9\beta^2 r-32)^2}$
p_{IN}^*	$\frac{2a(10-8\theta-3\beta^2 r+3\beta^2 r\theta)}{32-9\beta^2 r}$	π_L^{I*}	$\frac{16ra^2\theta^2-8ra^2\theta+ra^2-18br\beta^2+64b}{2(32-9\beta^2 r)}$
π_R^{I*}	$\frac{a^2(16\theta+3\beta^2 r-3\beta^2 r\theta-12)^2+16a^2(4\theta-1)^2(1-r)}{(9\beta^2 r-32)^2} - b$		

Lemma 3 shows that the equilibrium solutions are mainly affected by the market size (a), the live-streaming consumer purchase rate (θ) and the influencer's sales ability (β).

Corollary 2. The impact of the main parameters on the optimal results in influencer live-streaming mode are given in Table 7.

Table 7. The impact of main parameters on the optimal results in influencer live-streaming mode.

Parameter	e_I^*	w_I^*	p_{IN}^*	p_I^*	D_{IN}^*	D_I^*	π_M^{I*}	π_R^{I*}	π_R^{I*}
β	+	+	+	+	−	+	+	$-\frac{1}{4}<\theta<\widetilde{\theta_2}$ $+\widetilde{\theta_2}<\theta<\theta_2$	+
θ	+	+	$-\beta<\widetilde{\beta_1}$ $+\widetilde{\beta_1}<\beta<\beta_1$	+	−	+	+	$-\frac{1}{4}<\theta<\widetilde{\theta_3}$ $+\widetilde{\theta_3}<\theta<\theta_2$	+
r	+	+	+	+	−	+	−	−	+

+: increase; -: decrease. $\theta_2=\frac{12-3\beta^2 r}{16-3\beta^2 r}$, $\widetilde{\theta_2}=\frac{24-12r-3\beta^2 r}{64-48r-3\beta^2 r}$, $\widetilde{\theta_3}=\frac{-9\beta^4 r^2+84\beta^2 r+64r-256}{-9\beta^4 r^2+96\beta^2 r+256r-512}$.

Table 7 indicates the influence of the influencer's sales ability (β) on each equilibrium solution. The influencer's sales effect level, the equilibrium retail price, and the wholesale price will rise with the enhancement of the influencer's sales ability. The demand of the traditional channel is inversely proportional to the influencer's sales ability. This is because when the influencer's sales ability is strengthened, consumers can obtain more details, which increases the consumers' intent to purchase on the live-streaming channel. The profits of the retailer will be affected by the live-streaming consumer purchase rate. When the live-streaming consumer purchase rate is low, the retailer's profits will decrease as the influencer's sales ability improves because when the live-streaming consumer purchase rate is low, the profits that the retailer can obtain through live streaming cannot cover the cost of live streaming.

Table 7 indicates the influence of the live-streaming consumer purchase rate (θ) on each equilibrium solution. The influencer's sales effort level, equilibrium wholesale price, and the retail price of the live-streaming channel will rise with the growth of the live-streaming consumer purchase rate. When the influencer's sales ability is low, the retail price is negatively correlated with the influencer's sales effort level. At this point, the retailer will reduce the retail price to obtain greater demand from the traditional channel. Therefore, the retailer should take measures with the influencer to increase the live-streaming consumer purchase rate to increase profits.

Table 7 demonstrates the influence of the commission rate of the influencer (r) on each equilibrium solution. The profits of the retailer and the manufacturer are negatively associated with the commission rate of the influencer. As the commission rate rises, the fee that the retailer is required to pay to the influencer also increases, which directly results in higher costs for the retailer. For instance, if retailers sell products through platforms such as Tamall, they need to pay high commissions to both the Tamall platform and the influencer, and this cost might eventually be passed on to consumers, leading to an increase in the price of products.

4. COMPARISONS

To further analyze whether the retailer opts for live-streaming mode, a comparative analysis is conducted from the perspectives of the retailer and the manufacturer to discuss under what circumstances it is beneficial for the retailer, and countermeasures are proposed for the retailer to launch live-streaming. The following theorems provide a comparison of different modes.

Theorem 1. The comparison among the demand and prices

(i) The total demand in different modes satisfies the following relationship: $D_{IN}^*+D_I^*>D_N^*$, $D_{SN}^*+D_S^*>D_N^*$;

(ii) The equilibrium wholesale price satisfies the following relationship: $w_I^*<w_N^*$, $w_S^*<w_N^*$;

(iii) The equilibrium retail price in both two channels satisfies the following relationship. When $\frac{3}{8}<\theta<\overline{\theta_1}$, $p_{SN}^*>p_S^*$, *when* $\overline{\theta_1}<\theta<\theta_1$, $p_{SN}^*<p_S^*$; *when* $\frac{1}{4}<\theta<\overline{\theta_2}$, $p_{IN}^*>p_I^*$, *when* $\overline{\theta_2}<\theta<\theta_2$, $p_{IN}^*<p_I^*$.

