

MIXED-INTEGER LINEAR PROGRAMMING MODEL FOR INTEGRATED LINER SHIPPING ROUTE AND SCHEDULE OPTIMIZATION WITH WEEKLY PORT TIME WINDOWS

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Designing an efficient and cost-effective port network requires accounting for practical constraints, such as port time operation windows. Ignoring these constraints can lead to impractical schedules. This paper presents a method for optimizing liner shipping routes and schedules while accommodating multiple weekly port time windows. A mathematical model is developed to minimize overall costs by optimizing the sequence of port calls, schedules, and sailing speeds. The model is linearized as a mixed-integer program and validated through numerical experiments using commercial optimization solvers. A case study based on SITC Container Lines validates the proposed model and confirms its cost-saving effectiveness, showing total cost reductions of 13.28%, 32.74%, and 16.26% across three port time window scenarios compared with the original port call sequence. Sensitivity analyses further examine the effects of bunker prices, port time windows, and shipping demand volume.

Keywords: Liner Shipping Route Design; Weekly Multiple Port Time Windows; Container Lines; Port Call Sequence; Ship Schedule; Sailing Speed

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1. INTRODUCTION

Maritime transport is the backbone of global trade, facilitating the movement of goods across continents and connecting economies worldwide. UNCTAD (2024) reported that the containerized trade volume was projected to grow by 3.5% in 2024, with an average annual growth rate of 2.7% expected for 2025–2029. The efficiency of maritime transport is heavily dependent on the operational performance of ports, which serve as critical nodes in the global shipping network. Port operations play a pivotal role in determining the overall efficiency of shipping routes.

Port operations face a range of challenges, which include restricted service times, congestion, and draft limitations, all of which could significantly impact the efficiency of the maritime transport system—many ports close on weekends and at night (Christiansen and Fagerholt, 2002). As a result, ships cannot be serviced during these periods, even if they arrive at the port. Moreover, delayed port expansions in some emerging nations contribute to port congestion, limiting their daily service capacity. As a result, ships are not guaranteed to be serviced upon arrival (Wang *et al.*, 2014; Wang *et al.*, 2015). Furthermore, draft limitations at specific ports restrict the cargo that can be loaded onto ships and affect the timing of ships entering the ports (Furman *et al.*, 2011; Song and Furman, 2013). For instance, at Shahid Rajaei Port in Iran, ships with deep drafts must

wait for the tide before entering (Sheikholeslami *et al.*, 2020).

This paper investigates the design of port call sequences, shipping schedules, and sailing speeds for a given set of ports, all under the constraints of weekly multiple port time windows. The goal is to minimize overall costs, including ship operating expenses, bunker consumption, and penalties for berth waiting. This issue is called the liner shipping route and schedule design problem with port time windows (RSDPTW). Under the constraints of port time windows, the RSDPTW problem simultaneously addresses the following three issues:

- (1) **Sequence:** The sequence of port calls plays a crucial role in determining how effectively a shipping route operates (Jiang, Mao, and Zhang, 2020; Wang and Meng, 2013). Given a set of ports, the RSDPTW problem seeks to identify the optimal order of port calls by balancing operating and bunker consumption costs.
- (2) **Schedule:** The RSDPTW problem also requires optimizing the visit schedule for all ports along the route. To maintain consistent weekly service, the arrival times of ships on the route must be synchronized at each port.
- (3) **Speed:** The RSDPTW problem involves selecting the optimal sailing speed for each leg. Additionally, all vessels on the route must maintain uniform speeds to uphold a consistent weekly service frequency.

The three issues mentioned above are interrelated and influenced by port time windows. The port time windows directly affect ship arrival times and sailing speeds on the route, influencing the sequence of port calls. In summary, this study seeks to identify the best port call sequence and optimized shipping schedule and sailing speeds for a service route subject to port time window constraints. The RSDPTW problem aims to offer a support framework for container liner companies, helping them enhance service quality while lowering operational expenses.

We contribute to the literature by addressing a liner shipping route design problem. The model formulation effectively incorporates port time windows to determine the port rotation, the shipping schedule at each port call, the number of vessels deployed, and the sailing speed for each shipping leg. The problem is transformed into a mixed-integer linear programming problem using integer programming techniques, the penalty function method, and outer linear approximation. To evaluate the efficiency and applicability of the proposed model, we conduct a series of numerical experiments using SITC Container Lines as a case study. The results demonstrate the efficacy of our model in finding an optimal shipping route that minimizes the total cost. Furthermore, we conduct sensitivity analyses to examine the impact of variations in bunker price, port time windows, and shipping demand volume. The paper provides managerial insights that could help design port scheduling routed by container companies.

The structure of this paper is as follows: Section 2 reviews the literature. Section 3 introduces the problem and develops the mathematical model. Section 4 provides the linearization process of the model. Section 5 applies the model to an empirical case study. Finally, Section 6 concludes the paper and suggests directions for future research.

2. LITERATURE REVIEW

This study addresses topics within the liner shipping network design problem (LSNDP) and ship speed optimization problem, both of which have been explored in the literature, e.g., Christiansen *et al.* (2013), Meng *et al.* (2014), Psaraftis and Kontovas (2013), Christiansen *et al.* (2020), and Dulebenets *et al.* (2021). Traditionally, the two problems have been studied as separate and independent issues (Koza *et al.*, 2020). This is because the former belongs to a strategic decision problem, and the latter is a tactical-level decision problem (Christiansen *et al.*, 2013; Meng *et al.*, 2014).

2.1. Liner Shipping Network Design

Existing studies either assume a constant speed across all routes or select varying sailing speeds for different routes in the LSNDP. Moreover, fleet deployment is generally incorporated into the shipping network design problem. Shintani *et al.* (2007) developed a two-stage modeling framework for designing a single shipping line, considering the repositioning and leasing of empty containers. Agarwal and Ergun (2008) are the first to include container transshipment and weekly service frequency in the LSNDP involving two or more routes. Each containership type deployed in their model has a different but predetermined sailing speed. Alvarez (2009) jointly determines the optimal routing and fleet deployment while treating ships with varying speeds as different types.

Meng and Wang (2011) explore the LSNDP by integrating multiple port calls, hub-and-spoke systems, and the management of empty container repositioning operations. In their model, the authors select the most efficient routes from a set of potential shipping lines, with the sailing time for each ship type on each leg being predefined. Wang and Meng (2013) examine the effects of altering port rotation sequences, and case studies demonstrate that the rotation can substantially decrease the operational expenses of a liner shipping network. Karsten *et al.* (2017) explore the LSNDP, considering limitations on transit times affecting cargo movement. Their model uses an average sailing speed for each shipping route. Under the assumption of sailing at design speed for vessels, Lee *et al.* (2024) present a novel application of approximate

dynamic programming to address the LSNDP, highlighting its potential for optimizing complex shipping networks by minimizing operating costs and transit times.

