

OPTIMAL SELLING AND LEASING STRATEGY FOR A DOMINANT DURABLE GOODS MANUFACTURER UNDER SEQUENTIAL MARKET ENTRY AND RENTAL PLATFORM COMPETITION

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Recent advances in digital and information technologies have significantly enhanced the attractiveness of product leasing through online rental platforms, especially in durable product markets. Prominent manufacturers such as BMW, IBM, GM, and Epson have embraced the leasing model as a means to promote their products. However, limited studies have addressed the impacts of a competitor's encroachment on the strategic decisions of the dominant manufacturer. In this paper, we develop a two-period dual-channel analytical framework where the dominant manufacturer exists in period 1, and an entrant encroaches on the market share in period 2. Our analysis shows that, no matter how the market size changes, the dominant manufacturer can always benefit from the leasing channel, and it is never in its best interest to adopt a pure leasing strategy in period 1. If the market size shrinks in period 2, the dominant manufacturer prefers selling products directly to consumers in period 1, resulting in a decline in leasing ratios. Moreover, our analysis indicates that the entrant's encroachment may increase the dominant manufacturer's profit under specific conditions.

Keywords: Supply Chain Structure, Durable Products, Competition, Rental Channel, Market Size

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1. INTRODUCTION

In recent years, the development of information and mobile communication technologies has made leasing products through online rental platforms a trend in the economy, especially in the durable product market. In marketing durable products, e.g., cars, televisions, and personal computers, manufacturers usually adopt a mixed strategy of leasing and selling. Online platforms enable dealers and individuals to lease their products more conveniently. For example, Turo, one of the leading car-sharing platforms, has continued to expand in recent years. According to the company's recent reports, Turo closed 2024 with 3.6 million active guests, 140,000 active hosts, and 340,000 active vehicles, and operations across more than 28,000 cities.

In the leasing model, the right to use, rather than ownership, governs the relationship between manufacturers and consumers (Mao and Zaleski, 2011). Such a business model has attracted interest in a variety of contexts. For instance, the car-sharing program transfers the fixed cost associated with the ownership of a vehicle, such as purchase cost, depreciation, and insurance, to the variable cost that depends on vehicle usage. According to Statista, the number of car-sharing users

worldwide reached approximately 57 million in 2024 and is expected to exceed 68 million by 2029. This growth shows that accessing durable goods through rental platforms is becoming increasingly common in practice.

Companies often choose to lease durable products instead of selling them since these products can be used over multiple periods, enabling companies to derive continuous benefits (Goering, 2010; Hunsaker, 2000). For example, the old Bell system monopolized the telephone market and refused to sell telephones to consumers (Desai and Purohit, 1999). However, with the popularisation of telephone technology and the intensification of competition among telephone manufacturers, most manufacturers began to sell their telephones. This shows that the encroachment of competitors offering similar products would significantly influence the decision-making of the previous monopolist (Wan *et al.*, 2023). In this paper, we use a two-period framework to distinguish between the pre-entry and post-entry stages of the durable-goods market. The dominant manufacturer is the sole supplier in period 1, while an entrant introduces a substitute product and competes with the dominant manufacturer in period 2. These two periods represent strategic stages within a longer product life cycle, rather than the product's physical lifetime. Products sold or leased in period 1 continue to affect the market in period 2. At the same time, the two-period setting avoids complexity associated with repeated lease renewals and product replacement cycles.

The leasing of durable products through an intermediary can be characterized in terms of three forms: (i) an independent dealer brokers a lease from the manufacturer to the consumer (Bhaskaran and Gilbert, 2009); (ii) the manufacturer provides an e-channel through which the consumer can rent the product, with effective supervisory mechanisms in place (Xiao and Cao, 2023); (iii) a lease contract can be established on a third-party online platform. In this paper, we focus on the last form, which is widely used in practice and allows the manufacturer to concentrate on the core business instead of building and maintaining a rental platform. Such platforms are becoming increasingly popular in reality. For example, IKEA launched its rental service, IKEA RENTAL, which offers flexible access to furniture. However, there has been little research on leasing durable products through a third-party platform in a competitive environment.

Although there is a considerable body of research on two-period dual-channel supply chains, they often assume that manufacturers enter the market simultaneously and all have the option to lease and sell. This study contributes to the literature on durable-goods selling and leasing by developing a two-period, dual-channel model that considers sequential market entry, leasing through a third-party platform, and changes in market size. Whereas most existing studies assume that competing manufacturers enter the market at the same time, we consider a dominant manufacturer that operates in period 1 and an entrant that introduces a substitute product in period 2. The model also includes a third-party rental platform that sets a unit service fee, thus differing from the conventional setting in which the manufacturer leases products directly. In addition, the second-period market potential may be either larger or smaller than that in period 1, allowing us to examine how changes in future demand affect the dominant manufacturer's selling and leasing decisions across the two periods. The study is also relevant to industrial engineering. For durable-goods manufacturers, the choice between selling and leasing influences not only market strategy, but also production planning, channel design, cooperation with rental platforms, and the allocation of products over time. Selling enables the manufacturer to meet demand and establish its market position earlier, whereas leasing allows it to retain ownership and resell returned products in the following period. The model therefore helps manufacturers plan production and distribution when market conditions change and a competitor enters the market.

We examine three distinct supply chain structures in this study: the monopoly model, the duopoly model with pure selling, and the dual-channel competition model. We highlight several findings by comparing the optimal strategies of all players in these models: (i) In the monopoly model, the monopolist would abandon the second-period market if the potential market size in period 2 shrinks sharply, that is, the monopolist would only sell directly to consumers in period 1; (ii) if there are no significant changes in the market size, the total quantities of products provided by the two manufacturers are getting closer with the increase of the market size; (iii) the dominant manufacturer would never adopt a pure lease strategy in period 1; (iv) the dominant manufacturer always realises a higher profit in the dual-channel competition model, indicating that the emergence of a rental platform is always beneficial to him, regardless of changes in the market size; (v) our analysis shows that the entrant's encroachment may increase the dominant manufacturer's profit when the two products are weak substitutes, because anticipated encroachment encourages more selling and leasing in period 1 while the resulting competitive loss remains limited.

The remainder of this paper is organized as follows. In Section 2, we review the literature on durable products and supply channel structures. Section 3 introduces the basic assumptions and notations. Section 4 details the monopoly model, the duopoly model with pure selling, and the dual-channel competition model, respectively. We then generalize our model to consider the impacts of some important factors on each player's optimal strategy. We extend our study to sequential games in Section 5 and conclude this paper in Section 6. All technical proofs are relegated to the appendix.

2. LITERATURE REVIEW

Business models for durable products have been extensively studied in both academia and practice. With the deep integration of the Internet and traditional industries, innovative online trading platforms are emerging and gaining recognition among

consumers, such as Furniture Share and Airbnb. In the fields of management and business administration, platforms are typically associated with business innovation. Research on rental platforms has received more and more attention (Zervas *et al.*, 2017; Tian and Jiang, 2018).

2.1. Servicization and leasing models

An emerging paradigm among manufacturers involves the implementation of innovative business models where consumers no longer regard ownership as the best way to obtain a product. Alternatively, they prefer a more collaborative approach and are inclined to obtain products temporarily (Ye *et al.*, 2024). Wang *et al.* (2024a) explore optimal service modes for a monopolistic recycling platform under the trade-in-for-new mode, leasing mode, or a mixed mode. Agrawal and Bellos (2017) argue whether the servicization business model, under which a firm sells the right to use a product rather than the product itself, is environmentally beneficial. They show that the environmental performance of the servicization model depends on structural characteristics such as pay-per-use pricing and pooling degree. Based on the participation of Daimler and BMW in the car-sharing market, Bellos *et al.* (2017) study the interaction between different business models and the design of production lines. Agrawal *et al.* (2012) analyze the impacts of product leasing and purchasing on the environment and show that if used products are disposed of by requesting processing fees or remanufacturing, the leasing model can obtain not only higher economic benefits but also higher environmental benefits. Orsdemir *et al.* (2019) conclude that servicization results in a win-win outcome where it can simultaneously increase the firm's profit and decrease the environmental impact, compared with selling products. In the context of carbon reduction, Liu *et al.* (2024) examine optimal financing decisions for a capital-constrained manufacturer, comparing Carbon Emission Permits Repurchase Financing (CEPRF) and Green Credit Financing (GCF) under selling and leasing modes.

2.2. Durable goods in operations management

The second stream of literature relevant to our study deals with durable products in the operations management and marketing domains. Mantena *et al.* (2012) present a broad overview of research on durable product markets and propose an integrative framework for modeling durable products. In marketing durable products, manufacturers usually adopt a mix of leases and sales. The relationship between leases and sales has been studied for a long time, with most studies showing that leasing is a better option (Orsdemir *et al.*, 2019; Coase *et al.*, 1972). However, Chien and Chu (2008) find that, contrary to the existing literature, the profit from selling durable products might be higher than that from leasing when products exhibit network effects. Desai and Purohit (1999) consider competition in durable products markets and get two important findings: (i) the fraction of leased products decreases if the competing manufacturers' products are highly substitutable; (ii) the extent of leasing also depends on product reliability. Li *et al.* (2022) investigate the impacts of consumers' capital constraints on a monopoly manufacturer's choice of operation patterns, including the pure-selling, pure-leasing, and hybrid strategies. Kirkizoglu and Karaer (2022) discuss the after-sales service and warranty decisions of a durable goods manufacturer, and demonstrate that it is better for the manufacturer to grasp the after-sales market by itself, rather than leaving it to the retailer. Li *et al.* (2023) observe a counter-intuitive finding that partial centralization may yield more benefits for supply chain players than a fully centralized or decentralized supply chain structure. Wang *et al.* (2024b) examine how consumer purchase patterns influence a firm's retail decisions and profitability regarding extended warranties sold with durable products.

2.3. Dual-channel supply chains and market competition.

Some studies assume that a manufacturer interacts directly with consumers, neglecting the role of intermediaries in the distribution channel; much research has focused on investigating the structure of the durable products supply chain. For example, Bhaskaran and Gilbert (2009) study how product durability affects the interaction between the manufacturer and its dealers, assuming that the manufacturer delivers durable products to consumers through independent dealers. When competition among dealers is fierce, the manufacturer prefers to use a lease-brokering arrangement. The dual-channel supply chain model, which usually contains two distinct types of channel competition (i.e., vertical competition between the upstream and downstream supply chain players and horizontal competition between different channels), has received much attention (Tsao *et al.*, 2022; Yue *et al.*, 2025). Yin *et al.* (2010) consider both the retail used goods market and the P2P used goods market. They find that the initial introduction of the retail used goods channel discourages the introduction of new versions and restrains the rise in the selling price. Xiong *et al.* (2012) consider the supply chain competition model in which a manufacturer sells a durable product through both a manufacturer-owned e-channel and an independent dealer who adopts a mix of selling and leasing. Esenduran *et al.* (2020) assume manufacturers distribute products through dealers and rental agencies. The buyback programs are developed for manufacturers to repurchase used rental cars from rental agencies and redistribute them through dealers.

Unlike the previous assumption that the manufacturer sells and leases durable products directly to consumers or through independent retailers, we introduce an online rental platform for manufacturers to lease their products. In reality, a lot of leasing activities are done through third-party rental platforms (Liu *et al.*, 2023). For example, Erent is the largest platform for renting and sharing worldwide, where both manufacturers and individuals can provide products or services to consumers. Cai *et al.* (2023) analyze the sharing platform ecosystems in the car-sharing industry, with a particular focus on examining the two operational modes of the platform, namely the asset-heavy mode and the asset-light mode, under an uncertain market. It is worth noting that the roles performed by the independent retailer and the platform vary significantly. Specifically, the retailer can announce selling and leasing prices to consumers, but the platform cannot. The leasing price released by the platform is determined by the manufacturer, and the platform only collects the unit service fee for each successful transaction. Wang *et al.* (2020) investigate the online rental platform's decision-making in choosing between the one-side brokerage model, which charges hosts only, and the both-side brokerage model, which charges both hosts and renters. Ren *et al.* (2020) demonstrate that cooperation between the upstream manufacturer and the downstream platform can result in higher profits and a better service experience. From the practice of platform leasing and sharing business models in the Netherlands, Vaskelainen *et al.* (2022) reveal that crucial factors such as awareness of opportunities, position within the value chain and available resources play pivotal roles in determining which supply chain players are pioneers, early followers and late entrants. Tahirov and Glock (2022) provide a systematic review on manufacturer encroachment and channel conflicts. Our research also contributes to the literature concerning changes in the market size (Zhao and Jagpal, 2009; Zhu *et al.*, 2016; Ivanov, 2019), as most existing studies ignore the fact that if consumers hold a low evaluation of the product, demand may decrease in the following periods. To fill this gap, we consider all potential fluctuations in market demand.

Our paper differs from the aforementioned studies in three main respects. First, unlike the settings examined by Desai and Purohit (1999), who consider direct selling and leasing under durable goods competition, and Xiong *et al.* (2012), who study a dual-channel structure involving a manufacturer-owned e-channel and a dealer, we introduce a third-party rental platform into the durable goods selling and leasing model. The platform does not own the products but sets a unit service fee for each successful rental transaction. Second, we focus on a sequential market entry scenario in which the dominant manufacturer enters the market in period 1, followed by an entrant in period 2, thereby capturing how anticipated encroachment affects the dominant manufacturer's first-period selling and leasing decisions. Third, the market size in period 2 may be either larger or smaller than that in period 1. We examine how changes in market size, together with product substitutability, affect the optimal decisions of the dominant manufacturer, the entrant, and the rental platform.

3. MODEL ASSUMPTIONS

In this section, we present the basic setup of our model and outline our assumptions regarding products, manufacturers, and the rental platform. Let us begin by considering the monopoly model, in which a single manufacturer determines the mix of leasing and selling according to changes in the market size during two periods. We then move on to the duopoly model with pure selling, in which a dominant manufacturer sells durable products in both periods while an entrant encroaches on the market by providing substitutes in period 2. The two manufacturers compete with each other in terms of quantity. Finally, we consider the dual-channel competition model, in which the dominant manufacturer sells products to consumers directly or leases them through a platform by announcing selling and leasing prices. As in the duopoly model, the existence of substitutes is limited to period 2.