Theorem 1 (i) presents the comparison of equilibrium demand between the retailer with and without the live-streaming channel. When the retailer opts for the live-streaming channel, the overall demand will escalate. This is because the opening of the live-streaming channel boosts consumer purchases and enhances supply chain efficiency.

Theorem 1 (ii) demonstrates the comparison of the equilibrium wholesale price between the retailer with and without the live-streaming channel. After the retailer initiates the self-live-streaming mode, the equilibrium wholesale price for the live-streaming channel will be lower. This is because the manufacturer merely serves as the provider of the product, while

the retailer is responsible for selling the product. This leads to a reduction in promotional fees as well as the cost of selling profitable products for the manufacturer.

Theorem 1 (iii) demonstrates the comparison of the equilibrium retail price between the retailer with and without live streaming. Owing to the operating costs, marketing strategies, and consumer psychology in live-streaming e-commerce, the price of the live-streaming channel will be higher than that of the traditional channel.

Theorem 2. The comparison of the retailer's profits

(i) The following relationships are presented about the retailer's profits between mode N and mode S: there is a boundary point $\widehat{\theta_1}$ *such that* $\pi_R^{S*} < \pi_R^{N*}$ *if* $\theta \in \left\{\frac{3}{8}, \widehat{\theta_1}\right\}$ *otherwise* $\pi_R^{S*} > \pi_R^{N*}$ *if* $\theta \in \left\{\theta_1, \theta_1\right\}$.

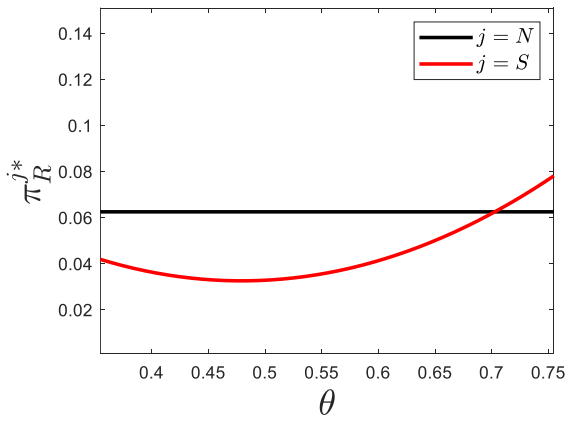
(ii) The retailer's profits in mode N and mode I satisfy the following relationship: $\pi_R^{I*} < \pi_R^{N*}$.

Theorem 2 (i) presents the profit comparison under the no live-streaming mode and the retailer self-live-streaming mode. The profit of the retailer will be affected by the live-streaming consumer purchase rate (θ). With a lower live-streaming consumer purchase rate, it is more beneficial for the retailer not to opt for live-streaming mode. Since the retailer selects a self-live-streaming mode, the sales effort level of the employee-streamer is positively correlated with the live-streaming consumer purchase rate. Consumers will obtain a comprehensive understanding of the product details from the retailer's self-live-streaming mode, increasing their purchase intentions and ultimately resulting in an increase in overall demand.

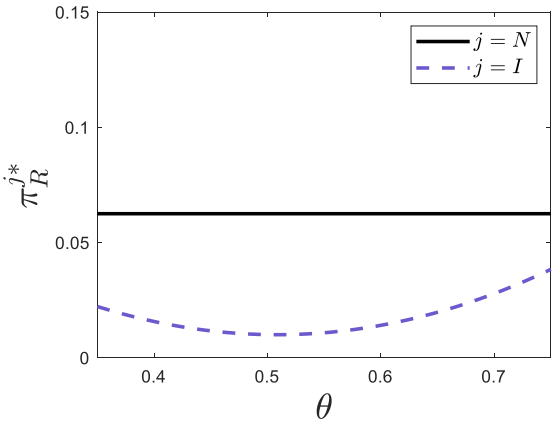
Theorem 2 (ii) presents the profit comparison under the no live-streaming mode and the influencer live-streaming mode. The retailer's profit margins are lower in the influencer live-streaming mode than they would be otherwise. By choosing the influencer live-streaming mode, the retailer will provide the influencer with a fixed participation fee and commission, which leads to lower profit margins.