Wang *et al.* (2019) design an intercontinental shipping service, treating the sailing speed for each shipping leg as a decision variable during model formulation. Jiang, Mao, and Zhang (2020) consider the constraints of port time windows while optimizing the liner shipping route and sailing speeds. Their focus is limited to a circular service route, allowing each port to be called only once during the voyage, thereby restricting their practical application. Cheng and Wang (2021) consider shippers' inertia and non-inertia preferences for time and cost and optimize shipping routes, ship type, and ship speed under a general route structure. However, neither study incorporates port time windows. Xin *et al.* (2023) introduce an LSNDP with alliance considerations while accounting for shippers' choice inertia and empty container relocation. This provides a novel profit allocation method among alliance members and an efficient solution. In their model, the sailing speed remains constant. Cheaitou *et al.* (2021) incorporate speed optimization for each leg based on speed discretization into the LSNDP. However, they disregard vessel capacity constraints, and their model only applies to circular service routes.

More recent studies have further extended the liner shipping network design problem by incorporating route selection, environmental considerations, and advanced solution methods. Hamdan *et al.* (2025) investigate the Asia–Europe container network under changing market and disruption conditions, comparing the Suez Canal route with the Cape of Good Hope route while considering demand changes, fuel prices, freight rates, and edge-based speed optimization. Xiang *et al.* (2025) develop an Asia–Europe liner shipping network design model considering the Northern Sea Route and black carbon taxes, in which port-of-call selection, calling sequences, liner class, calling frequency, and slot allocation are jointly optimized.

2.2. Speed Optimization

Another research area related to this article is speed optimization, which is often integrated with other research aspects such as berth allocation, schedule design, and fleet deployment. Since even a slight rise in sailing speed leads to a considerable increase in fuel consumption, vessel speeds play a crucial role in determining overall operating costs (Notteboom and Vernimmen, 2009; Ronen, 2011; Tran and Lam, 2022). Sailing speed optimization aims to identify the most efficient speeds for all legs of a given route or shipping network, where port rotations are predetermined inputs to the models (Dulebenets *et al.*, 2021). Additionally, fleet deployment is sometimes incorporated into the model formulation, either explicitly or implicitly. Cheaitou and Cariou (2012) investigate the slow steaming strategy by combining a semi-elastic demand while maximizing total profit. Their findings indicate that this strategy may not always yield optimal results.

Wang and Meng (2012) determine the optimal sailing speed and quantity of deployed ships for a given set of shipping routes. Giovannini and Psaraftis (2019) find that a flexible service frequency could be superior to a weekly service frequency through a model optimizing sailing speed, fleet size, and service frequency. Zhen *et al.* (2019) jointly determine cargo allocation, shipping schedule, ship deployment, and sailing speed for multiple shipping routes considering transshipment. The contribution of Reinhardt *et al.* (2020) lies in considering piracy areas and port work shifts in speed optimization. Zheng *et al.* (2021) examine the issue of tightness regarding the limits on the frequency of weekly service. Du *et al.* (2023) optimize container liner shipping schedules by incorporating shipper selection behavior. Golsefidi *et al.* (2025) optimize containership port sequences and sailing speeds under uncertain weather conditions to reduce fuel consumption and delivery delays. Li *et al.* (2025) study liner ship scheduling and refueling for cold chain shipping, considering sailing speed, port handling, refueling decisions, and cargo deterioration.

Time windows represent a fundamental constraint in the classical Vehicle Routing Problem (VRP) and urban logistics. Recent research has extensively explored time-window management in diverse scenarios, such as the joint scheduling of electric buses to synchronize charging and resting windows (Hu *et al.*, 2025), and the optimization of last-mile fresh-food delivery routes that account for time-dependent traffic and customer service windows (Qi *et al.*, 2025). While these studies underscore the maturity of time-window optimization in land-based transportation, applying such constraints to the maritime domain remains particularly challenging due to the inherent periodicity and global scale of liner services. Aydin *et al.* (2017) incorporate random port times, time windows, and refueling strategies into the speed optimization problem, while Dulebenets (2018) examines heterogeneous fleets and time windows. Neither Aydin *et al.* (2017) nor Dulebenets (2018) account for periodic time windows. Alharbi *et al.* (2015) consider weekly time windows in the schedule design problem; however, their model uses ‘day’ as the unit of time, lacking precision down to the hour, which imposes limitations on practical applications.

2.3. Research Contributions

To make the research gap more explicit, Table 1 compares representative studies that are closely related to the integration of route design, sailing speed optimization, service frequency, ship deployment, and port time windows.

Table 1. Comparison of representative studies related to this study

Reference	Route type	SO	SFC	SDO	PTWC	Solution approach
Meng and Wang (2011)	Candidate routes	–	✓	✓	–	MILP + CPLEX
Alharbi <i>et al.</i> (2015)	Fixed route	✓	✓	✓	✓	ILP + iterative heuristic
Karsten <i>et al.</i> (2017)	Butterfly	–	✓	✓	–	Matheuristic
Aydin <i>et al.</i> (2017)	Fixed route	✓	–	–	✓	Discretized dynamic programming
Dulebenets (2018)	Fixed route	✓	✓	✓	✓	MINLP + BARON
Zhen <i>et al.</i> (2019)	Fixed route	✓	✓	✓	–	MIP + heuristic
Wang <i>et al.</i> (2019)	General	✓	✓	✓	–	MILP + branch-and-cut
Reinhardt <i>et al.</i> (2020)	Fixed route	✓	✓	–	✓	MILP + CPLEX
Jiang, Mao and Zhang (2020)	Simple cyclic	✓	✓	✓	✓	MILP + CPLEX
Cheaitou <i>et al.</i> (2021)	Selective cyclic	✓	✓	✓	–	GA
Du <i>et al.</i> (2023)	Fixed route	✓	✓	✓	✓	PSO + CPLEX
Lee <i>et al.</i> (2024)	General	–	✓	✓	–	approximate dynamic programming, deep Q-network
Xiang <i>et al.</i> (2025)	General	✓	✓	✓	–	GA + linear programming
Hamdan <i>et al.</i> (2025)	Selective cyclic	✓	✓	✓	–	Hybrid heuristic
Our work	General	✓	✓	✓	✓	MILP + CPLEX

Note: “✓” indicates that the feature is considered in the corresponding study, whereas “–” indicates that it is not explicitly considered. SO = speed optimization; SFC = service frequency consideration; SDO = ship deployment optimization; PTWC = port time window consideration.

The above review shows that liner shipping network design and sailing speed optimization have been extensively investigated, but these two streams of research are often treated separately. Table 1 summarizes representative studies related to this study in terms of route type, speed optimization, service frequency consideration, ship deployment optimization, and port time window consideration. Existing studies on liner shipping network design have addressed candidate routes, butterfly routes, selective cyclic routes, and general route structures, and some of them further consider service frequency and ship deployment. However, many of these studies either do not optimize sailing speed or fail to incorporate port time windows. In contrast, studies on schedule design and speed optimization often consider port time windows, but they are usually based on a fixed route or a fixed port rotation. Therefore, the interaction among port call sequence optimization, sailing speed optimization, weekly service frequency, ship deployment, and port time windows has not been sufficiently addressed.