We examine the dominant manufacturer's incentives to sell and lease products in period 1, as well as how the degree of competition and changes in market size affect the decisions of the supply chain players. Market size may increase or decrease over time for two main reasons. First, consumers' valuations change when upgraded or substitute products become available. Second, changes in the economic environment influence consumer demand. In our model, β represents the expected market potential in period 2. It is assumed to be known to the platform and both manufacturers before the platform sets its service fee. In practice, manufacturers may estimate future market size using historical demand, product adoption patterns, and industry information. Therefore, β represents an anticipated change in future market size.

Assumption 1. *To simplify the analysis, we normalize the marginal cost $c = 0$, of producing a durable product.*

This assumption allows us to focus on the strategic effects of channel structure, platform service fees, and sequential market entry. It is most appropriate when marginal production costs are low relative to consumers' willingness to pay, or when the main concern is how products are allocated between selling and leasing rather than differences in production costs. Since the same production cost applies to products supplied through either channel, introducing a positive marginal cost would reduce the net value of supplying new products and shift the thresholds separating the equilibrium regimes. We normalize marginal cost to zero, while acknowledging that positive production costs may change the threshold conditions (Li *et al.*, 2023).

Assumption 2. *To represent durability, we assume that products are perfectly durable, which means there is no depreciation of value provided by a durable product between different periods.*

The assumption of no depreciation is a commonly used feature in modeling durable products, as seen in prior studies such as Bhaskaran and Gilbert (2009) and Sudhir (2022). We incorporate multiple periods in our model since durability is the distinction between durable and nondurable products. The perfect durability assumption means that the product retains its functional value over the strategic stage of the model, rather than that it experiences no physical wear over its entire lifetime. This assumption is suitable for durable products whose performance can be preserved through regular maintenance, refurbishment and standardization. As is common in most papers on durable products, we assume that the length of the lease contract is exactly one period. Consequently, there is no distinction between selling and leasing in period 2 because both provide one period of use.

Assumption 3. *The platform collects the unit service fee m for each successful transaction from the manufacturer who rents out his products through the platform.*

This assumption is consistent with many rental markets operated through platforms. Platforms often provide services such as product listing, customer matching, payment processing, insurance and transaction support, while product providers retain control over pricing. For example, on car-sharing platforms such as Turo, vehicle owners and professional hosts list their vehicles and make pricing decisions, while the platform facilitates each transaction and collects a service charge (Jiang and Tian, 2018; Tian and Jiang, 2018). In practice, this charge may take the form of a fixed fee or a commission based on rental revenue. These arrangements support our assumption that the platform sets the service charge m while the manufacturer chooses the leasing quantity and the rental price.

Since the model ends in period 2, selling and leasing both provide one period of use. Neither manufacturer has an incentive to lease through the rental platform in this period for two reasons. First, leasing incurs the unit service fee m whereas direct selling does not. As leasing provides no additional use value in the final period, direct selling is more profitable. This justifies restricting the entrant to selling in period 2. Second, manufacturers will likely produce upgraded products at the end of period 2 to adapt to diverse and changing market demand. This means that if the manufacturers continue to lease the current products, after being returned from consumers at the end of the lease period, the old products will become out of date and harder to sell or dispose of (Bo, 2015).

Assumption 4. *The market consists of two competing manufacturers selling partially substitutable products. The dominant manufacturer enters the market in period 1, while the entrant encroaches on the market share in period 2.*

In period 1, the dominant manufacturer has three choices: (i) selling products directly to consumers; (ii) leasing products through a rental platform; (iii) adopting a mixed strategy of leasing and selling.

Assumption 5. *The selling price is the discounted sum of the two one-period prices, i.e., $P_{1s} = P_{1l} + \rho P_2$.*

Without loss of generality, we normalize the potential market size in period 1 to one. In period 2, the potential market size is denoted by β , where $\beta > 0$, and β captures the changes in the market size. Let Q_i and q_i be the quantities of products provided by the dominant manufacturer and the entrant, respectively; the market-clearing prices in period i would be

$$\begin{aligned} p_i(q_i, Q_i) &= a_i - q_i - eQ_i, \\ P_i(q_i, Q_i) &= a_i - eq_i - Q_i. \end{aligned} \quad (1)$$

In the above Equations, we use uppercase letters to denote the dominant manufacturer and lowercase letters to denote the entrant. The parameter a_i denotes the market demand potential in period i , with $a_1 = 1$ and $a_2 = \beta$. The degree of competition between their products is captured by the parameter e , where $0 < e < 1$. A higher value of e implies more substitutability between the products, leading to more intense competition, where each manufacturer's quantity choice affects the rival's behavior.

In our model, consumers are fully informed and have rational expectations. Therefore, the first-period selling price should reflect their potential use of the product over both periods. To achieve this, we express the first-period selling price as the discounted sum of the two one-period prices, i.e., $P_{1s} = P_{1l} + \rho P_2$, where ρ is a discount factor representing the value of cash flows received in period 2 (Purohit and Staelin, 1994; Bhaskaran and Gilbert, 2009; Xiong *et al.*, 2012). This ensures that consumers are indifferent between leasing and selling and allows us to focus on the manufacturers' incentives to choose between leasing and selling. For simplicity, we assume a zero discount rate, i.e., $\rho = 1$ which does not affect our qualitative results (Li *et al.*, 2023).

4. ANALYSIS

4.1. Monopoly

In this section, we discuss the monopolist's problem of determining the optimal mixed strategy of leasing and selling. The key feature of leasing is that the ownership of the product still belongs to the dominant manufacturer. Previous studies have found that the optimal strategy is to lease all products (Xiong *et al.*, 2012; Bhaskaran and Gilbert, 2009). However, these studies assume that the manufacturer directly leases products to consumers or through some independent dealers, while in reality, many leasing activities are implemented through platforms. We solve the monopoly model by first determining the manufacturer's selling quantity in period 2, and then deciding on the optimal strategy of selling and leasing in period 1. Finally, the unit service fee for the platform is determined, which takes place before the selling season.

In particular, the monopoly model captures two essential aspects: (i) changes in the market size between the two periods; (ii) both selling and leasing channels. Let Q_{ij} and q_{ij} be the quantities of products provided by the dominant manufacturer and the entrant, P_i and p_i be the one-period prices, where $i = 1, 2$ denotes the period and $j = s, l$ denotes the business model, i.e., selling and leasing, respectively. We assume that the platform charges a unit service fee, denoted by $m > 0$. The inverse demand functions in both periods are

$$\begin{aligned} P_{1l} &= 1 - Q_{1s} - Q_{1l}, \\ P_{1s} &= \beta + 1 - 2Q_{1s} - 2Q_{1l} - Q_2, \\ P_2 &= \beta - Q_{1s} - Q_{1l} - Q_2. \end{aligned} \quad (2)$$

Consumers who purchase the product in period 1 receive use value in both periods. Under rational expectations, they anticipate the equilibrium price in period 2, which is determined by the manufacturers' quantity decisions in that period. The first-period selling price includes both the current use value and the use value in period 2.

4.1.1. Second-period analysis

In period 2, the monopolist's objective is to determine the optimal selling and leasing quantities that maximize his own profit, taking into account the decisions made in period 1. We use uppercase Π_i^k to represent the monopolist's profit in period i under model k , where $k = m, d, c$ refers to the monopoly model, duopoly model with pure selling, and dual-channel competition model, respectively. In the monopoly model, the monopolist's optimization problem in period 2 is

$$\max_{Q_2} \Pi_2^m = (\beta - Q_{1s} - Q_{1l} - Q_2)Q_2 + (\beta - Q_{1s} - Q_{1l} - Q_2)Q_{1l}, \quad (3)$$

where the first term is the monopolist's profit from selling products in period 2, and the second term is the profit from remarketing off-lease products in period 2. Taking the second-order derivative of Π_2^m with respect to Q_2 , we observe that the monopolist's profit is concave in Q_2 . Hence, the first-order condition (FOC) is sufficient to yield the equilibrium quantity

$$Q_2^{m*} = \frac{\beta - 2Q_{1l} - Q_{1s}}{2}. \quad (4)$$

4.1.2. First-period analysis

After deriving the optimal quantity of products sold in period 2, we now focus on the monopolist's problem in period 1. Recalling that the monopolist can adopt a mix of selling and leasing to maximize his total two-period profit, it is crucial to consider the impacts of the monopolist's decisions in period 1 on his second-period behavior. The monopolist's optimization problem in period 1 can be formulated as follows:

$$\begin{aligned} \max_{Q_{1s}, Q_{1l}} \quad & \Pi^m = (\beta + 1 - 2Q_{1s} - 2Q_{1l} - Q_2)Q_{1s} + (1 - Q_{1s} - Q_{1l})Q_{1l} - mQ_{1l} + \Pi_2^m, \\ \text{s. t.} \quad & \begin{cases} Q_{1s} \geq 0 \\ Q_{1l} \geq 0 \\ Q_2^{m*} \geq 0 \end{cases} \end{aligned} \quad (5)$$

From the above function, we can readily obtain the K-T optimality conditions and solve this problem. After that, the platform determines the optimal unit service fee charged to the manufacturer, i.e., $\max_m \pi_p^m = mQ_{1l}$. Table 1 summarises the optimal decisions of the monopolist and the platform in the monopoly model.

Lemma 1. *In the monopoly model, the equilibrium outcomes are as follows:*

Table 1. Equilibrium outcomes in the monopoly model

β	$0 < \beta \leq \frac{1}{3}$	$\frac{1}{3} < \beta \leq \frac{5}{11}$	$\frac{5}{11} < \beta \leq \frac{7}{10}$	$\beta > \frac{7}{10}$
m^{m*}	N/A	$\frac{3\beta - 1}{2}$	$\frac{1 - \beta}{3}$	$\frac{1}{10}$
Q_{1s}^{m*}	β	$\frac{1 + \beta}{4}$	$\frac{2(1 - \beta)}{3}$	$\frac{1}{5}$
Q_{1l}^{m*}	N/A	$\frac{3\beta - 1}{8}$	$\frac{5\beta - 2}{6}$	$\frac{1}{4}$
Q_2^{m*}	0	0	0	$\frac{10\beta - 7}{20}$
Π^{m*}	$(1 - \beta)\beta$	$\frac{1 + 26\beta - 23\beta^2}{32}$	$\frac{4 - 2\beta + 7\beta^2}{18}$	$\frac{17 + 20\beta^2}{80}$
π_p^{m*}	N/A	$\frac{(1 - 3\beta)^2}{16}$	$\frac{(1 - \beta)(-2 + 5\beta)}{18}$	$\frac{1}{40}$

The proof of the equilibrium outcomes in this model follows a similar approach to that in the dual-channel competition model in Section 4.3, which is explained in Appendix B. We can see from Table 1 that the unit service fee charged by the platform increases as β increases if $\frac{1}{3} < \beta \leq \frac{5}{11}$, but decreases as β increases if $\frac{5}{11} < \beta \leq \frac{7}{10}$. The monopolist gives up the second-period market if $\beta \leq \frac{7}{10}$. Moreover, the profit of the platform increases in the market size if $\frac{1}{3} < \beta \leq \frac{7}{10}$. Note that when $\beta > \frac{7}{10}$, the unit service fee charged by the platform remains constant at $\frac{1}{10}$ and its profit is $\frac{1}{40}$. The monopolist would adopt a pure selling strategy in period 1 if the market size shrinks significantly in period 2, i.e., $0 < \beta \leq \frac{1}{3}$. The profits of the monopolist and the platform are illustrated in Figures 1-2.

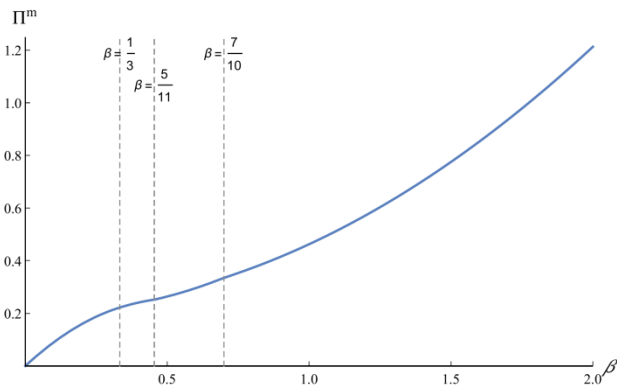


Figure 1. The monopolist's profit

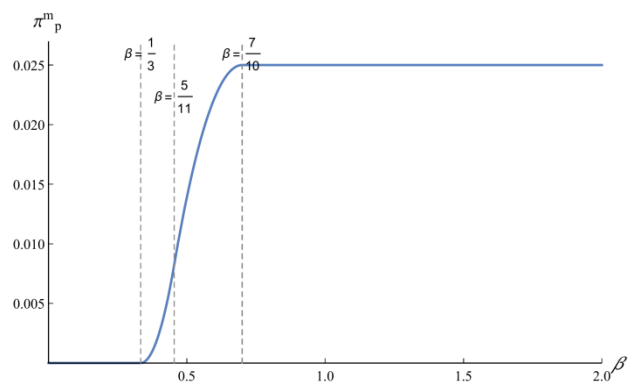


Figure 2. The platform's profit

Corollary 1. *The effects of the second-period market size β on the equilibrium decisions and profits of the supply chain players are summarised as follows: when $0 < \beta \leq \frac{1}{3}$, $\frac{\partial Q_{1s}^*}{\partial \beta} > 0$, $\frac{\partial Q_{1l}^*}{\partial \beta} = 0$, $\frac{\partial Q_2^*}{\partial \beta} = 0$, $\frac{\partial \pi_p^*}{\partial \beta} = 0$, $\frac{\partial \Pi^{m*}}{\partial \beta} > 0$; when $\frac{1}{3} < \beta \leq \frac{5}{11}$, $\frac{\partial Q_{1s}^*}{\partial \beta} > 0$, $\frac{\partial Q_{1l}^*}{\partial \beta} > 0$, $\frac{\partial Q_2^*}{\partial \beta} = 0$, $\frac{\partial \pi_p^*}{\partial \beta} > 0$, $\frac{\partial \Pi^{m*}}{\partial \beta} > 0$; when $\frac{5}{11} < \beta \leq \frac{7}{10}$, $\frac{\partial Q_{1s}^*}{\partial \beta} < 0$, $\frac{\partial Q_{1l}^*}{\partial \beta} > 0$, $\frac{\partial Q_2^*}{\partial \beta} = 0$, $\frac{\partial \pi_p^*}{\partial \beta} > 0$, $\frac{\partial \Pi^{m*}}{\partial \beta} > 0$; when $\beta > \frac{7}{10}$, $\frac{\partial Q_{1s}^*}{\partial \beta} = 0$, $\frac{\partial Q_{1l}^*}{\partial \beta} = 0$, $\frac{\partial Q_2^*}{\partial \beta} > 0$, $\frac{\partial \pi_p^*}{\partial \beta} = 0$, $\frac{\partial \Pi^{m*}}{\partial \beta} > 0$.*

Corollary 1 describes how the dominant manufacturer reallocates product supply between the two periods as the market size in period 2 changes. When $0 < \beta \leq 1/3$, the future market is too small to support leasing or the supply of new products in period 2. The dominant manufacturer relies entirely on selling in period 1. An increase in β raises both his selling quantity and profit, while the rental platform remains inactive. When $1/3 < \beta < 5/11$, leasing becomes profitable. The dominant manufacturer increases both his selling and leasing quantities in period 1, raising the profits of both the manufacturer and the platform. When $5/11 < \beta \leq 7/10$, the dominant manufacturer shifts part of his period 1 supply from selling to leasing. The selling quantity falls, while the leasing quantity rises because leased products can be returned and sold in period 2. No products are supplied in period 2 when $0 < \beta < 7/10$, as demand can be served by products supplied in period 1. When $\beta > 7/10$, the selling and leasing quantities in period 1 remain unchanged. Further market growth is met by supplying additional new products in period 2, and the dominant manufacturer's profit continues to rise, while the platform's profit remains unchanged.