In light of the actual situation, we employ a numerical example to elucidate Theorem 2 and obtain Figure 3(Xin *et al.,* 2023; Zhang *et al.,* 2023). If we change the parameter values, the main results are consistent with the analytical solution. *Theorem 2* commences from the profit of the retailer. It is more beneficial for the retailer to initiate self-live-streaming mode when the live-streaming consumer purchase rate is higher. If the retailer opts for the influencer live-streaming mode, the retailer will be less profitable compared to the situation without the live-streaming channel.

Nevertheless, the high traffic effort of the influencer is beyond the capacity of the employee-streamer. In the early stage of product promotion, the retailer still needs to promote products through the influencer live-streaming mode to enhance product visibility. Hence, the influencer live-streaming mode is a live-streaming mode requisite for the retailer.



(a) The profit comparison of the retailer in mode N and mode S with $\lambda = 0.5a = 1$



(b) The profit comparison of the retailer in mode N and mode I with $\beta = 0.5r = 0.1$, $b = 0.02$

Figure 3. The relationship between the retailer's profit and θ

Theorem 3. The comparison of the manufacturer's profit

(i) *In mode N and S, the relationships between the manufacturer's profit are as follows. When* $\lambda < \widetilde{\lambda_1}$, $\pi_M^{S*} < \pi_M^{N*}$; *when* $\widetilde{\lambda_1} < \lambda < \lambda_1$ *there is a boundary point* $\overline{\theta_3}$ *such that* $\pi_M^{S*} < \pi_M^{N*}$ *if* $\theta \in \left\{\frac{3}{8}, \overline{\theta_3}\right\}$, *otherwise* $\pi_M^{S*} > \pi_M^{N*}$ *if* $\theta \in \left\{\overline{\theta_3}, \theta_1\right\}$.

(ii) *In mode N and I, the relationship between the manufacturer's profit is as follows:* $\pi_M^{I*} < \pi_M^{N*}$.

Theorem 3 (i) presents the profit comparison under the retailer's self-live-streaming mode and no live-streaming mode. The most favorable mode for the manufacturer is determined by the sales ability of the employee-streamer (λ) and the live-streaming consumer purchase rates (θ). The retailer's self-live-streaming mode is disadvantageous to the manufacturer when the sales ability of the employee-streamer and the live-streaming consumer purchase rate are low. This is because when the live-streaming consumer purchase rate is high, both the retail and wholesale prices in the live-streaming channel increase in retailer self-live-streaming mode. Consequently, the manufacturer prefers the retailer's self-live-streaming mode. From *Theorem 1*, it can be observed that with the introduction of live streaming, the demand for the product is higher than that without live streaming. As a consequence, the manufacturer favors the retailer's self-live-streaming mode.

Theorem 3 (ii) presents the profit comparison of the manufacturer between the influencer live-streaming mode and no live-streaming mode. It is the influencer live-streaming mode that leads to a decrease in the wholesale price and the profit of the manufacturer.

In light of the actual situation, we employ a numerical example to elucidate *Theorem 3* and obtain Figure 4.