To fill this gap, this study develops an integrated model for the route schedule design problem of a container liner shipping service with port time windows. The proposed model simultaneously optimizes the port call sequence, sailing speed, and number of deployed ships while satisfying weekly service frequency requirements and port time windows. Compared with studies based on fixed routes, the proposed model can adjust the port rotation to reduce berth waiting time and total cost. Compared with general liner shipping network design studies that do not consider port time windows, this study explicitly incorporates port time window constraints into the route schedule design framework. In this way, the proposed model provides a more flexible and practical decision-support tool for liner shipping companies when designing services under berth availability restrictions.

3. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

Consider a container liner company planning to design a shipping route for a given set of ports. While minimizing costs, our model considers the sailing speed, shipping schedule, sequence of port visits, fleet size, and shipping volume. Weekly port time windows for ports are also considered to make the model more realistic. Table 2 lists all the variables and parameters.

Table 2. Notation.

Sets	Definition
I	Set of physical ports
I_0	Set of dummy ports
P	Set of ports, including dummy and physical ports, $P = I \cup I_0$.
A	Set of shipping legs
K	Set of container shipping demands
Ψ_i	Set of port time windows at port i

Parameters	Definition
L_{ij}	Distance from port i to j
$V_{ij}^{\max}$	Maximum sailing speed on shipping leg (i, j)
$V_{ij}^{\min}$	Minimum sailing speed on shipping leg (i, j)
c_{ij}^1, c_{ij}^2	Coefficients for bunker consumption function
γ	Unit bunker price
N	Number of physical ports on the designed shipping route
t_i^{port}	Fixed dwell time a ship spends at port i
$m^{\max}$	Maximum number of ships deployed on the designed shipping route
c_j^{wait}	Fixed waiting cost for the ship to wait for a berth at port j
C^{opr}	Fixed weekly operating cost per week per ship
D_k	Volume of container shipping demand k in twenty-foot equivalent units
o_k, d_k	Origin and destination ports for shipping demand k
Cap	Ship capacity
$M_1 \sim M_8$	A very large positive constant
Variables	Definition
x_{ij}, y_{ij}	Binary variables representing the usage of the leg (i, j) are used in the head-haul and back-haul directions on the designed shipping route, respectively
α_i, β_i	Binary variables equal to 1 if and only if port $i \in I$ is called in the head-haul and back-haul directions, respectively
v_{ij}^h, v_{ij}^b	Sailing speeds on the leg (i, j) in the head-haul and back-haul directions, respectively
u_{ij}^h, u_{ij}^b	Reciprocals of sailing speeds
T_{ij}^h, T_{ij}^b	Sailing times on leg (i, j) in the head-haul and back-haul directions, respectively
t_i^h, t_i^b	Ship arrival times at port i in the head-haul and back-haul directions, respectively
tw_{ij}^h, tw_{ij}^b	Waiting times for the ship to wait for a berth at port j after using leg (i, j) in the head-haul and back-haul directions, respectively
m	Fleet size on the designed shipping route
$\hat{t}_i^h, \hat{t}_i^b$	Transformed ship arrival times within $[0, 168]$ at port i in the head-haul and back-haul directions, respectively
D_k^{oh}, D_k^{ob}	Numbers of containers loaded at the port of origin o_k in the head-haul and back-haul directions, respectively
D_k^{dh}, D_k^{db}	Numbers of containers discharged at the port of destination d_k in the head-haul and back-haul directions, respectively
F_{ijk}, G_{ijk}	Quantities of containers for shipping demand $k \in K$ transported on the shipping leg (i, j) in the head-haul and back-haul directions, respectively
$w_{is}^h, z_{is}^h, w_{is}^b, z_{is}^b$	Auxiliary variables to linearize penalty functions
$\bar{P}_i^h, \bar{P}_i^b, B_{ij}^h, B_{ij}^b, g_{ij}^h, g_{ij}^b$	Auxiliary variables to linearize the objective function

Given a set of ports, we use index i and j to denote the N number of physical ports from 2 to $N + 1$, as well as two dummy ports, 1 and $N + 2$. Let I represent all the physical ports and I_0 represent the two dummy ports. All ports are denoted as $P = \{1, 2, \dots, N + 2\}$. Hence, we can have $P = I \cup I_0$. Let $A = \{(i, j) | i, j \in P\}$ represent all possible shipping legs. The length of any leg connected to a dummy port is set to 0. As the shipping route forms a loop service, we divide the round-trip journey into two directions: the head-haul direction from 1 to $N + 2$ and the back-haul direction from $N + 2$ to 1, as shown in Figure 1.

To aid model development, we start with these assumptions:

- (1) Each shipping leg can be used a maximum of once in either direction.
- (2) Each port is visited at most once per direction and at least once overall.
- (3) Vessels deployed have identical technical attributes.
- (4) The weekly shipping demand for each origin-to-destination (O-D) port pair is fixed, and the time a ship stays at each port is fixed.
- (5) All containers have been converted to twenty-foot equivalent units (TEU).
- (6) Each port has strict port time windows. If a ship arrives outside an available time window, it must wait until the next feasible window opens.

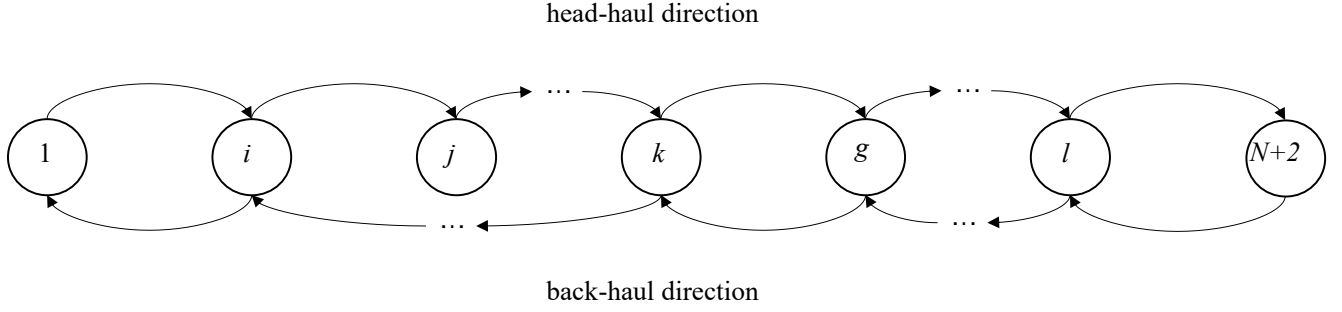


Figure 1. Shipping route structure.