4.2. Duopoly with pure selling

In the duopoly model with pure selling, the dominant manufacturer can only adopt a pure selling strategy because there is no rental channel for leasing products. Our analysis of this model serves to establish a benchmark for comparison with the dual-channel competition model, wherein products can be either leased through a platform or sold directly to buyers. By conducting this comparison, we can gain a better understanding of the significance of leasing flexibility and highlight its implications. The inverse demand functions for each manufacturer are given by

$$\begin{aligned} P_1 &= 1 - Q_1, \\ P_2 &= \beta - Q_1 - Q_2 - eq_2, \\ p_2 &= \beta - e(Q_1 + Q_2) - q_2. \end{aligned} \quad (6)$$

Proceeding with the standard backward induction approach, the duopoly model with pure selling can be solved by first determining both players' optimal quantities for sale in period 2, and then solving the dominant manufacturer's problem in period 1.

4.2.1. Second-period analysis

In period 2, both manufacturers seek to maximize their own profits by determining the optimal quantities simultaneously. The inverse demand functions for the dominant manufacturer and the entrant are given by Equation (6). The optimization problems of the two manufacturers in period 2 can be expressed as follows:

$$\max_{Q_2} \Pi_2^d = (\beta - Q_1 - Q_2 - eq_2)Q_2, \quad (7)$$

$$\max_{q_2} \pi_2^d = (\beta - eQ_1 - eQ_2 - q_2)q_2. \quad (8)$$

Solving Equations (7) and (8) simultaneously yields

$$Q_2^{d*} = \frac{(2-e^2)Q_1 + (e-2)\beta}{e^2-4}, \quad (9)$$

$$q_2^{d*} = \frac{-2\beta + e(Q_1 + \beta)}{e^2-4}. \quad (10)$$

From Equations (9) and (10), one can readily observe that the quantity of products produced by each manufacturer in period 2 decreases with the quantity of products offered by the dominant manufacturer in period 1, and increases with the market size in period 2, which is consistent with our economic intuition.

4.2.2. First-period analysis

Given the optimal solutions in period 2, we now look at the dominant manufacturer's first-period concerns. Recall that in period 1, the dominant manufacturer employs a pure selling strategy and determines the first-period quantity to maximize the aggregate profit Π^d across both periods. The optimization problem is formulated as follows:

$$\begin{aligned} \max_{Q_1} \quad & \Pi^d = \Pi_1^d + \Pi_2^d = (1 - Q_1)Q_1 + \Pi_2^d, \\ \text{s. t.} \quad & \begin{cases} Q_1 \geq 0 \\ q_2^{d*} \geq 0 \\ Q_2^{d*} \geq 0 \end{cases}. \end{aligned} \quad (11)$$

From Equation (11), we derive the following important lemma.

Lemma 2. *The equilibrium quantities and profits in the duopoly model with pure selling are as follows:*

Table 2. Equilibrium outcomes in the duopoly model with pure selling

β	$0 < \beta \leq \beta_4$ (C1)	$\beta_4 < \beta \leq \beta_7$ (C2)	$\beta > \beta_7$ (C3)
Q_1^{d*}	$\frac{(2-e)\beta}{2-e^2}$	$\frac{(2-e)(4e-2e^2-e^3-4\beta+8+2e^2\beta)}{8(3-e^2)}$	0
Q_2^{d*}	0	$\frac{(4-e^2)(-2+e^2+4\beta-2e\beta)}{8(3-e^2)}$	$\frac{\beta}{2+e}$
q_2^{d*}	$\frac{(1-e)\beta}{2-e^2}$	$\frac{-4e+e^3+12\beta-4e\beta-2e^2\beta}{8(3-e^2)}$	$\frac{\beta}{2+e}$
Π^{d*}	$\frac{(e-2)(-2+e^2+2\beta-e\beta)\beta}{(-2+e^2)^2}$	$\frac{(e-2)(e^3+e^2(2-4\beta)+4e(\beta^2-1)-8(1-\beta+\beta^2))}{16(3-e^2)}$	$\frac{\beta^2}{(2+e)^2}$
π_2^{d*}	$\frac{(e-1)^2\beta^2}{(e^2-2)^2}$	$\frac{(-4e+e^3+12\beta-4e\beta-2e^2\beta)^2}{64(e^2-3)^2}$	$\frac{\beta^2}{(2+e)^2}$

Note: $\beta_1 = \frac{4-2e^2}{12-6e-2e^2+e^3}$, $\beta_2 = \frac{4+2e-2e^2-e^3}{10-3e^2}$, $\beta_3 = \frac{20-22e^2+8e^4-e^6}{44-22e-26e^2+13e^3+4e^4-2e^5}$, $\beta_4 = \frac{2-e^2}{4-2e}$, $\beta_5 = \frac{28-30e^2+10e^4-e^6}{40-20e-26e^2+13e^3+4e^4-2e^5}$, $\beta_6 = \frac{2-e^2}{2-e}$, $\beta_7 = \frac{8+4e-2e^2-e^3}{4-2e^2}$, $\beta_8 = \frac{4-2e^2}{2e^2-e^3}$.

For the special case where the potential market size remains unchanged, i.e., $\beta = 1$, the equilibrium outcomes are shown in the second column (C2) of Table 2. If the market size shrinks in period 2, i.e., $0 < \beta \leq \beta_4$, the optimal strategy for the dominant manufacturer is to give up pursuing the market share in period 2 and focus on capturing consumers in period 1. The quantities of products provided by the two manufacturers increase as β increases, as well as their profits. When the market size in period 2 is at an intermediate level, i.e., $\beta_4 < \beta \leq \beta_7$, the dominant manufacturer engages in product sales during both periods. The quantity of products provided by the entrant is less than that provided by the dominant manufacturer, i.e., $q_2^{d*} < Q_1^{d*} + Q_2^{d*}$. As β increases, the quantities of products provided by the two manufacturers get closer.

However, if the market size expands rapidly, i.e., $\beta > \beta_7$, the dominant manufacturer would abandon the first-period market and concentrate on competing with the entrant in period 2. In equilibrium, the two manufacturers would adopt the same strategy to divide the entire market equally. Figures 3 and 4 present the profits of the dominant manufacturer and the entrant, respectively.

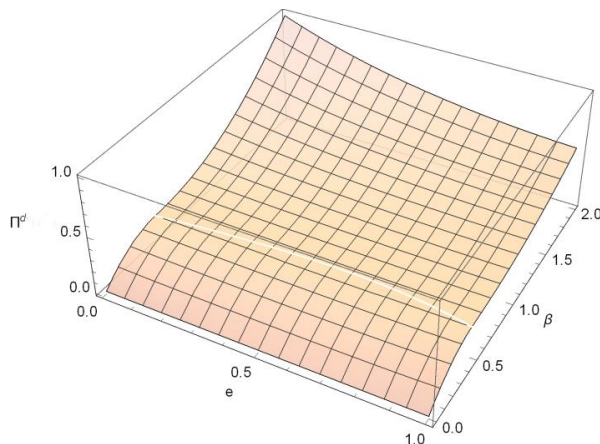


Figure 3. The dominant manufacturer's profit

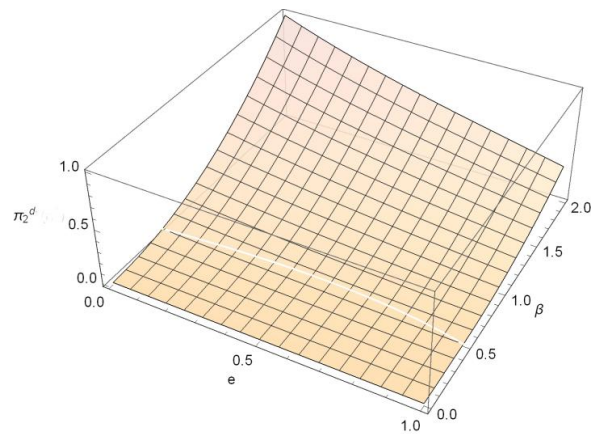


Figure 4. The entrant's profit

As e approaches 1, the two products become increasingly substitutable, and the difference between the dominant manufacturer's and the entrant's second-period quantities gradually decreases. Substituting $e \rightarrow 1^-$ into the equilibrium expressions reported in Table 2 shows that $Q_2^{d*} = q_2^{d*}$ in each market size regime. The reason is that, when the products are perfectly substitutable, product differentiation no longer has an asymmetric effect on the two manufacturers' market-clearing prices. After allowing for the dominant manufacturer's decision in period 1, the equilibrium in period 2 is therefore symmetric. As the manufacturers compete in quantities rather than prices, this limiting case corresponds to Cournot competition with homogeneous products.

4.3. Dual-channel competition

In this subsection, we examine a scenario in which the dominant manufacturer engages in both direct sales to buyers and leasing to renters through a rental platform in period 1, while the entrant enters the market in period 2 by providing substitute products. The structure of the dual-channel competition model is illustrated in Figure 5. In this figure, solid arrows represent direct sales, while dashed arrows indicate rental transactions. The rental platform charges a unit service fee, so the manufacturer receives the rental price less this fee. The notation on each arrow is the relevant transaction price. The market size is normalized to 1 in period 1 and changes to β in period 2.

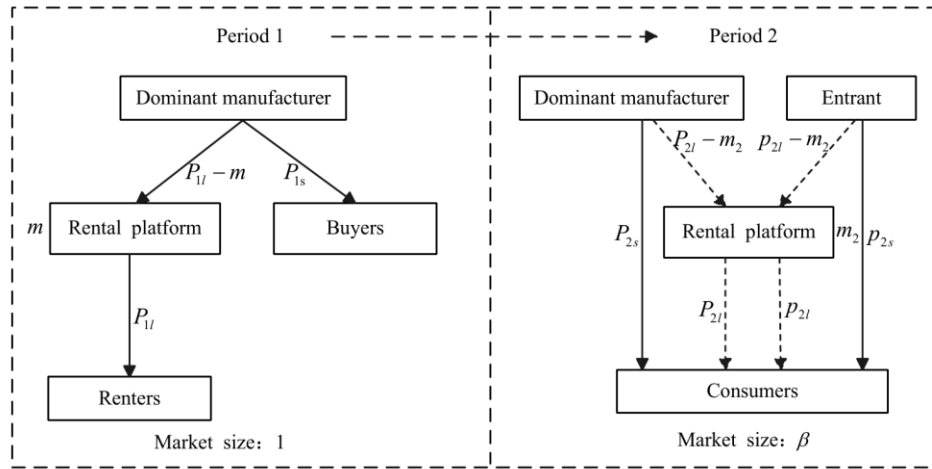


Figure 5. Structure of dual-channel competition

The sequence of decisions is shown in Figure 6. At the beginning of period 1, the rental platform announces the unit service fee m . After observing this fee, the dominant manufacturer determines the quantities sold directly and leased through the platform, denoted by Q_{1s} and Q_{1l} , respectively. In period 2, after the entrant has entered the market, the dominant manufacturer and the entrant simultaneously choose their selling quantities, Q_2 and q_2 , respectively.

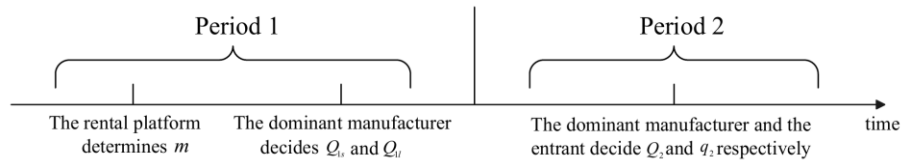


Figure 6. Sequence of events

From Equation (1), the inverse demand functions in the dual-channel competition model are as follows:

$$\begin{aligned}
 P_{1l} &= 1 - Q_{1s} - Q_{1l}, \\
 P_{1s} &= P_{1l} + P_2 = \beta + 1 - 2Q_{1s} - 2Q_{1l} - Q_2 - eq_2, \\
 P_2 &= \beta - Q_{1s} - Q_{1l} - Q_2 - eq_2, \\
 p_2 &= \beta - e(Q_{1s} + Q_{1l} + Q_2) - q_2.
 \end{aligned} \tag{12}$$

4.3.1. Second-period analysis

In period 2, both manufacturers aim to maximize their own profits by simultaneously determining the quantities of products sold to consumers. The optimization problem faced by the dominant manufacturer is as follows:

$$\max_{Q_2} \Pi_2^c = P_2 Q_2 + P_2 Q_{1l}, \quad (13)$$

where $P_2 Q_2$ reflects the profit from selling new products, and $P_2 Q_{1l}$ is the profit from selling products that were leased in period 1. It is worth noting that the price of new and leased products is the same because products are perfectly durable (they do not deteriorate over time) and both provide one period of use.