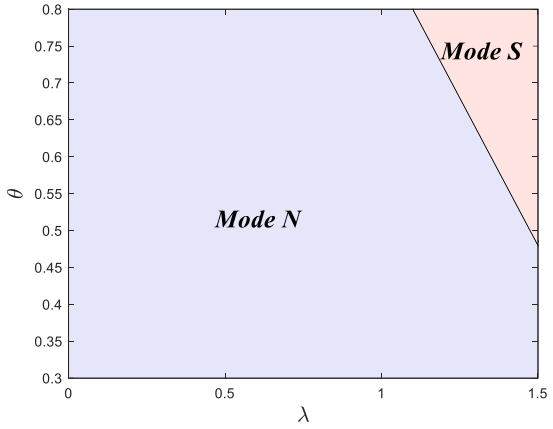


Figure 4. The manufacturer's mode preference according to θ and λ with $a = 1$

Theorem 4. The comparison of the total supply chain's profit

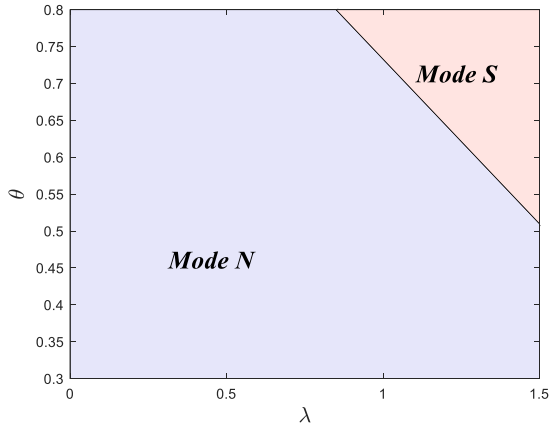
(i) *The total supply chain's profit in mode N and mode S satisfy the following relations. When* $\lambda < \widehat{\lambda_2}$, $\pi^{S*} < \pi^{N*}$; *when* $\widehat{\lambda_2} < \lambda < \lambda_1$ *there is a boundary point* $\overline{\theta_4}$ *such that* $\pi^{S*} < \pi^{N*}$ *if* $\theta \in \left\{\frac{3}{8}, \overline{\theta_4}\right\}$, *otherwise* $\pi^{S*} > \pi^{N*}$ *if* $\theta \in \left\{\overline{\theta_4}, \theta_1\right\}$.

(ii) *The following relationships are presented about the supply chain's profits between mode N and mode I. When* $0 < \beta < \widetilde{\beta_2}$, $\pi^{I*} < \pi^{N*}$; *when* $\widetilde{\beta_2} < \beta < \beta_1$ *there is a boundary point* $\overline{\theta_5}$ *such that* $\pi^{I*} < \pi^{N*}$ *if* $\theta \in \left\{\frac{1}{4}, \overline{\theta_5}\right\}$, *otherwise* $\pi^{I*} > \pi^{N*}$ *if* $\theta \in \left\{\overline{\theta_5}, \theta_2\right\}$.

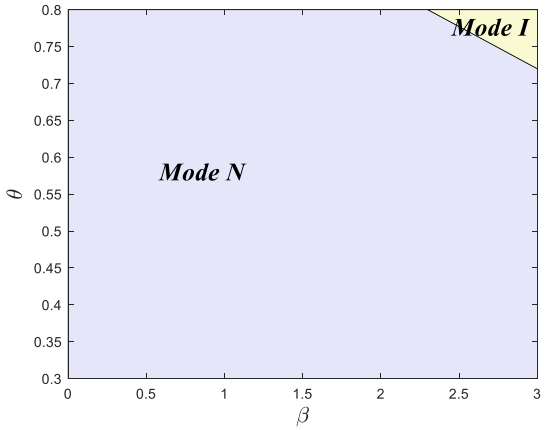
Theorem 4 (i) presents the profit comparison of the total supply chain between the retailer's self-live-streaming mode and the no-live-streaming mode. The profit of the total supply chain will be affected by the sales ability of the employee-streamer (λ) and the live-streaming consumer purchase rate (θ). In the retailer self-live-streaming mode, it is detrimental for the retailer to choose the live-streaming channel when the sales ability of the employee-streamer is low. When the live-streaming consumer purchase rate is low, and the sales ability of the employee-streamer is high, the number of consumers is relatively small, and the overall demand decreases in the retailer's self-live-streaming mode. The relatively high cost of the

retailer's self-live-streaming will impact the profitability of the supply chain. Therefore, the retailer chooses no live-streaming mode.

Theorem 4 (ii) presents the total supply chain profit comparison in retailer self-live-streaming mode and that of no live-streaming mode. Similar to the above, the profitability of the entire supply chain is related to the sales ability of the influencer (β) and the live-streaming consumer purchase rate (θ). In influencer live-streaming mode, the total supply chain profit increases with a higher live-streaming consumer purchase rate and the influencer's sales ability. This is because, at this point, the costs for the manufacturer and the retailer are reduced, optimizing the supply chain structure. In light of the actual situation, we employ a numerical example to elucidate Theorem 4 and obtain Figure 5. When the retailer initiates the influencer live-streaming, the retailer and the manufacturer will be less profitable than without the live-streaming channel. Therefore, from the profit perspective, initiating the influencer live-streaming is disadvantageous for the retailer.



(a) The total supply chain's mode preference according to $\lambda = 0.5$



(b) The total supply chain's mode preference according to $\beta = 0.5, r = 0.1, b = 0.02$

Figure 5. The total supply chain's mode preference with $a = 1$

We have ascertained that for the retailer, it is invariably more advisable not to initiate the influencer live-streaming mode. Nevertheless, in comparison with no live-streaming mode, the combined profits of the retailer and the influencer are higher than the retailer's profit without the live-streaming channel. Hence, by (Huang *et al.,* 2020; Li *et al.,* 2024), we propose *Theorem 5*.