3.1. Port Call Sequence

Let α_i be a binary decision variable that takes the value 1 if port $i \in I$ is visited in the head-haul direction, and 0 otherwise. Similarly, let β_i be a binary decision variable equal to 1 if port $i \in I$ is visited in the back-haul direction, and 0 otherwise. According to assumption 2, we have

$$\alpha_i + \beta_i \geq 1, \forall i \in I \quad (1)$$

The binary variables x_{ij} and y_{ij} represent the usage of leg (i, j) in head-haul and back-haul directions, respectively. If port $i \in I$ is visited in the head-haul direction, it must be connected by two legs in that direction; thus,

$$\sum_{j \in I, j \neq i} x_{ij} = \sum_{j \in I, j \neq i} x_{ji} = \alpha_i, \forall i \in I \quad (2)$$

Likewise, we can obtain a similar constraint in the back-haul direction:

$$\sum_{j \in I, j \neq i} y_{ij} = \sum_{j \in I, j \neq i} y_{ji} = \beta_i, \forall i \in I \quad (3)$$

Each dummy port must link to only one physical port per direction; thus, we have

$$\sum_{i \in I} x_{1,i} = 1; \quad \sum_{i \in I} x_{i,1} = 0 \quad (4)$$

$$\sum_{i \in I} x_{i,(N+2)} = 1; \quad \sum_{i \in I} x_{(N+2),i} = 0 \quad (5)$$

$$\sum_{i \in I} y_{1,i} = 0; \quad \sum_{i \in I} y_{i,1} = 1 \quad (6)$$

$$\sum_{i \in I} y_{i,(N+2)} = 0; \quad \sum_{i \in I} y_{(N+2),i} = 1 \quad (7)$$

Each dummy port must connect to the same physical port in both directions, with no links between the two dummy ports. Then, we have the following constraints

$$x_{1,i} = y_{i,1}, \forall i \in I \quad (8)$$

$$x_{i,(N+2)} = y_{(N+2),i}, \forall i \in I \quad (9)$$

$$x_{1,(N+2)} = x_{(N+2),1} = y_{1,(N+2)} = y_{(N+2),1} = 0 \quad (10)$$

3.2. Sailing Speed and Fuel Cost

Let the variables v_{ij}^h and T_{ij}^h represent the sailing speed and time on the shipping leg (i, j) in the head-haul direction, respectively, and let L_{ij} be the distance from port i to j . We define $V_{ij}^{\max}$ and $V_{ij}^{\min}$ as the maximum and minimum sailing speeds of the homogeneous vessel fleet on the leg (i, j) . By introducing a very large positive constant M_1 , the sailing time on each shipping leg in the head-haul direction is given by:

$$0 \leq T_{ij}^h \leq M_1 x_{ij}, \quad \forall i, j \in P \quad (11)$$

$$\frac{L_{ij}}{v_{ij}^h} - M_1(1 - x_{ij}) \leq T_{ij}^h \leq \frac{L_{ij}}{v_{ij}^h}, \quad \forall i, j \in P \quad (12)$$

$$V_{ij}^{\min} \leq v_{ij}^h \leq V_{ij}^{\max}, \quad \forall i, j \in P \quad (13)$$

where constraints (11) and (12) guarantee that when ports i and j are connected, $T_{ij} = L_{ij}/v_{ij}$, otherwise $T_{ij} = 0$. Constraint (13) sets the sailing speed limits for each shipping leg in the head-haul direction.

Similarly, let the variables v_{ij}^b and T_{ij}^b represent the sailing speed and time on the shipping leg (i, j) in the back-haul direction, respectively, and the constraints for sailing time and speed in the back-haul direction can be derived.

$$0 \leq T_{ij}^b \leq M_1 y_{ij}, \quad \forall i, j \in P \quad (14)$$

$$\frac{L_{ij}}{v_{ij}^b} - M_1(1 - y_{ij}) \leq T_{ij}^b \leq \frac{L_{ij}}{v_{ij}^b}, \quad \forall i, j \in P \quad (15)$$

$$V_{ij}^{\min} \leq v_{ij}^b \leq V_{ij}^{\max}, \quad \forall i, j \in P \quad (16)$$

We define $f_{ij}(v_{ij})$ as the fuel consumption in tons per nautical mile at sailing speed v_{ij} on leg (i, j) . Note that this distance-based fuel consumption function differs from the hourly fuel consumption function. In general, vessel fuel consumption per unit time is often assumed to be proportional to the third power of sailing speed. Since fuel consumption per nautical mile is obtained by dividing hourly fuel consumption by sailing speed, the corresponding distance-based fuel consumption is proportional to the second power of sailing speed. Therefore, following previous studies (Psaraftis and Kontovas, 2013; Wang and Meng, 2012), $f_{ij}(v_{ij})$ is modeled as a power function of the form below:

$$f_{ij}(v_{ij}) = c_{ij}^1 (v_{ij})^{c_{ij}^2}, c_{ij}^1 > 0, c_{ij}^2 \geq 2 \quad (17)$$

where c_{ij}^1 and c_{ij}^2 are coefficients derived from practical data. Specifically, $c_{ij}^2 = 2$ corresponds to the commonly used quadratic distance-based fuel consumption function, while larger values can be used to capture more nonlinear speed–fuel relationships when supported by vessel-specific data. Let γ represent the bunker price (US\$/ton). Hence, the weekly fuel cost for the designed route can be determined by $\gamma \sum_{i \in P} \sum_{j \in P} (x_{ij} L_{ij} c_{ij}^1 (v_{ij}^h)^{c_{ij}^2} + y_{ij} L_{ij} c_{ij}^1 (v_{ij}^b)^{c_{ij}^2})$.

3.3. Liner Shipping Schedule

Without loss of generality, we define 00:00 on a Sunday as the starting point. We denote by t_i^h the ship arrival time at port i in the head-haul direction. According to assumption 6, ships are not allowed to berth outside the time windows and must wait until the time windows open again. Hence, let the variable tw_{ij}^h be the time for the ship to wait for a berth at port j after using the shipping leg (i, j) in the head-haul direction, and let t_i^{port} represent the fixed dwell time a ship spends at port i . Then, we can obtain the following relation in the head-haul direction.

$$t_i^h + t_i^{port} + T_{ij}^h + tw_{ij}^h - t_j^h \leq M_2(1 - x_{ij}), \quad \forall i, j \in P \setminus \{N+2\} \quad (18)$$

$$t_i^h + t_i^{port} + T_{ij}^h + tw_{ij}^h - t_j^h \geq -M_2(1 - x_{ij}), \quad \forall i, j \in P \setminus \{N+2\} \quad (19)$$

where M_2 is a very large positive constant. Constraints (18) and (19) jointly establish the relationship between the ship arrival times in the head-haul direction and eliminate subtours. The two constraints suggest that if the shipping leg (i, j) is used in the head-haul direction, the ship arrival time at port j equals the sum of the arrival time at port i , the fixed dwell time a ship spends at port i , the sailing time on the shipping leg (i, j) , and the waiting time for the ship to wait for a berth at port j after using the shipping leg (i, j) , as shown in Figure 2.