The entrant's optimization problem can be expressed as follows:

$$\max_{q_2} \pi_2^c = p_2 q_2. \quad (14)$$

Solving both manufacturers' problems simultaneously yields

$$q_2^{c*} = \frac{eQ_{1s} - 2\beta + e\beta}{e^2 - 4}, \quad (15)$$

$$Q_2^{c*} = \frac{2Q_{1s} - 2\beta - e^2 Q_{1s} + e\beta}{e^2 - 4} - Q_{1l}. \quad (16)$$

Two interesting characteristics of the result can be observed: (i) the quantity of products leased by the dominant manufacturer has no impact on the entrant's second-period decision, although an increase in the leasing quantity would result in a reduction of the dominant manufacturer's sales volume in period 2; (ii) an expansion (reduction) in the market size leads to an increase (decrease) in the quantities of products provided by the two manufacturers in period 2, which aligns with our intuition.

4.3.2. First-period analysis

The dominant manufacturer determines the first-period quantities of selling and leasing to maximize his profit, considering the effects on the second-period behaviors of the two manufacturers. In this period, the dominant manufacturer not only sells products directly to consumers but also offers rental services through a rental platform, which improves market segmentation. Therefore, the profit function of the dominant manufacturer is

$$\Pi^c = P_{1l} Q_{1l} - m Q_{1l} + Q_{1s} P_{1s} + \Pi_2^c. \quad (17)$$

Plugging q_2^{c*} , Q_2^{c*} back into Π_2^c yields

$$\Pi_2^c = \frac{((e^2 - 2)Q_{1s} - \beta(e - 2))^2}{(e^2 - 4)^2}. \quad (18)$$

After some algebra, the dominant manufacturer's optimization problem is as follows:

$$\begin{aligned} \max_{Q_{1s}, Q_{1l}} \quad & \Pi^c = \frac{-(e^2 - 4)^2(m + 2Q_{1s} - 1)Q_{1l} - (e^2 - 4)^2 Q_{1l}^2 - (e^4 - 10e^2 + 20)Q_{1s}^2}{(-4 + e^2)^2} \\ \text{s. t.} \quad & \begin{cases} \frac{eQ_{1s} - 2\beta + e\beta}{e^2 - 4} \geq 0 \\ \frac{2Q_{1s} - 2\beta - e^2 Q_{1s} + e\beta}{e^2 - 4} - Q_{1l} \geq 0 \\ Q_{1s} \geq 0 \\ Q_{1l} \geq 0. \end{cases} \end{aligned} \quad (19)$$

The platform seeks to maximize its profit by solving the following problems:

$$\max_m \pi_p^c = mQ_{11}. \quad (20)$$

From Equations (18), (19), and (20), we can derive the following results.

Lemma 3. *In the dual-channel competition model, the equilibrium outcomes are shown in Table 3.*

Table 3. Equilibrium outcomes in the dual-channel competition model

<i>Range</i>	<i>Strategy</i>	<i>Description</i>
$0 < \beta \leq \beta_1$	S6	Pure selling in period 1; only the entrant sells in period 2
$\beta_1 < \beta \leq \beta_3$	S3	Selling and leasing in period 1; only the entrant sells products in period 2
$\beta_3 < \beta \leq \beta_5$	S2	Selling and leasing in period 1; only the entrant sells products in period 2
$\beta_5 < \beta \leq \beta_8$	S1	Selling and leasing in period 1; both manufacturers sell products in period 2
$\beta > \beta_8$	S5	Pure selling in period 1; both manufacturers sell in period 2

The detailed results of strategies S1-S6 are shown in the appendix. Lemma 3 indicates that the dominant manufacturer can respond to changes in the market size in two ways: (i) adjusting the quantity of products provided for selling and leasing in period 1, or (ii) strategically enhancing market penetration or withdrawing from the market in period 2. We introduce several critical cutoffs, namely β_1 , β_3 , β_5 and β_8 , which determine the dominant manufacturer's course of action. Our findings are summarised in the following remark.

Remark 1. (i) When $\beta_1 < \beta \leq \beta_8$, the dominant manufacturer adopts a mixed strategy of selling and leasing in period 1. Otherwise, he adopts a pure selling strategy. (ii) When the market size shrinks sharply, i.e., $0 < \beta \leq \beta_5$, the dominant manufacturer withdraws from the market in period 2 and only concentrates on occupying a larger share of the first-period market.

Lemma 3 reveals that changes in the market size β have non-monotonic effects on the optimal service fee m of the rental platform. Specifically, anticipating that the market size shrinks sharply in period 2, i.e., $0 < \beta \leq \beta_1$, the dominant manufacturer has the incentive to capture as much of the consumer market as possible in period 1. Consequently, in equilibrium, no leasing transactions take place.

In the scenario where the market size experiences a moderate decline in period 2, i.e., $\beta_1 < \beta \leq \beta_3$, the unit service fee m charged by the platform increases along with the market size. This outcome arises from the dominant manufacturer's recognition of the impossibility of achieving substantial profit in period 2, so he places greater emphasis on capturing the market by selling and leasing more products in period 1. This situation presents the platform with an opportunity to raise its unit service fee.

Conversely, if β exceeds the threshold $\beta > \beta_3$, the unit service fee m is non-increasing in β ; that is, if the market size remains steady or expands rapidly, the dominant manufacturer shifts its focus to competing with the entrant in period 2. To attract the dominant manufacturer to lease more products in period 1, the rental platform lowers its charge, which is rather intuitive.

It is worth noting that the conclusion is consistent with that of the monopoly model. The difference lies in that in the monopoly model, the rental platform charges a constant unit service fee when the market size exceeds $\frac{7}{10}$ in period 2. However, in the dual-channel competition model, the dominant manufacturer faces competition from the entrant and does not participate in the rental market if the second-period market size is extremely large, i.e., $\beta > \beta_8$.

Proposition 1. *The quantities of products provided by the two manufacturers increase with the second-period market size, as well as their profits. Meanwhile, the leasing ratio of the dominant manufacturer decreases with the market size if $\beta_5 < \beta \leq \beta_8$, but increases with the market size if $\beta_1 < \beta \leq \beta_5$.*

Proposition 1 suggests that both the dominant manufacturer and the entrant prefer to increase their product supply to attract more purchases or leases rather than resort to price hikes. Our calculations of the supply chain players' profits draw an intuitive conclusion that market expansion benefits the competing manufacturers.

Our results also indicate that if the market size is relatively large in period 2, i.e., $\beta_5 < \beta \leq \beta_8$, the proportion of leasing declines with the market size. One possible explanation for this phenomenon is that the dominant manufacturer could expect significant profit from the second-period market and thus focuses on occupying the market in advance in period 1. Moreover, leasing is less profitable than selling products due to the need to pay the service fee to the platform, making it less attractive when the market size becomes larger.

To illustrate the equilibrium outcomes, we consider three levels of product substitutability, i.e., $e = 0.1, 0.5, 0.9$, and four levels of the market size in period 2, i.e., $\beta = 0.2, 0.5, 1, 1.5$. These values represent low, moderate, and high product substitutability, together with a sharply reduced, moderately reduced, unchanged, or expanded market size in period 2 relative to period 1. Table 4 reports the equilibrium quantities and profits in the dual-channel competition model.

Table 4. Numerical equilibrium outcomes under representative parameter settings

β	e	Strategy	Q_{1s}^{c*}	Q_{1l}^{c*}	Q_2^{c*}	q_2^{c*}	Π^{c*}	π_2^{c*}
0.2	0.1	S6	0.191	0.000	0.000	0.090	0.154	0.008
0.2	0.5	S6	0.171	0.000	0.000	0.057	0.142	0.003
0.2	0.9	S6	0.185	0.000	0.000	0.017	0.151	0.000
0.5	0.1	S2	0.349	0.064	0.000	0.229	0.258	0.053
0.5	0.5	S3	0.371	0.027	0.000	0.151	0.247	0.023
0.5	0.9	S2	0.406	0.021	0.000	0.058	0.251	0.003
1.0	0.1	S1	0.201	0.249	0.127	0.471	0.440	0.222
1.0	0.5	S1	0.232	0.223	0.068	0.369	0.381	0.136
1.0	0.9	S1	0.314	0.156	0.071	0.253	0.363	0.066
1.5	0.1	S1	0.202	0.248	0.365	0.709	0.723	0.503
1.5	0.5	S1	0.248	0.210	0.274	0.567	0.585	0.321
1.5	0.9	S1	0.370	0.110	0.270	0.413	0.529	0.171

Several findings emerge from the numerical results. First, a larger market in period 2 increases the quantities supplied and the profits of both manufacturers. For example, when $e = 0.5$, the dominant manufacturer's total supply, $Q_{1s}^{c*} + Q_{1l}^{c*} + Q_2^{c*}$, increases from 0.171 at $\beta = 0.2$ to 0.732 at $\beta = 1.5$. Over the same interval, the entrant's quantity increases from 0.057 to 0.567. The profits of the dominant manufacturer and the entrant also increase, from 0.142 and 0.003 to 0.585 and 0.321, respectively. Changes in β also lead to distinct channel configurations. When $\beta = 0.2$, the dominant manufacturer adopts pure selling in period 1 and supplies no products in period 2. When $\beta = 0.5$, leasing becomes profitable, but $Q_2^{c*} = 0$. The dominant manufacturer serves the second-period market only by remarketing products leased in period 1. When $\beta = 1$ or 1.5, both Q_{1l}^{c*} and Q_2^{c*} are positive, indicating that the dominant manufacturer combines selling and leasing in period 1 with sales in period 2. Second, higher product substitutability reduces the dominant manufacturer's reliance on leasing. For instance, when $\beta = 1.5$, increasing e from 0.1 to 0.9 raises Q_{1s}^{c*} from 0.202 to 0.370 but reduces Q_{1l}^{c*} from 0.248 to 0.110. Meanwhile, the entrant's quantity decreases from 0.709 to 0.413, and the profits of the dominant manufacturer and the entrant decrease from 0.723 and 0.503 to 0.529 and 0.171, respectively. Thus, market expansion encourages both manufacturers to supply more products, whereas higher substitutability increases competitive pressure and leads the dominant manufacturer to place more emphasis on direct selling.

4.4. Monopoly vs. Duopoly with pure selling vs. Dual-channel competition

In the previous subsections, we have derived the equilibrium outcomes under the three supply chain structures. We now compare these models and examine how the parameters β and e affect the optimal strategies of the two manufacturers.

Proposition 2. *Regardless of how the market size changes, the dominant manufacturer can always earn a higher profit in the dual-channel competition model than in the duopoly model with pure selling.*

Before explaining this proposition, we briefly analyze the dominant manufacturer's profit, which consists of two parts in the dual-channel competition model: (i) leasing products through a rental platform in period 1 and selling them in period 2; (ii) selling products to consumers directly in both periods. Despite the fact that the total profit from leasing a product in period 1 and selling it to consumers in period 2 is lower than that from selling directly to consumers in period 1, the rental channel benefits the dominant manufacturer in two ways. First, the rental platform provides an additional transaction channel, which enables the dominant manufacturer to capture more market share. Second, the dominant manufacturer can flexibly adjust the quantities of products leased and sold in period 1 to maximise his aggregated profit in response to changes in the market size.

Proposition 3. *When the market size in period 2 remains unchanged, i.e., $\beta = 1$, there is a critical threshold e^* , $0 < e^* < 1$, under which the dominant manufacturer can benefit from the entrant's encroachment; that is, the dominant manufacturer's profit in the dual-channel competition model is greater than that in the monopoly model.*

Proposition 3 shows that if the degree of substitutability between the products of the two manufacturers is extremely low, the encroachment of the entrant can increase the dominant manufacturer's profit. The threshold e^* is obtained by comparing the dominant manufacturer's equilibrium profits in the monopoly and dual-channel competition models, which gives $e^* \approx 0.126$ (see Appendix B for details). In the dual-channel competition model, the dominant manufacturer recognizes the entry of a competitor in period 2; in response, he strategically increases the quantity of products leased to consumers because they can provide two periods of use. This proactive approach can help the dominant manufacturer gain a market advantage. Consequently, the total quantity of products provided in period 1 increases compared to the monopoly case. To clarify this point, we show that $Q_{1s}^c + Q_{1l}^c - Q_1^m = \frac{3-5e+4e^2}{60-20e^2} > 0$. However, increased product competition results in a reduction in the market-clearing price and the dominant manufacturer's profit. Figure 7 illustrates the dominant manufacturer's total profit in the three models when the market size remains unchanged in period 2.

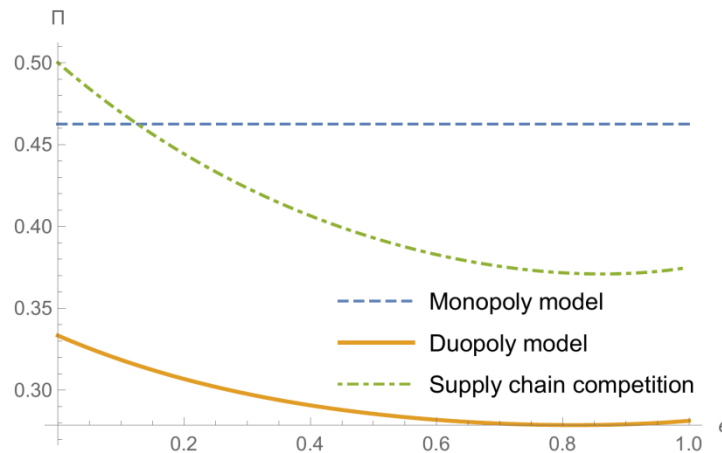


Figure 7. The dominant manufacturer's profit in different models

Proposition 4. *Compared with the duopoly model, the existence of the rental channel encourages the entrant to provide more products when $0 < \beta \leq \beta_7$. However, when $\beta > \beta_7$, the entrant prefers reducing the supply of products.*

Proposition 4 discusses the impacts of the rental channel on the entrant's decision. Our findings indicate that in the dual-channel competition model, if the market size in period 2 is no larger than the threshold, β_7 , the entrant supplies a greater quantity of products than in the duopoly model.

In the dual-channel competition model, the dominant manufacturer seeks to capture the market in period 1 when the market size is small in period 2, i.e., $0 < \beta \leq \beta_7$. To occupy more market share and pursue a higher profit, the entrant would provide more products. However, when the market size exceeds β_7 in period 2, both the dominant manufacturer and the entrant shift their focus towards competing in period 2.