Theorem 5. *The profit-sharing mechanism is as follows:* $\pi_R^{I*} + \pi_L^{I*} > \pi_R^{N*}$ *indicates that there is always a profit-sharing range A and* $A \in (\pi_R^{N*} - \pi_R^{I*}, \pi_L^{I*})$. *It makes* $\pi_L^{I*} - A > \pi_L^{N*} = 0$ *and* $\pi_R^{I*} + A > \pi_R^{N*}$ *constant.*

Theorem 5 presents the comparison of the total profit between the retailer and the influencer when the retailer initiates the influencer live-streaming mode and no live-streaming mode. The retailer's profit from no live-streaming mode is lower than the sum of the retailer's and the influencer's profits from the influencer live-streaming mode. Hence, the influencer can compensate for the profit loss of the retailer by choosing the influencer live-streaming mode by sharing profit to achieve Pareto improvement for both the retailer and the influencer.

The influencer can share profit A with the retailer to achieve Pareto improvement in the profits of the retailer and the influencer; that is, the subsidized retailer is more profitable than that without the live-streaming channel. Also, the influencer remains profitable. At this juncture, the retailer will choose the influencer live-streaming mode, attaining a Pareto improvement for both the retailer and the influencer compared to no live-streaming mode.

5. EXTENSIONS

This section presents three extended studies. Firstly, non-zero production cost is considered. Secondly, the influence of consumers' cross-price sensitivity between the channels is explored.

5.1. Non-zero production cost

In the actual production process, the profit of the manufacturer is affected by the production cost. Taking into account the effect of production costs, if the retailer selects the no live-streaming mode, the demand function is$D_N = a - p_N$, the profit

functions are$\pi_R^N = (p_N - w)D_N$for the retailer and$\pi_M^N = D_N(w_N - c)$for the manufacturer, where$c$is the unit production cost of the product. Through the inverse solution method, Table 8 shows the equilibrium solution.

Table 8. Equilibrium solutions for no live-streaming mode when production cost is non-zero

	w_N^*	p_N^*	D_N^*	π_M^{N*}	π_R^{N*}
Equilibrium solution	$\frac{a+c}{2}$	$\frac{3a+c}{4}$	$\frac{a-c}{4}$	$\frac{a^2-c^2}{8}$	$\frac{a^2-c^2}{16}$

In the retailer self-live-streaming mode, $\pi_M^S = (w_S - c)(D_{SN} + D_S)$is used to illustrate the profit function of the manufacturer. The other functions are the same as in Section 3.2. The equilibrium solutions are shown in Table 9.

$\pi_M^I = (w_I - c)(D_{IN} + D_I)$is used to illustrate the profit function of the manufacturer, and the other functions are the same as in section 3 in influencer live-streaming mode. Table 10 shows the equilibrium solutions.

This section is more parameterized. Hence, we have employed numerical analysis in the following section to verify and extend the conclusions. Therefore, in accordance with the relevant literature and the actual situation of the e-commerce industry, we adopt a numerical example to obtain Figures 6 and 7.

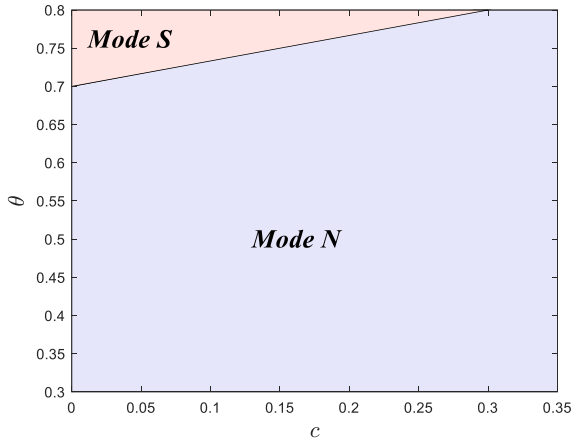


Figure 6. The retailer's selection in relation to θ and c according to $a = 1,\ \lambda = 0.8$

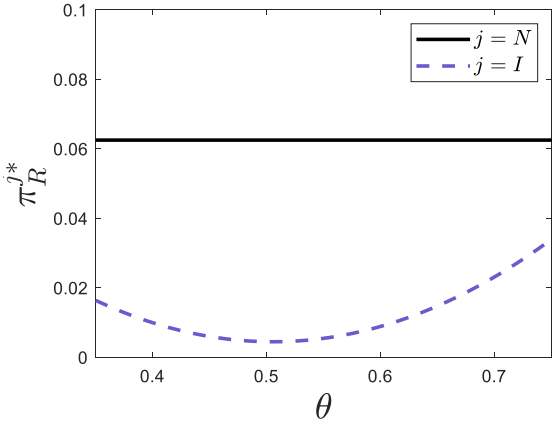


Figure 7. The relationship between the profit of the retailer and θ in mode *N* and mode *I* with $a = 1, \beta = 1, r = 0.1, b = 0.02,\ c = 0.05$