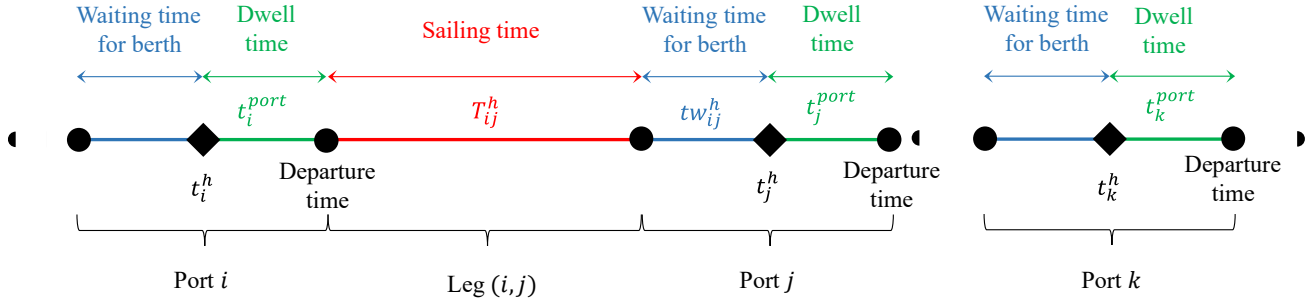


Figure 2. Schematic diagram of the relationship between the arrival times at port i and port j .

Likewise, we denote by t_i^b the ship arrival time at port i in the back-haul direction, and by tw_{ij}^b represent the waiting time for a ship to wait for a berth at port j after using leg (i, j) in the back-haul direction. Hence, we have the following constraints for the back-haul direction.

$$t_i^b + t_i^{port} + T_{ij}^b + tw_{ij}^b - t_j^b \leq M_2(1 - y_{ij}), \quad \forall i, j \in P \setminus \{1\} \quad (20)$$

$$t_i^b + t_i^{port} + T_{ij}^b + tw_{ij}^b - t_j^b \geq -M_2(1 - y_{ij}), \quad \forall i, j \in P \setminus \{1\} \quad (21)$$

The ship arrival time at a dummy port must match the ship arrival time at the connected port $i \in I$. By introducing a very large positive constant M_3 , we have the following relation.

$$t_i^h - t_{N+2}^h \leq M_3(1 - x_{i,(N+2)}), \quad \forall i \in I \quad (22)$$

$$t_i^h - t_{N+2}^h \geq -M_3(1 - x_{i,(N+2)}), \quad \forall i \in I \quad (23)$$

$$t_i^b - t_1^b \leq M_3(1 - y_{i1}), \quad \forall i \in I \quad (24)$$

$$t_i^b - t_1^b \geq -M_3(1 - y_{i1}), \quad \forall i \in I \quad (25)$$

where constraints (22)–(25) can jointly avoid double counting of the fixed dwell time t_i^{port} at port $i \in I$ that is connected to the dummy port. If leg (i, j) is not employed in either direction, the waiting time is 0. By introducing a very large positive constant M_4 , thus, we have

$$0 \leq tw_{ij}^h \leq M_4 x_{ij}, \quad \forall i, j \in P \quad (26)$$

$$0 \leq tw_{ij}^b \leq M_4 y_{ij}, \quad \forall i, j \in P \quad (27)$$

Based on real-world practices in liner shipping, a container liner company must uphold a weekly service schedule; therefore, we can have

$$t_1^b = t_1^h + 168m \quad (28)$$

$$m \in \{1, 2, 3, \dots, m^{\max}\} \quad (29)$$

$$0 \leq t_1^h \leq 168 \quad (30)$$

where 168 represents the total hours in a week, the integer variable m represents the number of ships deployed on the designed shipping route, with $m^{\max}$ being the maximum allowed. Constraint (30) indicates that the time for the first arrival at the dummy port 1 is defined between 0 and 168 hours. Constraints (28)–(30) impose the weekly service schedule at the complete route-cycle level. Specifically, $t_1^b = t_1^h + 168m$ requires the total cycle time of the designed shipping route to be equal to m weeks, so that one ship can depart from the initial port every 168 hours. Different sailing speeds on different shipping legs do not contradict this requirement, because they only affect the sailing time on individual legs and the resulting total cycle time. The model adjusts the sailing speed, waiting time, and port time-window arrangement to ensure that the complete route cycle satisfies the $168m$ -hour requirement.

The arrival time at the dummy port $N + 2$ in the head-haul direction should be equal to that in the back-haul direction; then, we can have

$$t_{N+2}^h = t_{N+2}^b \quad (31)$$

The waiting time between the dummy ports and physical ports should be 0. Hence, we can have

$$tw_{1i}^h = tw_{i,(N+2)}^h = tw_{(N+2),i}^b = tw_{i1}^b = 0, \quad \forall i \in I \quad (32)$$

Let C^{opr} represent the fixed weekly operating cost per ship, measured in US dollars. Thus, the weekly operating cost of the designed route can be calculated by $C^{opr} \cdot m$. We denote by c_j^{wait} (in US dollars per hour) the fixed waiting cost for the ship to wait for a berth at port j . Thus, the weekly waiting cost for the designed route is $\sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b)$.

3.4. Weekly Multiple Port Time Windows

We use $\Psi_i \subseteq [0, 168]$ to represent the time windows available at port i . For example, we assume $\Psi_i = [0, 24] \cup [48, 72] \cup [96, 132]$, which means that port i can provide services for the ships on the designed route at all hours on Sunday and Tuesday, and from 0:00 Thursday to 12:00 Friday. Owing to the weekly service pattern, ships should visit port i at the same time every week. We assume port i is visited in the head-haul direction; thus, the arrival time is feasible only if $(t_i^h \bmod 168) \in \Psi_i$. For instance, a ship can visit port i at the 288th hour, as $(288 \bmod 168 = 120) \in \Psi_i$. We use $\hat{t}_i^h$ to denote the transformed ship arrival time within $[0, 168]$ at port i in the head-haul direction. Then, we can have the following time window constraints for each port call in the head-haul direction.

$$\hat{t}_i^h = t_i^h \bmod 168, \forall i \in I \quad (33)$$

$$\text{if } \alpha_i = 1, \text{ then } \hat{t}_i^h \in \Psi_i, \forall i \in I \quad (34)$$

Constraint (34) indicates that if port i is called in the head-haul direction, the ship's arrival time must fall within its time windows. Similarly, we define $\hat{t}_i^b$ as the transformed ship arrival time within $[0, 168]$ at port i in the back-haul direction, and the time window constraints for each port call in the back-haul direction can be obtained:

$$\hat{t}_i^b = t_i^b \bmod 168, \forall i \in I \quad (35)$$

$$\text{if } \beta_i = 1, \text{ then } \hat{t}_i^b \in \Psi_i, \forall i \in I \quad (36)$$