Proposition 5. *In the monopoly model and the dual-channel competition model, the rental platform's profit increases with the market size, respectively, when $0 < \beta \leq 0.7$ and $0 < \beta \leq \frac{14-11e^2+2e^4}{20-10e-6e^2+3e^3}$. Moreover, the rental platform's profit in the monopoly model is no lower than that in the competition model.*

Proposition 5 examines how changes in the market size affect the rental platform's profit. In the monopoly model, the rental platform's profit increases with the market size. This is because if the market size decreases in period 2, the monopolist will capture more market share in period 1 for the reason that the return on each product traded in period 1 is higher than in period 2. Therefore, the monopolist is willing to lease more products, resulting in a higher profit for the rental platform. However, if the market size is greater than 0.7, the monopolist prioritizes capturing the second-period market. The rental platform's charge drops to 0.1, and the quantity of products leased in period 1 is a constant 0.25.

In the dual-channel competition model, our analysis is similar to that in the monopoly model, but with the additional consideration of the dominant manufacturer's pressure from the entrant. When the market size is at a high level in period 2, both the dominant manufacturer and the entrant compete to provide more products to consumers, and the quantity of products leased by the dominant manufacturer in period 1 thus decreases. To encourage the dominant manufacturer to lease more products, the rental platform may reduce the service fee. However, despite the possibility of an increase in the quantity of products leased, it is not able to compensate for the reduction in the service fee. As a result, when the market size remains at a high level, the profit of the platform decreases as the market size increases.

5. EXTENSIONS

In this section, we consider a sequential game in which the two manufacturers make decisions in turn. The two manufacturers may also engage in a Stackelberg game if the dominant manufacturer has established an advantage that enables it to move first. The key difference from the simultaneous decision model is that the dominant manufacturer makes decisions before the entrant in period 2. All variables and parameters are the same as those in the dual-channel competition model in Section 4.3, and we thus do not go into details here. Note that the technical analysis for the sequential decision model is omitted here because it is a repetitive job.

Lemma 4. *The equilibrium outcomes of the sequential games in the duopoly model with pure selling and the dual-channel competition model are shown in Tables 5 and 6.*

Table 5. Equilibrium outcomes in the duopoly model with pure selling

β	$0 < \beta \leq \frac{2-e^2}{4-2e}$	$\frac{2-e^2}{4-2e} < \beta \leq \frac{4}{2-e}$	$\beta > \frac{4}{2-e}$
$\tilde{Q}_1^{d*}$	$\frac{\beta e - 2\beta}{e^2 - 2}$	$\frac{-2\beta + \beta e + 4}{e^2 + 6}$	0
$\tilde{Q}_2^{d*}$	0	$\frac{2(4\beta + e^2 - 2\beta e - 2)}{(2 - e^2)(e^2 + 6)}$	$\frac{\beta e - 2\beta}{2(e^2 - 2)}$
$\tilde{q}_2^{d*}$	$\frac{\beta(1-e)}{2-e^2}$	$\frac{-6\beta + (\beta-1)e^3 + \beta e^2 + 2(\beta+1)e}{(e^2-2)(e^2+6)}$	$\frac{\beta(e^2+2e-4)}{4(e^2-2)}$
$\tilde{\Pi}^{d*}$	$\frac{\beta(e-2)(2\beta+e^2-\beta e-2)}{(e^2-2)^2}$	$\frac{-4(\beta^2-\beta+1) + \beta e^3 - (\beta^2+2\beta-2)e^2 + 2\beta(2\beta-1)e}{(e^2-2)(e^2+6)}$	$\frac{\beta^2(e-2)^2}{8(2-e^2)}$
$\tilde{\pi}_2^{d*}$	$\frac{\beta^2(e-1)^2}{(e^2-2)^2}$	$\frac{(-6\beta + (\beta-1)e^3 + \beta e^2 + 2(\beta+1)e)^2}{(e^2-2)^2(e^2+6)^2}$	$\frac{\beta^2(e^2+2e-4)^2}{16(e^2-2)^2}$

Table 6. Equilibrium outcomes in the dual-channel competition model

β	$0 < \beta \leq \frac{4-2e^2}{e^3-2e^2-6e+12}$	$\frac{4-2e^2}{e^3-2e^2-6e+12} < \beta \leq \frac{-6e^4+32e^2-40}{e^5-2e^4-16e^3+32e^2+44e-88}$
$\tilde{m}^{c*}$	N/A	$\frac{-12\beta - \beta e^3 + 2(\beta-1)e^2 + 6\beta e + 4}{4(e^2-2)}$
$\tilde{Q}_{1s}^{c*}$	$\frac{2\beta - \beta e}{2 - e^2}$	$\frac{\beta(e-2) - 2}{2(e^2-4)}$

β	$0 < \beta \leq \frac{4-2e^2}{e^3-2e^2-6e+12}$	$\frac{4-2e^2}{e^3-2e^2-6e+12} < \beta \leq \frac{-6e^4+32e^2-40}{e^5-2e^4-16e^3+32e^2+44e-88}$
$\bar{Q}_{1l}^{c*}$	N/A	$\frac{12\beta + \beta e^3 - 2(\beta - 1)e^2 - 6\beta e - 4}{4(e^4 - 6e^2 + 8)}$
$\bar{q}_2^{c*}$	0	$\frac{32\beta + \beta e^4 + (6\beta + 2)e^3 - 14\beta e^2 - 4(5\beta + 1)e}{8(e^4 - 6e^2 + 8)}$
$\bar{Q}_2^{c*}$	$\frac{\beta(1-e)}{2-e^2}$	0
$\bar{\Pi}^{c*}$	$\frac{\beta(e-2)(2\beta + e^2 - \beta e - 2)}{(e^2 - 2)^2}$	$\frac{16 + 416\beta - 80e^3(\beta - 2)\beta - 368\beta^2 + e^6\beta^2 - 4e^5\beta(7 + \beta) + 16e\beta(23\beta - 13) - 4e^2(4 + 80\beta + 3\beta^2) + 4e^4(1 + 14\beta + 6\beta^2)}{32(4 - e^2)(-2 + e^2)^2}$
$\bar{\pi}_2^{c*}$	$\frac{\beta^2(e-1)^2}{(e^2 - 2)^2}$	$\frac{(32\beta + \beta e^4 + (6\beta + 2)e^3 - 14\beta e^2 - 4(5\beta + 1)e)^2}{64(e^4 - 6e^2 + 8)^2}$
$\bar{\pi}_p^{c*}$	N/A	$\frac{(-12\beta - \beta e^3 + 2(\beta - 1)e^2 + 6\beta e + 4)^2}{16(4 - e^2)(e^2 - 2)^2}$

Table 6. Equilibrium outcomes in the dual-channel competition model (continued)

β	$\frac{-6e^4+32e^2-40}{e^5-2e^4-16e^3+32e^2+44e-88} < \beta \leq \frac{e^4-16e^2+28}{2e^3-4e^2-20e+40}$	$\frac{e^4-16e^2+28}{2e^3-4e^2-20e+40} < \beta$
$\bar{m}^{c*}$	$\frac{2\beta + e^2 - \beta e - 2}{e^2 - 6}$	$\frac{2 - e^2}{2(10 - e^2)}$
$\bar{Q}_{1s}^{c*}$	$\frac{4(-2\beta - e^2 + \beta e + 2)}{e^4 - 8e^2 + 12}$	$\frac{2}{10 - e^2}$
$\bar{Q}_{1l}^{c*}$	$\frac{20\beta + \beta e^3 - 2(\beta - 2)e^2 - 10\beta e - 8}{2(e^4 - 8e^2 + 12)}$	$\frac{1}{4}$
$\bar{q}_2^{c*}$	$\frac{-12\beta + \beta e^2 + 2(\beta + 2)e}{4(e^2 - 6)}$	$\frac{40\beta + \beta e^4 + 2(\beta + 1)e^3 - 14\beta e^2 - 4(5\beta + 1)e}{4(e^4 - 12e^2 + 20)}$
$\bar{Q}_2^{c*}$	0	$\frac{40\beta - e^4 + 2\beta e^3 - 4(\beta - 4)e^2 - 20\beta e - 28}{4(e^4 - 12e^2 + 20)}$
$\bar{\Pi}^{c*}$	$\frac{(e-2)[16(2\beta - 4 - 7\beta^2) + \beta^2 e^5 - 2\beta^2 e^4 - 2(7\beta^2 - 8)e^3]}{8(e^2 - 6)^2(2 - e^2)} + \frac{(e-2)[4(7\beta^2 - 4\beta + 8)e^2 + 8(7\beta^2 - 4)e]}{8(e^2 - 6)^2(2 - e^2)}$	$\frac{80\beta^2 + (1 - 2\beta^2)e^4 + 8\beta^2 e^3}{16(e^4 - 12e^2 + 20)} + \frac{12(\beta^2 - 3)e^2 - 80\beta^2 e + 68}{16(e^4 - 12e^2 + 20)}$
$\bar{\pi}_2^{c*}$	$\frac{(-12\beta + \beta e^2 + 2(\beta + 2)e)^2}{16(e^2 - 6)^2}$	$\frac{(40\beta + \beta e^4 + 2(\beta + 1)e^3 - 14\beta e^2 - 4(5\beta + 1)e)^2}{16(e^4 - 12e^2 + 20)^2}$
$\bar{\pi}_p^{c*}$	$\frac{(2\beta + e^2 - \beta e - 2)(20\beta + \beta e^3 - 2(\beta - 2)e^2 - 10\beta e - 8)}{2(e^2 - 6)^2(e^2 - 2)}$	$\frac{2 - e^2}{8(10 - e^2)}$

Proposition 6. *In both the duopoly model with pure selling and the dual-channel competition model, sequential decisions increase the dominant manufacturer's total quantity supplied over the two periods and reduce the entrant's quantity relative to simultaneous decisions.*

Proposition 6 shows the commitment effect arising from sequential decisions. When the dominant manufacturer chooses his quantity in period 2 before the entrant, it can influence the entrant's subsequent response. The advantage of moving first allows the dominant manufacturer to supply a larger total quantity over the two periods, leaving less demand for the entrant. The entrant's equilibrium quantity is no greater than that under simultaneous decisions.

This effect is also reflected in the timing of the dominant manufacturer's supply. Relative to the simultaneous setting, the dominant manufacturer generally supplies less in period 1 and more in period 2. The ability to commit to a larger quantity in period 2 reduces the need to secure demand in advance. In the dual-channel competition model, this adjustment involves changes in both the selling and leasing quantities in period 1.

Proposition 7. *When the market size in period 2 is sufficiently low, the simultaneous and sequential games yield the same equilibrium outcomes. Specifically, the equilibrium outcomes coincide when $0 < \beta \leq \frac{2-e^2}{4-2e}$ in the duopoly model with pure selling, and when $0 < \beta \leq \frac{4-2e^2}{e^3-2e^2-6e+12}$ in the dual-channel competition model.*

Proposition 7 shows that, when the market size in period 2 is below the relevant threshold, the market is too small for the dominant manufacturer to supply products in that period. The non-negativity constraint on the period 2 quantity is therefore binding in both decision settings. As the dominant manufacturer supplies no products in period 2, moving before the entrant does not alter the entrant's response. The simultaneous and sequential games consequently yield the same equilibrium quantities and profits for all supply chain players.

6. CONCLUDING REMARKS

The leasing model is increasingly recognized as an environmentally sustainable way to access product usage rights, including manufacturer-to-consumer leasing and consumer-to-consumer sharing. This innovative business model is mainly aimed at durable goods because they can provide multiple use periods. Some well-known companies, such as BMW and GM, have already embraced rental activities. Although the literature on durable products has primarily focused on whether to sell or lease products, the competition between substitutes and the distribution channel does not receive sufficient attention. This paper aims to fill this gap by examining a two-period dual-channel competition model where a dominant manufacturer can sell products directly to consumers or lease them through the online rental platform. We characterize the optimal strategies of all supply chain players in the monopoly model, the duopoly model, and the dual-channel competition model, respectively, and address the impacts of the changing market size and competition between products on both manufacturers' decisions. The essential findings of this paper are as follows:

- In the monopoly and dual-channel competition models, it is never in the best interests of the dominant manufacturer to adopt a pure leasing strategy in period 1. Instead, the dominant manufacturer would either adopt a mixed strategy of leasing and selling or a pure selling strategy. The underlying reason for this is that the pure selling strategy and the mix of leases and sales allow the dominant manufacturer to gain a larger market share and gain a competitive edge. A similar result was found in Desai and Purohit (1999), but in their assumption, two manufacturers enter the market simultaneously.
- The existence of the rental platform is always beneficial to the dominant manufacturer because it provides an additional transaction channel. The dominant manufacturer can adjust the leasing fraction, especially when the market size changes drastically. We find that the dominant manufacturer's aggregate profit from both periods in the dual-channel competition model is always no lower than that in the duopoly model with pure selling, regardless of the changes in the market size.
- If the market size decreases, the dominant manufacturer would rather reduce the leasing fraction in period 1. This is because leasing products through the rental platform is less effective than selling in occupying the second-period market, and unlike selling directly to consumers in period 1, leasing is at a price disadvantage.
- Our analysis shows that the entrant's encroachment may increase the dominant manufacturer's profit when the market size remains unchanged, and the two products are only weak substitutes. Anticipated encroachment encourages the dominant manufacturer to increase leasing in period 1, while the resulting competitive loss remains limited. This is a boundary case that holds only within a narrow range of product substitutability.

The results offer several practical implications for durable goods manufacturers that operate or consider rental services, such as BMW, GM, and IKEA. Selling allows the dominant manufacturer to serve demand and establish its market position earlier, whereas leasing enables it to retain ownership and resell returned products in the following period. The choice between these two modes affects production planning, channel configuration, cooperation with rental platforms, and the allocation of products across periods. When the future market is expected to contract, the dominant manufacturer should place greater emphasis on direct sales in period 1 to secure demand before the entrant increases competitive pressure. When the future market size is relatively stable, combining selling and leasing can serve different types of demand and provide greater flexibility in product allocation. When the future market is expected to expand, the manufacturer should limit leasing in period 1 and retain sufficient supply for direct sales in period 2.

Future research may develop the present model in several directions. First, consumer sharing could be introduced by allowing buyers in period 1 to resell or rent their products in period 2, creating a secondary source of supply that competes with the manufacturer's rental channel. Second, information asymmetry could be examined under a setting in which the manufacturers or the platform observe only an imperfect signal of future market potential before making their decisions.

Third, moral hazard could be considered by linking renters' care effort to product depreciation and the residual value of returned products, which would in turn influence the manufacturer's selling and leasing decisions.