Table 9. Equilibrium solutions of retailer self-live-streaming mode when production cost is non-zero

	Equilibrium solution		Equilibrium solution
e_S^*	$\frac{3a\lambda+2\lambda c-8a\lambda\theta}{5\lambda^2-16}$	D_{SN}^*	$\frac{12a-8c-16a\theta-3a\lambda^2+3\lambda^2c+3a\lambda^2\theta}{2(16-5\lambda^2)}$
w_S^*	$\frac{4a+8c-2a\lambda^2-3\lambda^2c+2a\lambda^2\theta}{16-5\lambda^2}$	D_S^*	$\frac{4a+8c-16a\theta+a\lambda^2-\lambda^2c-a\lambda^2\theta}{10\lambda^2-32}$
p_S^*	$\frac{4a+8c+16a\theta-5a\lambda^2-5\lambda^2c+5a\lambda^2\theta}{2(16-5\lambda^2)}$	p_{SN}^*	$\frac{20a+8c-16a\theta-7a\lambda^2-3\lambda^2c+7a\lambda^2\theta}{2(16-5\lambda^2)}$
π_M^{S*}	$\frac{4(2a-4c-a\lambda^2+\lambda^2c+a\lambda^2\theta)^2}{(5\lambda^2-16)^2}$		
π_S^{R*}	$\frac{a^2\lambda^2\theta^2-2a^2\lambda^2\theta+a^2\beta^2-16a^2\theta^2+16a^2\theta-5a^2+2a\lambda^2c\theta-2a\lambda^2c+4ac+\lambda^2c^2-4c^2}{10\lambda^2-32}$		

Table 10. Equilibrium solutions of influencer live-streaming mode when production cost is non-zero

	Equilibrium solution		Equilibrium solution
e_I^*	$\frac{3\beta r(a+2c-4a\theta)}{9\beta^2r-32}$	D_{IN}^*	$\frac{12a-8c-16a\theta-3a\beta^2r+3\beta^2cr+3a\beta^2r\theta}{32-9\beta^2r}$
w_I^*	$\frac{8a+16c-3a\beta^2r-6\beta^2cr+3a\beta^2r\theta}{32-9\beta^2r}$	D_I^*	$\frac{4(a+2c-4a\theta)}{9\beta^2r-32}$
p_I^*	$\frac{4a+8c+16a\theta-3a\beta^2r-6\beta^2cr+3a\beta^2r\theta}{32-9\beta^2r}$	p_I^*	$\frac{20a+8c-16a\theta-6a\beta^2r-3\beta^2cr+6a\beta^2r\theta}{32-9\beta^2r}$
π_M^{I*}	$\frac{(8a-16c-3a\beta^2r+3\beta^2cr+3a\beta^2r\theta)^2}{(9\beta^2r-32)^2}$		
π_R^{I*}	$\frac{\begin{pmatrix}18a^2\beta^4r^2\theta-9a^2\beta^4r^2\theta^2-9a^2\beta^4r^2+96a^2\beta^2r\theta^2-168a^2\beta^2r\theta+72a^2\beta^2r\\+256a^2r\theta^2-128c^2-128a^2r\theta+16a^2r-512a^2\theta^2+512a^2\theta-160a^2\\-18a\beta^4cr^2\theta+18a\beta^4cr^2+144a\beta^2cr\theta-120a\beta^2cr-256acr\theta+64acr\\+128ac-9\beta^4c^2r^2+81b\beta^4r^2+48\beta^2c^2r-576b\beta^2r+64c^2r+1024b\end{pmatrix}}{(9\beta^2r-32)^2}$		
π_L^{I*}	$\frac{16ra^2\theta^2-8ra^2\theta+ra^2-16rac\theta+4rac-18br\beta^2+4rc^2+64b}{64-18\beta^2r}$		

From Figure 6 and Figure 7, the following results can be obtained. When θ is larger and c is smaller, $\pi_R^{S*} > \pi_R^{N*}$; otherwise, $\pi_R^{S*} < \pi_R^{N*}$. It indicates that when the production cost is low, and the live-streaming consumer purchase rate in the retailer's self-live-streaming mode is high, the retailer may opt for live-streaming mode. $\pi_R^{I*} < \pi_R^{N*}$ reveals that regardless of the production cost, the retailer's profit will decrease in the influencer live-streaming mode, which is consistent with the conclusion in Section 4. As can be observed from Figure 6 and Figure 7, with a higher production cost, the retailer will not choose live-streaming mode. The production costs directly influence the profit of the retailer in live-streaming sales. For instance, when the production cost of a product is low, the retailer may possess a greater price advantage in live-streaming sales, thereby attracting more consumers and increasing the sales demand. Conversely, if the production cost is higher, the retailer may need to maintain profits by increasing the selling price or reducing marketing investment, which might affect sales effectiveness and consumers' willingness to purchase.