3.5. Container Flow Balance

Container flow is represented by a series of container shipping demands K , where each demand $k \in K$ is transported from its port of origin o_k to its port of destination d_k . Let D_k be the volume of container shipping demand $k \in K$ in TEUs. Let D_k^{oh} and D_k^{ob} represent the number of containers loaded at the port of origin o_k in the head-haul and back-haul directions, respectively. Let D_k^{ah} and D_k^{ab} be the number of containers discharged at the port of destination d_k in the head-haul and back-haul directions, respectively. Then, we have

$$D_k = D_k^{oh} + D_k^{ob}, \forall k \in K \quad (37)$$

$$D_k^{oh} + D_k^{ob} = D_k^{dh} + D_k^{db}, \forall k \in K \quad (38)$$

$$0 \leq D_k^{oh} \leq M_5 \cdot \alpha_{o_k}, \forall k \in K \quad (39)$$

$$0 \leq D_k^{dh} \leq M_5 \cdot \alpha_{d_k}, \forall k \in K \quad (40)$$

$$0 \leq D_k^{ob} \leq M_5 \cdot \beta_{o_k}, \forall k \in K \quad (41)$$

$$0 \leq D_k^{db} \leq M_5 \cdot \beta_{d_k}, \forall k \in K \quad (42)$$

where M_5 is a very large positive constant. Constraint (37) indicates that for each container shipping demand, the sum of the containers loaded in the head-haul and back-haul directions at the port of origin o_k should be equal to its amount. Constraint (38) means that for each container shipping demand, the total containers loaded at the port of origin o_k in both directions should be equal to those discharged at the port of destination d_k in both directions. Constraints (39)–(42) indicate that if a port is not visited in a certain direction, there will be no loading or unloading activities in that direction.

Let F_{ijk} and G_{ijk} be the quantities of containers for shipping demand $k \in K$ transported on the shipping leg (i, j) in the head-haul and back-haul directions, respectively. Hence, the constraints ensuring container flow balance across all ports are expressed as follows:

$$\sum_j F_{jik} - \sum_j F_{ijk} = \begin{cases} -D_k^{oh}, & \forall i = o_k, \\ 0, & \forall i \neq o_k, d_k, \\ D_k^{dh}, & \forall i = d_k, \end{cases} \quad \forall k \in K \quad (43)$$

$$\sum_j G_{jik} - \sum_j G_{ijk} = \begin{cases} -D_k^{ob}, & \forall i = o_k, \\ 0, & \forall i \neq o_k, d_k, \\ D_k^{db}, & \forall i = d_k, \end{cases} \quad \forall k \in K \quad (44)$$

$$F_{ijk}, G_{ijk} \geq 0, \forall i, j \in P, \forall k \in K \quad (45)$$

$$\sum_k F_{ijk} \leq Cap \cdot x_{ij}, \forall i, j \in P \quad (46)$$

$$\sum_k G_{ijk} \leq Cap \cdot y_{ij}, \forall i, j \in P \quad (47)$$

$$F_{1ik} = G_{i1k}, \forall i \in I, \forall k \in K \quad (48)$$

$$F_{i,(N+2),k} = G_{(N+2),i,k}, \forall i \in I, \forall k \in K \quad (49)$$

where Cap is the ship capacity; constraints (46) and (47) impose the ship capacity constraints. Constraints (48) and (49) are the container flow balance constraints at the two dummy ports.

3.6. Mathematical Model

We now present the mathematical model for the RSDPTW problem, subject to constraints (1)–(49).

$$[M1] \min C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} \left(x_{ij} L_{ij} c_{ij}^1 (v_{ij}^h)^{c_{ij}^2} + y_{ij} L_{ij} c_{ij}^1 (v_{ij}^b)^{c_{ij}^2} \right) + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b) \quad (50)$$

4. MODEL LINEARIZATION

It is easy to see that the objective function of model [M1] is nonlinear, where the fuel consumption is expressed as a power function of speed. Besides, constraints (33)–(36) contain the ‘mod’ and ‘if-then’ operations and the discrete domain set Ψ_i leads to a nonconvex set. Moreover, variables v_{ij}^h and v_{ij}^b are in the denominator in constraints (12) and (15). Therefore, [M1] is classified as a mixed-integer nonlinear programming (MINLP) model. To utilize advanced solvers like CPLEX to address the developed model [M1], we aim to linearize both the objective function and the nonlinear constraints.

4.1. Linearization of Port Time Windows

As the discrete domain set Ψ_i leads to a nonconvex set, we attempt to approximate model [M1] using the penalty function method. The approximation is achieved by incorporating a term into the objective function that imposes a significant penalty for constraint violations. For instance, we assume the time windows at the port i are Monday, Wednesday, and Friday, as illustrated in Figure 3. By incorporating a significantly large positive constant M_6 , a penalty function $p_i^h(\hat{t}_i^h)$ at port i in the head-haul direction can be derived:

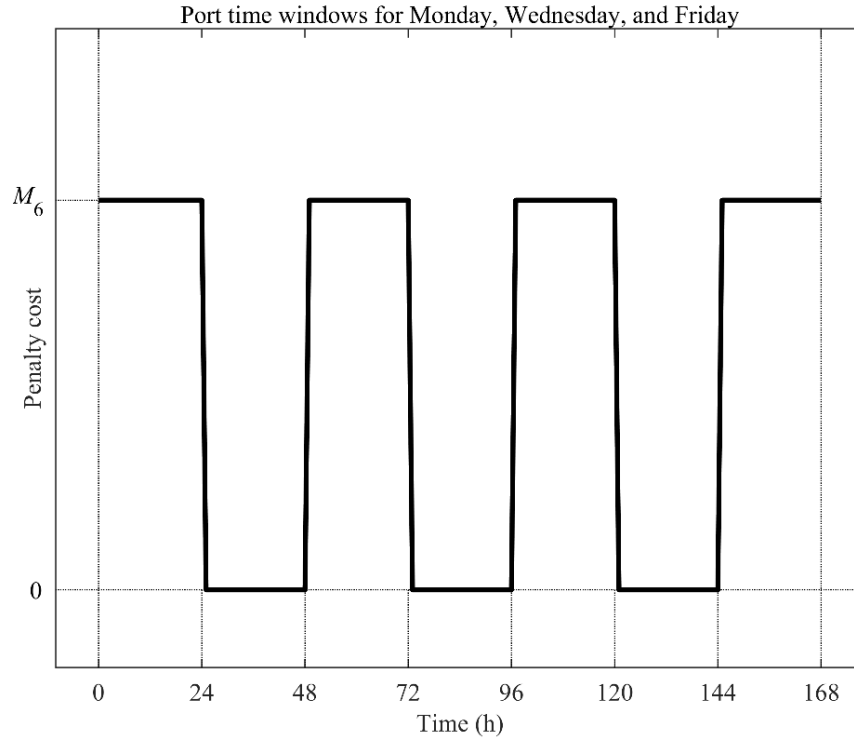


Figure 3. Port time windows for Monday, Wednesday, and Friday.

$$p_i^h(\hat{t}_i^h) = \begin{cases} M_6, & 0 \leq \hat{t}_i^h < 24 \\ 0, & 24 \leq \hat{t}_i^h < 48 \\ M_6, & 48 \leq \hat{t}_i^h < 72 \\ 0, & 72 \leq \hat{t}_i^h < 96 \\ M_6, & 96 \leq \hat{t}_i^h < 120 \\ 0, & 120 \leq \hat{t}_i^h < 144 \\ M_6, & 144 \leq \hat{t}_i^h < 168 \end{cases} \quad (51)$$

Let $p_i^b(\hat{t}_i^b)$ be the penalty function at port i in the back-haul direction. Thus, the objective function and constraints (34) and (36) of model [M1] can be rewritten in the following form.

$$\begin{aligned}
[M1] \min & C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} \left(x_{ij} L_{ij} c_{ij}^1 (v_{ij}^h)^{c_{ij}^2} + y_{ij} L_{ij} c_{ij}^1 (v_{ij}^b)^{c_{ij}^2} \right) \\
& + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b) + \sum_{i \in I} \alpha_i p_i^h(\hat{t}_i^h) + \sum_{i \in I} \beta_i p_i^b(\hat{t}_i^b)
\end{aligned} \tag{52}$$

where the objective function (52) also eliminates the ‘if-then’ operation in the model [M1]. It is easy to see that $p_i^h(\hat{t}_i^h)$ and $p_i^b(\hat{t}_i^b)$ are two piecewise functions that can be linearized.