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APPENDIX A. NOTATION

Table A1. Notation and meaning

Notation	Meaning
i	$i \in \{1,2\}$, index of the period
j	$j \in \{s, l\}$, index of the channel mode, where s and l denote selling and leasing, respectively
k	$k \in \{m, d, c\}$, index of the model, denoting the monopoly model, duopoly model with pure selling, and dual-channel competition model, respectively
Q_{ij}/q_{ij}	Quantity supplied by the dominant manufacturer/entrant in period i through mode j
P_{ij}/p_{ij}	price of the dominant manufacturer’s/entrant’s product in period i under mode j
a_i	market potential in period i , where $a_1 = 1$ and $a_2 = \beta$
m	unit service fee charged by the rental platform
β	second-period market size
e	degree of substitutability between the products of the dominant manufacturer and the entrant
Π_i^k	dominant manufacturer’s profit in period i under model k
π_i^k	entrant’s profit in period i under model k
π_p^k	rental platform’s profit under model k

APPENDIX B. MATHEMATICAL PROOFS

Proof of Lemma 1. The proof is similar to the proof of Lemma 3. Therefore, it is omitted.

Proof of Lemma 2. The proof is similar to the proof of Lemma 3. Therefore, it is omitted.

Proof of Lemma 3. For simplicity, we denote all players’ profits under the following six strategies by superscript $S1 - S6$ in the dual-channel competition model. The dominant manufacturer’s profit is given by:

$$\begin{aligned} \max \quad & \Pi^c = P_{1l}Q_{1l} - mQ_{1l} + Q_{1s}P_{1s} + \Pi_2^c \\ \text{s. t.} \quad & \begin{cases} \frac{eQ_{1s}-2\beta+e\beta}{e^2-4} \geq 0 \\ \frac{2Q_{1s}-2\beta-e^2Q_{1s}+e\beta}{e^2-4} - Q_{1l} \geq 0 \\ Q_{1s} \geq 0 \\ Q_{1l} \geq 0. \end{cases} \end{aligned}$$

It is easy to see that Π^c is a joint concave function of Q_{1s} and Q_{1l} , and it has a unique optimal solution. Its Hessian matrix can be easily obtained as follows:

$$A = \begin{bmatrix} -\frac{2(20-10e^2+e^4)}{(e^2-4)^2} - 2 & -2 \\ -2 & -2 \end{bmatrix}.$$

Hence, we conclude that Π^c can achieve its maximum. The K-T optimality conditions are as follows:

$$\begin{aligned} \frac{\partial L}{\partial Q_{1s}} &= \frac{16+e^4+2e^2(\beta-4)-e^3\beta-2(e^2-4)^2Q_{1l}-2(20-10e^2+e^4)Q_{1s}}{(e^2-4)^2} + \\ &\quad \frac{e}{e^2-4}\lambda_1 + \frac{2-e^2}{e^2-4}\lambda_2 + \lambda_3 \\ \frac{\partial L}{\partial Q_{1l}} &= 1 - m - 2Q_{1l} - 2Q_{1s} - \lambda_2 + \lambda_4 \\ \lambda_1 \frac{eQ_{1s}-2\beta+e\beta}{e^2-4} &= 0 \\ \lambda_2 \frac{2Q_{1s}-2\beta+e\beta-e^2Q_{1s}-e^2Q_{1l}+4Q_{1l}}{e^2-4} &= 0 \\ \lambda_3 Q_{1s} &= 0 \\ \lambda_4 Q_{1l} &= 0. \end{aligned}$$

We consider sixteen subcases according to the signs of the Lagrangian multipliers $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. With the constraints on all variables, four feasible sets of solutions satisfying the non-negativity conditions are taken into consideration, denoted by A1 ~ A4.

A1: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$. Solving the system, we have $Q_{1s} = \frac{\beta(2-e)e^2+(4-e^2)^2m}{4(2-e^2)}$, $Q_{1l} = \frac{4-e^4m+\beta e^3+e^2(10m-2\beta-2)-20m}{4(2-e^2)}$, $Q_2 = \frac{e^2(2m-1)+\beta e-2(\beta+3m-1)}{2(e^2-2)}$, $q_2 = \frac{4em-e^3m-4\beta+2e\beta+e^2\beta}{4(e^2-2)}$. In order to satisfy the non-negativity constraints ($Q_{1s} \geq 0, Q_{1l} \geq 0, Q_2 \geq 0, q_2 \geq 0$), we obtain that $\frac{2-e^2-2\beta+e\beta}{2(3-e^2)} \leq m \leq \min\{\frac{4-2e^2+e^3\beta-2e^2\beta}{20-10e^2+e^4}, \frac{4\beta-2e\beta-e^2\beta}{4e-e^3}\}$, which requires $\beta \geq \beta_2$.

A2: $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0, \lambda_4 = 0$. Solving the system, we have $Q_{1s} = \frac{(2-e^2)m-\beta(2-e)+2}{4}$, $Q_{1l} = \frac{(e^2-2)^2m+4+2e^2(\beta-1)-12\beta+6e\beta-e^3\beta}{4(e^2-4)}$, $\lambda_2 = \frac{2(3-e^2)m-2+e^2+2\beta-e\beta}{e^2-4}$, $q_2 = \frac{(2-e^2)em-8\beta+e^2\beta+2e(1+\beta)}{4(e^2-4)}$, $Q_2 = 0$. In order to satisfy the non-negativity constraints, we obtain that $\frac{2\beta-2-e\beta}{2-e^2} \leq m \leq \frac{\beta e^3+(2-2\beta)e^2-6\beta e+12\beta-4}{(2-e^2)^2}$ and $m < \frac{2-e^2-2\beta+e\beta}{6-2e^2}$, which requires $\beta < \frac{8+4e-2e^2-e^3}{8-3e^3}$.

A3: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 > 0$. Solving the system yields $Q_{1l} = 0$, $Q_{1s} = \frac{16+e^4-e^3\beta+2e^2(\beta-4)}{2(20-10e^2+e^4)}$, $\lambda_4 = \frac{(20-10e^2+e^4)m-4-e^3\beta+2e^2(1+\beta)}{20-10e^2+e^4}$. In order to satisfy the non-negativity constraints, we obtain that $m > \frac{\beta e^3-2e^2(1+\beta)+4}{e^4-10e^2+20}$ and $\beta > \frac{4+2e-2e^2-e^3}{10-3e^2}$.

A4: $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0, \lambda_4 > 0$. Solving the system yields $Q_{1l} = 0, Q_{1s} = \frac{(e-2)\beta}{e^2-2}, \lambda_2 = \frac{(2-e)(4-10\beta-e^3-(2-3\beta)e^2+2e)}{(e^2-2)^2}, \lambda_4 = \frac{(2-e^2)^2m+4-12\beta-\beta e^3-2(1-\beta)e^2+6\beta e}{(e^2-2)^2}$. In order to satisfy the non-negativity constraints, we obtain that $\beta < \frac{4+2e-2e^2-e^3}{10-3e^2}$ and $m > \frac{\beta e^3-2(\beta-1)e^2-6\beta e+12\beta-4}{(e^2-2)^2}$.

Now, we consider the problem of the rental platform, whose profit is

$$\max \pi_p = mQ_{1l} = \frac{(20-10e^2+e^4)m^2}{4(e^2-2)} - \frac{(4-2e^2-2e^2\beta+e^3\beta)m}{4(e^2-2)}.$$

Recall that our analysis is restricted to the case $m > 0$; then $m \geq 0$ is not binding. It can be seen that the rental platform's profit function is concave in m . Hence, the FOC is sufficient to characterize the platform's charge.

A1B1: Considering the platform's problem under case A1, we have that the optimal charge for the platform is $m^{c*} = \frac{4+e^3\beta-2e^2(1+\beta)}{2(20-10e^2+e^4)}$, which is derived from $\frac{\partial \pi_p}{\partial m} = 0$. To guarantee m^{c*} satisfies $m^{c*} > \frac{2-e^2-2\beta+e\beta}{2(3-e^2)}$, $m^{c*} \leq \min\{\frac{4-2e^2+e^3\beta-2e^2\beta}{20-10e^2+e^4}, \frac{4\beta-2e\beta-e^2\beta}{4e-e^3}\}$ and $m^{c*} > 0$, we have $\beta_5 \leq \beta < \beta_8$. Denote the equilibrium under case A1B1 as strategy S1; the results by anticipating the platform's response to other variables can be summarised in Table B1.

A1B2: Consistent with A1B1, we derive that the optimal service fee of the rental platform is $m^{c*} = \frac{4+e^3\beta-2e^2(1+\beta)}{2(20-10e^2+e^4)}$. However, if m^{c*} satisfies $m^{c*} \leq \frac{2-e^2-2\beta+e\beta}{2(3-e^2)}$, the platform would charge the service fee $\frac{2-e^2-2\beta+e\beta}{2(3-e^2)}$ to maximize its profit. In order to meet the constraints, β should satisfy the constraint $\beta_2 \leq \beta < \beta_5$. We denote the equilibrium under case A1B2 as strategy S2. The results by replacing m in case A1B2 can be summarised in Table B2.

A1B3: In the same manner, we have that if $m^{c*} > \frac{4\beta-2e\beta-e^2\beta}{4e-e^3}$, then $\beta < \frac{16e-12e^3+2e^5}{160-80e-120e^2+48e^3+24e^4-6e^5-e^6}$ holds, which is in contradiction with the previous constraint $\beta > \beta_2$. As a result, this case should be ruled out. Similarly, we can exclude some other cases in the following discussion using similar criteria.

A2B1: Now, we consider the platform's problem under case A2; we have that the optimal charge for the platform is $m^{c*} = \frac{\beta e^3+2(1-\beta)e^2-6\beta e+12\beta-4}{2(2-e^2)^2}$, which is derived from $\frac{\partial \pi_p}{\partial m} = 0$. If m^{c*} satisfies $\frac{2\beta-2-e\beta}{2-e^2} \leq m^{c*} \leq \frac{\beta e^3+2(1-\beta)e^2-6\beta e+12\beta-4}{(2-e^2)^2}$, $m^{c*} < \frac{2-e^2-2\beta+e\beta}{6-2e^2}$, we have $\beta_1 < \beta < \beta_3$, which guarantees the feasibility of our solution. Denote the equilibrium under case A2B1 as strategy S3 (see Table B3), and replace $m^{c*} = \frac{\beta e^3+2(1-\beta)e^2-6\beta e+12\beta-4}{2(2-e^2)^2}$. Under case A2B1, the following results can be obtained.

Table B1. Results under strategy S1

m^{c*}	$\frac{4+e^3\beta-2e^2(1+\beta)}{2(20-10e^2+e^4)}$
Q_{1s}^{c*}	$\frac{(e-2)(32+16e-12e^3+2e^5+e^4(4-12\beta)+24e^2(\beta-1)+e^6\beta)}{8(e^2-2)(20-10e^2+e^4)}$
Q_{1l}^{c*}	$\frac{4+e^3\beta-2e^2(1+\beta)}{8(2-e^2)}$
q_2^{c*}	$\frac{-160\beta+120e^2\beta-24e^4\beta+e^6\beta-12e^3(1+4\beta)+16e(1+5\beta)+e^5(2+6\beta)}{8(e^2-2)(20-10e^2+e^4)}$
Q_2^{c*}	$\frac{-28+e^6+e^2(30-26\beta)+40\beta-20e\beta+13e^3\beta-2e^5\beta+2e^4(2\beta-5)}{2(2-e^2)(20-10e^2+e^4)}$
Π^{c*}	$\frac{272-4e^5(\beta-3)\beta+320\beta^2-320e\beta^2+8e^3\beta(20\beta-3)+e^6(\beta^2-8)-12e^4(2\beta+3\beta^2-7)-16e^2(17-3\beta+5\beta^2)}{32(2-e^2)(20-10e^2+e^4)}$

m^{c*}	$\frac{4 + e^3\beta - 2e^2(1 + \beta)}{2(20 - 10e^2 + e^4)}$
π_2^{c*}	$\frac{(-160\beta + 120e^2\beta - 24e^4\beta + e^6\beta - 12e^3(1 + 4\beta) + 16e(1 + 5\beta) + e^5(2 + 6\beta))^2}{64(2 - e^2)^2(20 - 10e^2 + e^4)^2}$
π_p^{c*}	$\frac{(4 + e^3\beta - 2e^2(1 + \beta))^2}{16(2 - e^2)(20 - 10e^2 + e^4)}$

Table B2. Results under strategy S2

m^{c*}	$\frac{2 - e^2 - 2\beta + e\beta}{2(3 - e^2)}$
Q_{1s}^{c*}	$\frac{(2 - e)(e^3 + e^2(2 - 3\beta) - 4e - 8(1 - \beta))}{8(e^2 - 3)}$
Q_{1l}^{c*}	$\frac{(2 - e)(e^3 + e^2(2 - 3\beta) - 2e + 10\beta - 4)}{8(3 - e^2)}$
q_2^{c*}	$\frac{e^3 - 3e^2\beta - 2e(2 + \beta) + 12\beta}{8(3 - e^2)}$
Q_2^{c*}	0
Π^{c*}	$\frac{(2 - e)(e^5 + e^4(2 - 6\beta) + e^3(9\beta^2 - 8) - 2e^2(8 - 12\beta + 9\beta^2) - 4e(7\beta^2 - 4) + 8(4 - 2\beta + 7\beta^2))}{32(3 - e^2)^2}$
π_2^{c*}	$\frac{(e^3 - 3e^2\beta - 2e(2 + \beta) + 12\beta)^2}{64(e^2 - 3)^2}$
π_p^{c*}	$\frac{(2 - e)(2 - e^2 - 2\beta + e\beta)(e^3 + e^2(2 - 3\beta) - 2e + 10\beta - 4)}{16(3 - e^2)^2}$