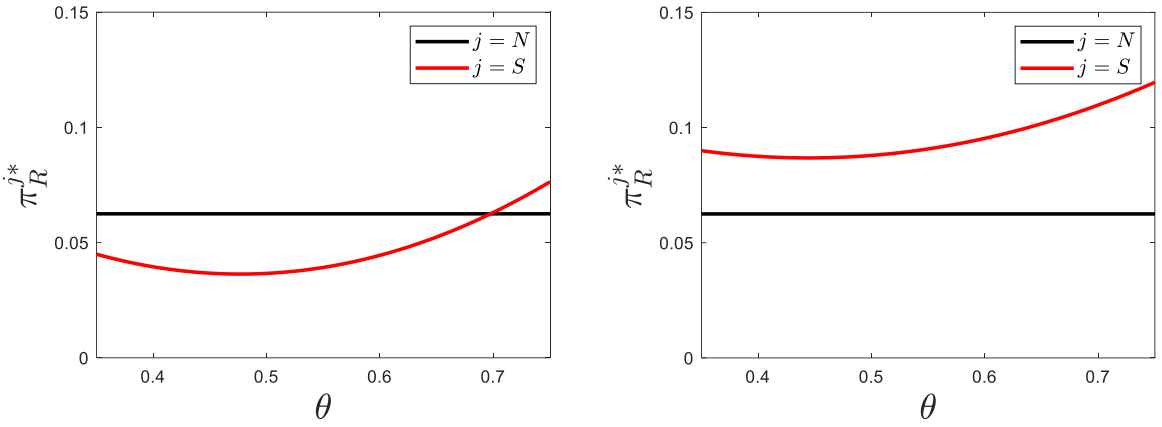
5.2. Impact of dual-channel cross-price sensitivity

When consumers decide to purchase products, they compare the prices of products from different channels. This implies that the retail price in the traditional channel will have an impact on the demand in the live-streaming channel. Hence, there is a strong imperative to study the impact of cross-price sensitivity. We use μ to represent cross-price sensitivity. In the retailer self-live-streaming mode, the demand functions in the traditional channel and the live-streaming channel are respectively denoted by $D_{SN} = (1-\theta)a - p_{SN} + \mu p_S$ and $D_S = \theta a - p_S + \lambda e_S + \mu p_{SN}$. Meanwhile, the manufacturer's profit function is $\pi_M^S = w_S(D_{SN} + D_S)$. $\pi_R^S = (p_{SN} - w_S)\, D_{SN} + (p_S - w_S)\, D_S - \frac{e_S^2}{2}$ is used to illustrate the profit of the retailer. The equilibrium solutions are shown in Table 11.

We conduct an analysis of the retailer's mode choice strategy through numerical analysis, as illustrated in Figure 9.

Figure 9 indicates that the profit of the retailer is related to the live-streaming consumer purchase rate (θ) and the sensitivity of cross-price (μ). When μ and θ are smaller, $\pi_R^{S*} < \pi_R^{N*}$; When μ is smaller but θ is larger, $\pi_R^{S*} > \pi_R^{N*}$. The results demonstrate that when the cross-price sensitivity is low, and the live-streaming consumer purchase rate is also low, that is,

the price competition between the two channels is not significant, the retailer may not perceive the necessity to open the live-streaming channel as it will not have a substantial impact on their sales strategy or increase sales. This conclusion is in line with the one in Section 4.



(a) $\mu = 0.1$ (b) $\mu = 0.6$

Figure 9. The influence of the cross-price sensitivity in different modes on the retailer's profit with $a = 1,\ \lambda = 0.8$

Table 11. Equilibrium solutions of the retailer self-live-streaming mode under cross-price sensitivity