The penalty function $p_i^h(\hat{t}_i^h)$ has a total of n turning points. Let $\lambda_{is}, i \in I, s \in S_i$ be the turning points. As an example, Figure 3 illustrates 14 turning points corresponding to the following coordinates: $(0, M_6), (24, M_6), (24, 0), (48, 0), (48, M_6), (72, M_6), (72, 0), (96, 0), (96, M_6), (120, M_6), (120, 0), (144, 0), (144, M_6)$, and $(168, M_6)$. Continuous auxiliary variables w_{is}^h and binary auxiliary variables z_{is}^h are introduced to realize a linear transformation, in which continuous variables w_{is}^h are used to linearize the penalty function $p_i^h(\hat{t}_i^h)$, and binary variables z_{is}^h are introduced to constrain the value range of w_{is}^h , thereby restricting the interval of $\hat{t}_i^h$. Hence, the piecewise function $p_i^h(\hat{t}_i^h)$ can be replaced by the following linear form:

$$p_i^h(w_{is}^h) = \sum_{s \in S_i} w_{is}^h p_i^h(\lambda_{is}), \forall i \in I \tag{53}$$

$$\hat{t}_i^h(w_{is}^h) = \sum_{s \in S_i} w_{is}^h \lambda_{is}, \forall i \in I \tag{54}$$

$$w_{i1}^h \leq z_{i1}^h, \forall i \in I \tag{55}$$

$$w_{is}^h \leq z_{i,s-1}^h + z_{is}^h, \forall i \in I, \forall s \in S_i \setminus \{1, n\} \tag{56}$$

$$w_{in}^h \leq z_{i,n-1}^h, \forall i \in I \tag{57}$$

$$\sum_{s \in S_i} w_{is}^h = 1, \forall i \in I \tag{58}$$

$$\sum_{s \in S_i \setminus \{n\}} z_{is}^h = 1, \forall i \in I \tag{59}$$

$$w_{is}^h \geq 0, \forall i \in I, \forall s \in S_i \tag{60}$$

$$z_{is}^h \in \{0, 1\}, \forall i \in I, \forall s \in S_i \setminus \{n\} \tag{61}$$

Likewise, by introducing variables w_{is}^b and z_{is}^b , the piecewise function $p_i^b(\hat{t}_i^b)$ can be substituted with the following linear expression and constraints:

$$p_i^b(w_{is}^b) = \sum_{s \in S_i} w_{is}^b p_i^b(\lambda_{is}), \forall i \in I \tag{62}$$

$$\hat{t}_i^b(w_{is}^b) = \sum_{s \in S_i} w_{is}^b \lambda_{is}, \forall i \in I \tag{63}$$

$$w_{i1}^b \leq z_{i1}^b, \forall i \in I \tag{64}$$

$$w_{is}^b \leq z_{i,s-1}^b + z_{is}^b, \forall i \in I, \forall s \in S_i \setminus \{1, n\} \tag{65}$$

$$w_{in}^b \leq z_{i,n-1}^b, \forall i \in I \tag{66}$$

$$\sum_{s \in S_i} w_{is}^b = 1, \forall i \in I \quad (67)$$

$$\sum_{s \in S_i \setminus \{n\}} z_{is}^b = 1, \forall i \in I \quad (68)$$

$$w_{is}^b \geq 0, \forall i \in I, \forall s \in S_i \quad (69)$$

$$z_{is}^b \in \{0,1\}, \forall i \in I, \forall s \in S_i \setminus \{n\} \quad (70)$$

Thus, the objective function (52) can be rewritten in the following form.

$$\begin{aligned} \min C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} \left(x_{ij} L_{ij} c_{ij}^1 (v_{ij}^h)^{c_{ij}^2} + y_{ij} L_{ij} c_{ij}^1 (v_{ij}^b)^{c_{ij}^2} \right) \\ + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (t w_{ij}^h + t w_{ij}^b) + \sum_{i \in I} \alpha_i P_i^h(w_{is}^h) + \sum_{i \in I} \beta_i P_i^b(w_{is}^b) \end{aligned} \quad (71)$$

To replace the nonlinear combinatorial components in the objective function (71), we introduce two auxiliary variables, $\bar{P}_i^h$ and $\bar{P}_i^b$. Specifically, we define $\bar{P}_i^h$ as $\alpha_i P_i^h(w_{is}^h)$ and $\bar{P}_i^b$ as $\beta_i P_i^b(w_{is}^b)$. Hence, if $\alpha_i = 1$, then $\bar{P}_i^h = P_i^h(w_{is}^h)$; otherwise, $\bar{P}_i^h = 0$. If $\beta_i = 1$, then $\bar{P}_i^b = P_i^b(w_{is}^b)$; otherwise, $\bar{P}_i^b = 0$. By introducing a very large positive constant M_7 , the aforementioned if-then complementary conditions can be equivalently represented by the big-M reformulations of the following constraints.

$$0 \leq \bar{P}_i^h \leq \alpha_i \cdot M_7, \forall i \in I \quad (72)$$

$$P_i^h(w_{is}^h) - M_7(1 - \alpha_i) \leq \bar{P}_i^h \leq P_i^h(w_{is}^h), \forall i \in I \quad (73)$$

$$0 \leq \bar{P}_i^b \leq \beta_i \cdot M_7, \forall i \in I \quad (74)$$

$$P_i^b(w_{is}^b) - M_7(1 - \beta_i) \leq \bar{P}_i^b \leq P_i^b(w_{is}^b), \forall i \in I \quad (75)$$

It should be emphasized that the penalty function method is introduced as a computational treatment for strict port time-window constraints, rather than as a representation of soft time windows or emergency berthing outside the prescribed time windows. In the original RSDPTW problem, ships are not allowed to berth outside the port time windows and must wait until the windows reopen. Therefore, the penalty factors M_6 and M_7 are set sufficiently large so that any solution violating the port time windows is heavily penalized and excluded from the final optimal solution. In this sense, the penalty function method improves computational tractability while maintaining the strict port time window assumption.