Table B3. Results under strategy S3

m^{c*}	$\frac{e^3\beta + 2e^2(1 - \beta) - 6e\beta + 12\beta - 4}{2(2 - e^2)^2}$
Q_{1s}^{c*}	$\frac{e^3\beta + 2e^2(1 - \beta) + 2e\beta - 4(1 + \beta)}{8(e^2 - 2)}$
Q_{1l}^{c*}	$\frac{4 - e^3\beta - 2e^2(1 - \beta) + 6e\beta - 12\beta}{8(e^2 - 4)}$
q_2^{c*}	$\frac{e^4\beta + e^3(2 + 6\beta) - 14e^2\beta - 4e(1 + 5\beta) + 32\beta}{8(8 - 6e^2 + e^4)}$
Q_2^{c*}	0
Π^{c*}	$\frac{16 + 416\beta - 80e^3(\beta - 2)\beta - 368\beta^2 + e^6\beta^2 - 4e^5\beta(7 + \beta) + 16e\beta(23\beta - 13) - 4e^2(4 + 80\beta + 3\beta^2) + 4e^4(1 + 14\beta + 6\beta^2)}{32(4 - e^2)(2 - e^2)^2}$
π_2^{c*}	$\frac{(32\beta + e^4\beta + e^3(2 + 6\beta) - 14e^2\beta - 4e(1 + 5\beta))^2}{64(8 - 6e^2 + e^4)^2}$
π_p^{c*}	$\frac{(4 - e^3\beta - 2e^2(1 - \beta) + 6e\beta - 12\beta)^2}{16(4 - e^2)(e^2 - 2)^2}$

sB2: Consistent with A2B1, we have that the optimal unit service fee for the platform is $m^{c*} = \frac{\beta e^3 + 2(1 - \beta)e^2 - 6e\beta + 12\beta - 4}{2(2 - e^2)^2}$. However, if m^{c*} satisfies $m^{c*} \geq \frac{2 - e^2 - 2\beta + e\beta}{6 - 2e^2}$, the platform would charge the service fee a little lower than $\frac{2 - e^2 - 2\beta + e\beta}{6 - 2e^2}$ to lead to its maximum profit, we denote it as $m^{c*} = \frac{2 - e^2 - 2\beta + e\beta}{6 - 2e^2} - \xi$, and ξ is positive and infinitely small. Note that the optimal unit service fee might be infinitely close but not equal to $\frac{2 - e^2 - 2\beta + e\beta}{6 - 2e^2}$ because of the discontinuity of the profit function, and this expression for equilibrium has been commonly adopted in the literature (Liu *et al.*, 2021). In order to meet all the constraints, it requires $\beta_3 \leq \beta < \beta_6$. We denote the equilibrium under case A2B2 as strategy S4. The equilibrium outcomes

by replacing $m = \frac{2-e^2-2\beta+e\beta}{2(3-e^2)} - \xi$ under case A2B2 can be summarised in Table B4. $f_1(\xi) \sim f_7(\xi)$ are all positive and are close to but not equal to zero. We solve the remaining cases in a similar manner, and the results under cases A3 and A4 can be summarised in Tables B5-B6.

Table B4. Results under strategy S4

m^{c*}	$\frac{2-e^2-2\beta+e\beta}{2(3-e^2)} - \xi$
Q_{1s}^{c*}	$\frac{(2-e)(e^3+e^2(2-3\beta)-4e-8(1-\beta))}{8(e^2-3)} - f_1(\xi)$
Q_{1l}^{c*}	$\frac{(e-2)(e^3+e^2(2-3\beta)-2e+10\beta-4)}{8(e^2-3)} + f_2(\xi)$
q_2^{c*}	$\frac{e^3-3e^2\beta-2e(2+\beta)+12\beta}{8(3-e^2)} + f_3(\xi)$
Q_2^{c*}	$f_4(\xi)$
Π^{c*}	$\frac{(2-e)(e^5+e^4(2-6\beta)-4e(7\beta^2-4)+e^3(9\beta^2-8)-2e^2(8-12\beta+9\beta^2)+8(4-2\beta+7\beta^2))}{32(e^2-3)^2} + f_5(\xi)$
π_2^{c*}	$\frac{(e^3-3e^2\beta-2e(2+\beta)+12\beta)^2}{64(e^2-3)^2} + f_6(\xi)$
π_p^{c*}	$\frac{(e-2)(e^2-e\beta+2\beta-2)(e^3+e^2(2-3\beta)-2e+10\beta-4)}{16(e^2-3)^2} - f_7(\xi)$

Table B5. Results under strategy S5

m^{c*}	N/A
Q_{1s}^{c*}	$\frac{16+e^4-e^3\beta+2e^2(\beta-4)}{2(20-10e^2+e^4)}$
Q_{1l}^{c*}	N/A
q_2^{c*}	$\frac{20\beta+e^3(1+2\beta)-5e^2\beta-2e(2+5\beta)}{2(20-10e^2+e^4)}$
Q_2^{c*}	$\frac{(2-e)(e^3+e^2(2-3\beta)-2e+10\beta-4)}{2(20-10e^2+e^4)}$
Π^{c*}	$\frac{(e-2)(e^3-2e^2(\beta-1)+e(5\beta^2-4)-2(4+5\beta^2))}{4(20-10e^2+e^4)}$
π_2^{c*}	$\frac{(20\beta+e^3(1+2\beta)-5e^2\beta-2e(2+5\beta))^2}{4(20-10e^2+e^4)^2}$
π_p^{c*}	N/A

Table B6. Results under strategy S6

m^{c*}	N/A
Q_{1s}^{c*}	$\frac{(e-2)\beta}{e^2-2}$
Q_{1l}^{c*}	N/A
q_2^{c*}	$\frac{(e-1)\beta}{e^2-2}$
Q_2^{c*}	0
Π^{c*}	$\frac{(e-2)(e^2-e\beta+2\beta-2)\beta}{(e^2-2)^2}$

m^{c*}	N/A
π_2^{c*}	$\frac{(1-e)^2\beta^2}{(2-e^2)^2}$
π_p^{c*}	N/A

We denote the equilibrium outcomes under cases A3 and A4 as strategies S5 and S6, respectively. Then, we perform the following analysis for the seven intervals.

The rental platform in the dual-channel model is assumed to play the role of the Stackelberg leader who announces a service fee to the dominant manufacturer. Thus, the platform first chooses its strategy with the highest profit, and then the dominant manufacturer decides the quantities of products leased through the rental platform and sold directly to consumers.

When $0 < \beta \leq \beta_1$, only case A4 falls within this interval, corresponding to strategy S6, one can refer to Table B6 for details.

When $\beta_1 < \beta \leq \beta_2$, because the revenue of the rental platform by selecting strategy S3 is positive, while it is zero under strategy S6. We conclude that strategy S3 is the optimal strategy for the platform, and its optimal service fee for each successful transaction is $m^{c*} = \frac{e^3\beta + 2e^2(1-\beta) - 6e\beta + 12\beta - 4}{2(2-e^2)^2}$.

When $\beta_2 < \beta < \beta_3$, strategies S2, S3 and S5 are feasible. The revenue of the rental platform by adopting strategy S5 is zero. We compare the platform's profit by adopting strategies S2 and S3, and easily come to the conclusion that S3 is better.

$$\pi_p^{S3} - \pi_p^{S2} = \frac{(e^6 - 2e^5\beta - 8e^4 + 4e^4\beta + 13e^3\beta + 22e^2 - 26e^2\beta - 22e\beta + 44\beta - 20)^2}{16(4-e^2)(6-5e^2+e^4)^2} > 0.$$

When $\beta_3 \leq \beta < \beta_5$, strategies S2, S4 and S5 are feasible. We find that strategy S4 is better by comparing the rental platform's revenues under the three strategies.

Similarly, when $\beta_5 \leq \beta < \beta_6$ and $\beta_6 \leq \beta < \beta_8$, strategy S1 is optimal for the rental platform. Only strategy S5 is available when $\beta \geq \beta_8$, and the results have been shown in Table B5.

Proof of Remark 1. This proof follows directly from the expressions of Q_{1l}^{c*} and Q_2^{c*} in Lemma 3.

Proof of Proposition 1. We now discuss the decisions of the dominant manufacturer and the entrant under different ranges of β . Taking the case $\beta_5 < \beta \leq \beta_8$ as an example, the total quantities of products sold and leased by the dominant manufacturer and the entrant in both periods are, respectively.

$$Q_{1s}^{c*} + Q_{1l}^{c*} + Q_2^{c*} = \frac{(2-e)(8+40\beta+4e-4e^2-24e^2\beta-2e^3+3e^4\beta)}{4(2-e^2)(20-10e^2+e^4)},$$

$$q_2^{c*} = \frac{-160\beta+16e(1+5\beta)+120e^2\beta-12e^3(1+4\beta)-24e^4\beta+e^5(2+6\beta)+e^6\beta}{8(e^2-2)(20-10e^2+e^4)}.$$

After simplification, we have

$$\frac{\partial(Q_{1s}^{c*}+Q_{1l}^{c*}+Q_2^{c*})}{\partial\beta} = \frac{(2-e)(40-24e^2+3e^4)}{4(2-e^2)(20-10e^2+e^4)^2},$$

$$\frac{\partial q_2^{c*}}{\partial\beta} = \frac{160-80e-120e^2+48e^3+24e^4-6e^5-e^6}{8(2-e^2)(20-10e^2+e^4)}.$$

Clearly, since $e \in (0,1)$, we can show that $\frac{\partial(Q_{1s}^{c*}+Q_{1l}^{c*}+Q_2^{c*})}{\partial\beta} > 0$. It is easy to see that $\frac{\partial(160-80e-120e^2+48e^3+24e^4-6e^5-e^6)}{\partial e} < 0$ and $160-80e-120e^2+48e^3+24e^4-6e^5-e^6 > 0$ achieves its minimum of 25 at $e = 1$. Therefore, $\frac{\partial q_2^{c*}}{\partial\beta} > 0$. Then, we discuss the impacts of changes in the market size on the profits of the dominant manufacturer and the entrant. From $\frac{\partial\pi^{c*}}{\partial\beta} = \frac{(2-e)(12e^2-6e^4+(160-80e-80e^2+40e^3+2e^4-e^5)\beta)}{16(2-e^2)(20-10e^2+e^4)}$, one can easily show that $160-80e-80e^2+40e^3+2e^4-e^5 > 0$ and thus the coefficient of β is positive, i.e., $\frac{(2-e)(160-80e-80e^2+40e^3+2e^4-e^5)}{16(2-e^2)(20-10e^2+e^4)} > 0$. Hence, $\frac{\partial\pi^{c*}}{\partial\beta}$ achieves its minimum at $\beta = \beta_5$ and the corresponding value is positive, implying $\frac{\partial\pi^{c*}}{\partial\beta} > 0$ always holds when $\beta_5 < \beta \leq \beta_8$. $\frac{\partial\pi_2^{c*}}{\partial\beta} =$

$\frac{(80e+120e^2-48e^3-24e^4+6e^5+e^6-160)(16e-12e^3+2e^5+(80e+120e^2-48e^3-24e^4+6e^5+e^6-160)\beta)}{32(e^2-2)^2(20-10e^2+e^4)^2}$ is a monotonically increasing function

of β . Similarly, $\frac{\partial \pi_2^C}{\partial \beta}$ achieves its minimum, which is positive when $\beta = \beta_5$, and we deduce that $\frac{\partial \pi_2^C}{\partial \beta} > 0$ if $\beta_5 < \beta \leq \beta_8$.

Similarly, we can obtain the conclusions when $0 < \beta \leq \beta_1$, $\beta_1 < \beta \leq \beta_3$, $\beta_3 < \beta \leq \beta_5$ and $\beta > \beta_8$.

Now, we consider the proportion of products leased through the rental platform. Let $f = \frac{Q_{1l}^{C*}}{Q_{1s}^{C*} + Q_{1l}^{C*}}$ denote the fraction of products the dominant manufacturer chooses to lease through the rental platform in period 1. One can readily calculate that when $\beta_1 < \beta \leq \beta_3$, $f = \frac{(2-e^2)(e^3\beta+2e^2-2e^2\beta-6e\beta+12\beta-4)}{8+40\beta-20e\beta+6e^3\beta-4e^2(1+3\beta)}$, $\frac{\partial f}{\partial \beta} = \frac{4(2-e)^2(2+e)(2-e^2)^2}{(4+20\beta-10e\beta+3e^3\beta-2e^2-6e^2\beta)^2} > 0$; when $\beta_3 < \beta \leq \beta_5$, $f = \frac{-4-2e+e^3+e^2(2-3\beta)+10\beta}{2(2+e+\beta)}$, $\frac{\partial f}{\partial \beta} = \frac{2(6+3e-2e^2-e^3)}{(2+e+\beta)^2} > 0$; when $\beta_5 < \beta \leq \beta_8$, $f = \frac{(20-10e^2+e^4)(4+e^3\beta-2e^2(1+\beta))}{2(2-e^2)(36+2e^4+2e^2(\beta-9)-e^3\beta)}$, $\frac{\partial f}{\partial \beta} = \frac{e^2(e-2)(20-10e^2+e^4)^2}{(2-e^2)(36+2e^4+2e^2(\beta-9)-e^3\beta)^2} < 0$.

Proof of Proposition 2. In the duopoly model with pure selling, we denote the profit of the dominant manufacturer under strategies C1, C2, C3 as Π^{C1} , Π^{C2} and Π^{C3} . We first compare β_7 and β_8 , both of which depend only on the product substitutability parameter e . Specifically, we examine the sign of $\beta_8 - \beta_7$ over $0 < e < 1$. The difference has a unique zero at $e \approx 0.806$, as further illustrated by the corresponding graphical comparison. Therefore, $\beta_8 > \beta_7$ when $0 < e < 0.806$ and $\beta_8 < \beta_7$ when $0.806 < e < 1$. Moreover, $\beta_3 < \beta_4 < \beta_5 < 1$, whereas both β_7 and β_8 are greater than 1. Based on these threshold relationships, the proof is divided into the following cases.

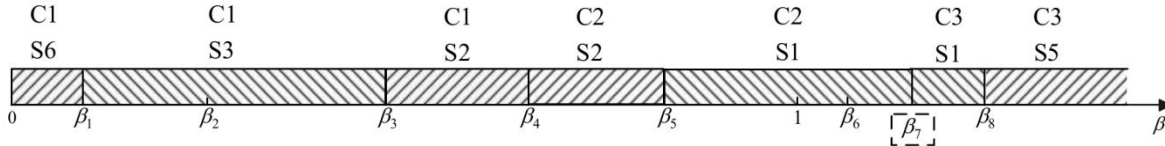


Figure B1. Strategies in the pure-selling duopoly and dual-channel competition models when $0 < e < 0.806$

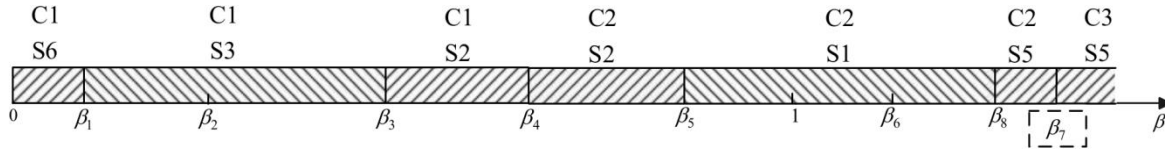


Figure B2. Strategies in the pure-selling duopoly and dual-channel competition models when $0.806 < e < 1$

Case 1: $0 < \beta \leq \beta_1$.