	Equilibrium solution		Equilibrium solution
e_S^*	$\frac{3a\lambda-5a\lambda\mu-8a\lambda\theta+8a\lambda\mu\theta}{5\lambda^2+16\mu^2-16-3\lambda^2\mu}$	p_{SN}^*	$\frac{a(4\mu-16\theta+16\mu\theta+7\lambda^2\theta-7\lambda^2+20)}{6\lambda^2\mu-10\lambda^2-32\mu^2+32}$
w_s^*	$\frac{2a(2\mu+\lambda^2\theta-\lambda^2+2)}{3\lambda^2\mu-5\lambda^2-16\mu^2+16}$	p_S^*	$\frac{a(20\mu+16\theta-16\mu\theta+5\lambda^2\theta-5\lambda^2+4)}{6\lambda^2\mu-10\lambda^2-32\mu^2+32}$
D_S^*	$\frac{a(16\theta+3\lambda^2\mu+\lambda^2\theta-16\mu^2\theta-\lambda^2+4\mu^2-3\lambda^2\mu\theta-4)}{6\lambda^2\mu-10\lambda^2-32\mu^2+32}$		
D_{SN}^*	$\frac{a(16\theta-\lambda^2\mu-3\lambda^2\theta-16\mu^2\theta+3\lambda^2+12\mu^2+\lambda^2\mu\theta-12)}{10\lambda^2+32\mu^2-32-6\lambda^2\mu}$		
π_M^{S*}	$\frac{4a^2(1-\mu)(2\mu+\lambda^2\theta-\lambda^2+2)^2}{(3\lambda^2\mu-5\lambda^2-16\mu^2+16)^2}$		
π_R^{S*}	$-\frac{a^2(3\mu+16\theta-16\mu\theta-2\lambda^2\theta+16\mu\theta^2+\lambda^2-16\theta^2+\lambda^2\theta^2-5)}{6\lambda^2\mu-10\lambda^2-32\mu^2+32}$		

When μ is larger, $\pi_R^{S*} > \pi_R^{N*}$. The results disclose that the retailer is more profitable when cross-price sensitivity is high. The retailer may select the live-streaming channel. This is because the live-streaming mode, as an emerging sales modality, can effectively engage consumers and boost sales conversion rates. Through the live-streaming channel, the retailer can directly showcase product details to consumers and employ live talk to influence consumer satisfaction rates, thereby reducing return behavior.

Additionally, the live-streaming channel has a positive spillover effect on traditional e-commerce channels, which suggests that through the live-streaming channel is for goods. The retailer can not only attract more potential consumers but also enhance the sales performance of traditional channels. The retailer's profit will increase. Consequently, the retailer may opt for live-streaming mode.

6. CONCLUSION

This paper examines the retailer's perspective regarding whether the retailer should introduce live-streaming channels, with a focus on the influence of the sales ability of the streamer and the live-streaming consumer purchase rate. There are three modes for online retailers: influencer live-streaming mode, no live-streaming mode, and retailer self-live-streaming mode. The equilibrium results are obtained. We compared the profits of the retailer, the manufacturer, the influencer, and the supply chain under the three modes.

Firstly, influenced by the fixed participation fee and commission rate of the influencer, the influencer live-streaming mode is always disadvantaged compared to the no-live-streaming mode for the online retailer. When the live-streaming consumer purchase rate is high, the online retailer will select the retailer self-live-streaming mode, otherwise, the retailer will choose no live-streaming mode. Secondly, the total profit of the retailer and the influencer in the influencer live-streaming mode is always higher than the profit of the retailer in the live-streaming mode. Considering the case where the live-streaming consumer purchase rate is low, the online retailer will choose no live-streaming mode. In this case, the influencer can share the profit with the retailer to ensure that the retailer's profit exceeds that under no live-streaming mode and the influencer guarantees the profitability. The profit-sharing mechanism successfully achieves a Pareto improvement for both the retailer and influencer in the influencer live-streaming mode compared to no live-streaming mode. Finally, we extend our conclusions by considering the case where the manufacturer has nonzero production costs and price cross-sensitivity exists.

Our research offers several practical insights for the manufacturer, the online retailer, and the influencer alike. Firstly, the online retailer enhances the training of the employee-streamer to enhance their sales ability and augment profit. Secondly, for the influencer, the appropriate reduction of the fixed participation fee and commission rate encourages the online retailer to opt for cooperation to maximize profit on both sides. Finally, the online retailer, the manufacturer, and the streamer are not isolated entities, and the three parties should intensify cooperation to maximize profits collectively. For retailers who have just entered the market or new brands in the early stage of the product, the brand can use the traffic of influencers to improve their influence. When the brand image is established, retailers or brands can sell products through retail employees. For example, the beauty brand Huaxizi initially stood out through the publicity of streamer Li Jiaqi.

In the future, we may focus on the following aspects. Firstly, in this study, we only focus on the situation of a single retailer. In the future, we will consider the competition of multiple retailers, or we will consider joining the manufacturer's live-streaming channel to discuss the problem of channel encroachment. Secondly, we treat the commission rate as an exogenous variable, and future research could use this as a decision variable.

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