Clearly, the value of $\Pi^{C1} - \Pi^{S6}$ equals zero. Therefore, there is no difference between the duopoly model with pure selling and the dual-channel competition model for the dominant manufacturer.

Case 2: $\beta_1 < \beta \leq \beta_3$.

We can easily see that $\Pi^{S3} - \Pi^{C1} = \frac{3(4+2e^2(\beta-1)-12\beta+6e\beta-e^3\beta)^2}{32(4-e^2)(e^2-2)^2} > 0$. It shows that the dominant manufacturer is better off from the leasing channel when $\beta_1 < \beta \leq \beta_3$.

Case 3: $\beta_3 < \beta \leq \beta_4$.

From $\Pi^{S2} - \Pi^{C1} = \frac{(2+e)(2-e)^3(-4-2e+e^3+e^2(2-3\beta)+10\beta)^2}{32(6-5e^2+e^4)^2} > 0$, we show that the dominant manufacturer's profit in the dual-channel competition model is higher.

Case 4: $\beta_4 < \beta \leq \beta_5$.

$\Pi^{S2} - \Pi^{C2} = \frac{(2-e)^2(2+e)(-4-2e+2e^2+e^3+(8-2e^2)\beta+(2-e)\beta^2)}{32(3-e^2)^2}$. Let $\Gamma = -4 - 2e + 2e^2 + e^3 + (8 - 2e^2)\beta + (2 - e)\beta^2$ and Γ is convex in β because the coefficient of β^2 is positive. The minimum of Γ is reached at $\beta = \beta_4$, which turns out to be $\frac{(-2+e^2)^2}{4(2-e)} > 0$; therefore, $\Pi^{S2} > \Pi^{C2}$, the dominant manufacturer is better off from the leasing channel.

Next, we discuss the case when $0 < e < 0.806$.

case 5a: $\beta_5 < \beta \leq \beta_7$.

$\Pi^{S1} - \Pi^{C2} = \frac{N_1 + N_2\beta + N_3\beta^2}{32(3-e^2)(2-e^2)(20-10e^2+e^4)}$, where $N_1 = -464 + 832e^2 - 580e^4 + 196e^6 - 32e^8 + 2e^{10}$, $N_2 = 1280 - 640e - 1776e^2 + 888e^3 + 904e^4 - 452e^5 - 200e^6 + 100e^7 + 16e^8 - 8e^9$, $N_3 = -320 + 320e + 400e^2 - 480e^3 - 92e^4 + 212e^5 - 25e^6 - 28e^7 + 7e^8$. Obviously, $N_3 = (e-2)^3(2+e)(20-25e^2+7e^4) < 0$ and the numerator of $\Pi^{S1} - \Pi^{C2}$ is a concave function of β , therefore, one can easily show that the minimum is achieved at one of the two boundary points. When $\beta = \beta_5$, the numerator of $\Pi^{S1} - \Pi^{C2}$ is $(2-e^2)^2(20-10e^2+e^4)(344-372e^2+140e^4-21e^6+e^8)$. Clearly, $344-372e^2+140e^4-21e^6+e^8$ is a monotonic non-increasing function, and its minimum occurs when $e = 0.806$ is greater than zero. Thus, the numerator of $\Pi^{S1} - \Pi^{C2}$ is positive. When $\beta = \beta_7$, the numerator of $\Pi^{S1} - \Pi^{C2}$ is $13056 - 25856e^2 + 22144e^4 - 10464e^6 + 2880e^8 - 448e^{10} + 35e^{12} - e^{14}$, which is a monotonic non-increasing function of e , and it is also greater than zero when $e = 0.806$. Consequently, the denominator and numerator of $\Pi^{S1} - \Pi^{C2}$ are both positive, implying that $\Pi^{S1} > \Pi^{C2}$.

case 6a: $\beta_7 < \beta \leq \beta_8$.

$\Pi^{S1} - \Pi^{C3} = \frac{1088 + 1088e - 816e^2 - 1088e^3 + 64e^4 + 336e^5 + 52e^6 - 32e^7 - 8e^8 + (192e^2 + 96e^3 - 144e^4 - 72e^5 + 24e^6 + 12e^7)\beta + (32e^4 - 16e^6 + e^8)\beta^2}{32(2+e)^2(2-e^2)(20-10e^2+e^4)}$, one can easily show that $1088 + 1088e - 816e^2 - 1088e^3 + 64e^4 + 336e^5 + 52e^6 - 32e^7 - 8e^8 > 0$, $192e^2 + 96e^3 - 144e^4 - 72e^5 + 24e^6 + 12e^7 \geq 0$, $32e^4 - 16e^6 + e^8 \geq 0$ and $32(2+e)^2(2-e^2)(20-10e^2+e^4) > 0$. Thus, $\Pi^{S1} - \Pi^{C3} > 0$.

case 7a: $\beta_8 > \beta$.

$$\Pi^{S5} - \Pi^{C3} = \frac{(8+4e-e^3-e^2(2-\beta))^2}{4(2+e)^2(20-10e^2+e^4)} > 0.$$

All proofs for the case $0.806 < e < 1$ are similar and omitted here. In fact, it can be shown that $\Pi^{S1} > \Pi^{C2}$ when $\beta_5 < \beta \leq \beta_8$, $\Pi^{S5} > \Pi^{C2}$ when $\beta_8 < \beta \leq \beta_7$ and $\Pi^{S5} > \Pi^{C3}$ when $\beta_7 < \beta$.

From the above analysis, we conclude that an additional leasing channel is always beneficial to the dominant manufacturer, no matter how the market size changes.

Proof of Proposition 3. When the market size remains unchanged between the two periods, i.e., $\beta = 1$, the profits of the dominant manufacturer in the monopoly model and the dual-channel competition model are, respectively.

$$\Pi^{m*} = \frac{37}{80},$$

$$\Pi^{C*} = \frac{(2-e)(e^5-4e^4+e^3-10e^2-12e+72)}{32(3-e^2)^2}.$$

After simplification, it is easy to see that $\Pi^{C*} - \Pi^{m*} = \frac{H}{160(3-e^2)^2}$, where $H = 54 - 480e + 404e^2 + 60e^3 - 119e^4 + 30e^5 - 5e^6$ and $e \in (0,1)$. Because $160(-3+e^2)^2 > 0$, we only need to confirm the sign of H . Since $\frac{d^2H}{de^2} = 808 + 360e - 1428e^2 + 600e^3 - 150e^4 > 0$, we have $\frac{dH}{de} = -480 + 808e + 180e^2 - 476e^3 + 150e^4 - 30e^5$ is a monotonically increasing function of e , and $\frac{dH}{de} > 0$ if $0.626457 < e < 1$, otherwise, $\frac{dH}{de} < 0$. One can show $H > 0$ at $e = 0$ and $H < 0$ at $e = 0.626457$. Thus, there exists a unique $e^* \in (0, 0.626457)$ such that $H = 0$. Since $H(e)$ is obtained from the difference between the dominant manufacturer's equilibrium profits in the dual-channel competition and monopoly models, numerically solving $H(e) = 0$ yields $e^* \approx 0.126$. We conclude that $\Pi^{C*} > \Pi^{m*}$ if $e < e^*$; otherwise, $\Pi^{C*} < \Pi^{m*}$. The result implies that the emergence of a rival does not always do harm to the dominant manufacturer.

Proof of Proposition 4. This proof follows from the expression of q_2^{d*} in Proposition 2 and q_2^{C*} in Proposition 3.

Proof of Proposition 5. We first examine how the rental platform's revenue changes with respect to β in the monopoly model. If $\frac{1}{3} < \beta \leq \frac{5}{11}$, $\pi_p^{m*} = \frac{(1-3\beta)^2}{16}$ and $\frac{\partial \pi_p^{m*}}{\partial \beta} = \frac{-3(1-3\beta)}{8} > 0$; if $\frac{5}{11} < \beta \leq \frac{7}{10}$, $\pi_p^{m*} = \frac{(1-\beta)(-2+5\beta)}{18}$ and $\frac{\partial \pi_p^{m*}}{\partial \beta} = \frac{7-10\beta}{18} \geq 0$; if $\beta > \frac{7}{10}$, the revenue of the rental platform is always 0.025.

Then, we examine the rental platform's revenue in the dual-channel competition model. If $\beta_5 < \beta \leq \beta_8$, strategy S1 is adopted, where $\pi_p^{C*} = \frac{(4+e^3\beta-2e^2(1+\beta))^2}{16(2-e^2)(20-10e^2+e^4)}$ and $\frac{\partial \pi_p^{C*}}{\partial \beta} = \frac{(-2e^2+e^3)(4-2e^2+(-2e^2+e^3)\beta)}{8(2-e^2)(20-10e^2+e^4)}$. Note that $8(2-e^2)(20-10e^2+e^4) > 0$, $-2e^2+e^3 < 0$, $4-2e^2+(-2e^2+e^3)\beta$ decreases with β and achieves its minimum 0 at $\beta = \beta_8$, thus, $\frac{\partial \pi_p^{C*}}{\partial \beta} \leq 0$.

If $\beta_3 < \beta \leq \beta_5$, strategy S2 is adopted. The platform's profit is $\pi_p^{c*} = \frac{(e-2)(e^2-e\beta+2\beta-2)(e^3+e^2(2-3\beta)-2e+10\beta-4)}{16(e^2-3)^2}$ and $\frac{\partial \pi_p^{c*}}{\partial \beta} = \frac{(2-e)(14+2e^4-20\beta+10e\beta-3e^3\beta-e^2(11-6\beta))}{8(e^2-3)^2}$. It is easy to verify that $\frac{\partial \pi_p^{c*}}{\partial \beta}$ is monotonously decreasing in β and $\frac{\partial \pi_p^{c*}}{\partial \beta} > 0$ when $\beta_3 < \beta \leq \frac{14-11e^2+2e^4}{20-10e-6e^2+3e^3}$, $\frac{\partial \pi_p^{c*}}{\partial \beta} \leq 0$ when $\frac{14-11e^2+2e^4}{20-10e-6e^2+3e^3} < \beta \leq \beta_5$.

If $\beta_1 < \beta \leq \beta_3$, strategy S3 is adopted, where $\pi_p^{c*} = \frac{(4+2e^2(\beta-1)-12\beta+6e\beta-e^3\beta)^2}{16(4-e^2)(e^2-2)^2}$, $\frac{\partial \pi_p^{c*}}{\partial \beta} = \frac{(6e+2e^2-e^3-12)(4-2e^2+(6e+2e^2-e^3-12)\beta)}{8(4-e^2)(e^2-2)^2} > 0$.

Finally, it has been shown that the dominant manufacturer would never lease products if $0 < \beta \leq \beta_1$ and $\beta > \beta_8$.

Now, we compare the platform's revenue in the monopoly model and the dual-channel competition model. The proof follows from the expressions of π_p^{m*} in the monopoly model and π_p^{c*} in the dual-channel competition model.

If $\beta_1 < \beta \leq \frac{5}{11}$, $\Delta\pi_p = \frac{(144e-168e^2-48e^3+80e^4+4e^5-10e^6)\beta^2-(48e-56e^2-32e^3+40e^4+4e^5-6e^6)\beta}{16(4-e^2)(e^2-2)^2} - \frac{4e^2-4e^4+e^6}{16(4-e^2)(e^2-2)^2}$ is a convex function about β . The symmetrical axis of $\Delta\pi_p$ is $\frac{48e-56e^2-32e^3+40e^4+4e^5-6e^6}{2(144e-168e^2-48e^3+80e^4+4e^5-10e^6)}$, which is smaller than β_1 . We can obtain that $\Delta\pi_p$ takes the minimum $\frac{e^2(6-4e-e^2)}{16(e-2)^2(e^2-6)^2} > 0$ at $\beta = \beta_1$. Thus, the rental platform's revenue in the monopoly model is larger than that in the dual-channel competition model.

Similarly, we can easily prove that $\Delta\pi_p$ is always no smaller than 0 when $\frac{5}{11} < \beta \leq \beta_3$, $\beta_3 < \beta \leq \frac{7}{10}$, $\frac{7}{10} < \beta \leq \beta_5$ and $\beta_5 < \beta \leq \beta_8$. The proofs of these cases are similar and thus omitted.

Proof of Proposition 6. The result follows immediately from the equilibrium under the simultaneous and sequential settings. The proof is therefore omitted.

Proof of Proposition 7. The result follows immediately from the equilibrium under the simultaneous and sequential settings. The proof is therefore omitted.

APPENDIX C. DERIVATION OF INVERSE DEMAND FUNCTION EQUATION (1)

To derive the inverse demand function, we adopt a standard quadratic utility specification for differentiated products, which is widely used in analytical models of marketing and supply chain competition. In period i , the representative consumer derives utility from consuming the dominant manufacturer's products and the entrant's products. Let Q_i and q_i denote the quantities supplied by the dominant manufacturer and the entrant, respectively. The consumer's net utility is given by

$$U_i(Q_i, q_i) = a_i(Q_i + q_i) - \frac{1}{2}(Q_i^2 + q_i^2 + 2eQ_iq_i) - P_iQ_i - p_iq_i,$$

where a_i denotes the potential market size in period i , with $a_1 = 1$ and $a_2 = \beta$. The parameter $e \in (0,1)$ captures the degree of substitutability between the two products. A larger e implies stronger substitutability and more intense competition between the two manufacturers.

The first-order condition with respect to Q_i is $\frac{\partial U_i}{\partial Q_i} = a_i - Q_i - eq_i - P_i = 0$, which yields the inverse demand function for the dominant manufacturer's product $P_i(q_i, Q_i) = a_i - Q_i - eq_i$. Similarly, the first-order condition with respect to q_i is $\frac{\partial U_i}{\partial q_i} = a_i - q_i - eQ_i - p_i = 0$, which gives the inverse demand function for the entrant's product $p_i(q_i, Q_i) = a_i - q_i - eQ_i$.