By introducing auxiliary integer variables θ_i^h and θ_i^b , constraints (33) and (35) can be equivalently transformed into the following linear form.

$$\hat{t}_i^h = t_i^h - 168\theta_i^h, \forall i \in I \quad (76)$$

$$0 \leq \hat{t}_i^h \leq 168 \cdot \alpha_i, \forall i \in I \quad (77)$$

$$\theta_i^h \in \mathbb{Z}^+, \forall i \in I \quad (78)$$

$$\hat{t}_i^b = t_i^b - 168\theta_i^b, \forall i \in I \quad (79)$$

$$0 \leq \hat{t}_i^b \leq 168 \cdot \beta_i, \forall i \in I \quad (80)$$

$$\theta_i^b \in \mathbb{Z}^+, \forall i \in I \quad (81)$$

Thus, we arrive at an equivalent MINLP model [M2]:

$$[M2] \min C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} \left(x_{ij} L_{ij} c_{ij}^1 (v_{ij}^h)^{c_{ij}^2} + y_{ij} L_{ij} c_{ij}^1 (v_{ij}^b)^{c_{ij}^2} \right) + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b) + \sum_{i \in I} \bar{P}_i^h + \sum_{i \in I} \bar{P}_i^b \quad (82)$$

subject to constraints (1)–(32), (37)–(49), (53)–(70), and (72)–(81).

4.2. Linearization of Bunker Consumption

We aim to transform the power functions in the objective function (82) and the reciprocal of the sailing speed in constraints (12) and (15) into linear forms. To this end, two new variables u_{ij}^h and u_{ij}^b are introduced as the reciprocals of the sailing speeds v_{ij}^h and v_{ij}^b , respectively.

$$u_{ij}^h = \frac{1}{v_{ij}^h}, \quad \forall (i, j) \in A \quad (83)$$

$$u_{ij}^b = \frac{1}{v_{ij}^b}, \quad \forall (i, j) \in A \quad (84)$$

Hence, constraints (12), (13), (15), and (16) can be transformed into the following linear forms:

$$L_{ij} u_{ij}^h - M_1(1 - x_{ij}) \leq T_{ij}^h \leq L_{ij} u_{ij}^h, \quad \forall i, j \in P \quad (85)$$

$$u_{ij}^{\min} = \frac{1}{V_{ij}^{\max}} \leq u_{ij}^h \leq \frac{1}{V_{ij}^{\min}} = u_{ij}^{\max}, \quad \forall i, j \in P \quad (86)$$

$$L_{ij} u_{ij}^b - M_1(1 - y_{ij}) \leq T_{ij}^b \leq L_{ij} u_{ij}^b, \quad \forall i, j \in P \quad (87)$$

$$u_{ij}^{\min} = \frac{1}{V_{ij}^{\max}} \leq u_{ij}^b \leq \frac{1}{V_{ij}^{\min}} = u_{ij}^{\max}, \quad \forall i, j \in P \quad (88)$$

By introducing auxiliary variables B_{ij}^h , B_{ij}^b , g_{ij}^h , and g_{ij}^b , as well as a very large positive constant M_8 , the objective function (82) can be linearized into the following form.

$$\min C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} (B_{ij}^h + B_{ij}^b) + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b) + \sum_{i \in I} \bar{P}_i^h + \sum_{i \in I} \bar{P}_i^b \quad (89)$$

$$0 \leq B_{ij}^h \leq M_8 x_{ij}, \quad \forall i, j \in P \quad (90)$$

$$L_{ij} g_{ij}^h - M_8(1 - x_{ij}) \leq B_{ij}^h \leq L_{ij} g_{ij}^h, \quad \forall i, j \in P \quad (91)$$

$$g_{ij}^h \geq c_{ij}^1 (u_{ij}^h)^{-c_{ij}^2}, \quad \forall i, j \in P \quad (92)$$

$$0 \leq B_{ij}^b \leq M_8 y_{ij}, \quad \forall i, j \in P \quad (93)$$

$$L_{ij} g_{ij}^b - M_8(1 - y_{ij}) \leq B_{ij}^b \leq L_{ij} g_{ij}^b, \quad \forall i, j \in P \quad (94)$$

$$g_{ij}^b \geq c_{ij}^1 (u_{ij}^b)^{-c_{ij}^2}, \quad \forall i, j \in P \quad (95)$$

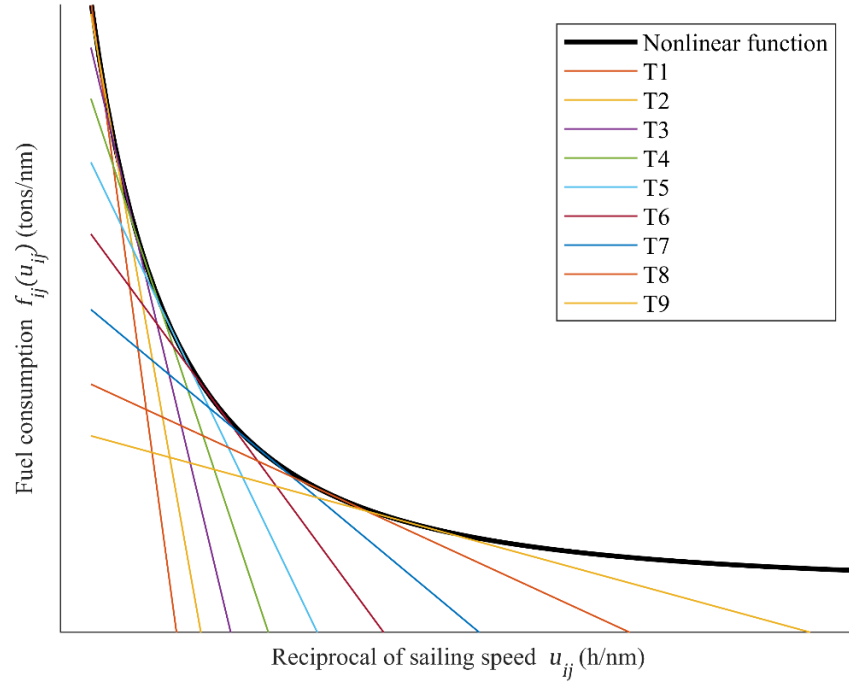


Figure 4. Tangent approximation of the nonlinear fuel consumption function $f_{ij}(u_{ij})$ after substituting $u_{ij} = 1/v_{ij}$. The black curve represents the nonlinear function, and T1–T9 denote the tangent lines generated by the iterative tangent approximation method.

Constraints (92) and (95) remain nonlinear, but they can be made linear using the outer linear approximation method, as shown in Figure 4, which has been extensively utilized in previous research (Jiang, Mao, Wang and Zhang, 2020; Wang and Meng, 2012; Wang *et al.*, 2019). Then, constraints (92) and (95) can be represented as

$$g_{ij}^h \geq c_{ij}^1 (u_{ij}^r)^{-c_{ij}^2} - c_{ij}^1 c_{ij}^2 (u_{ij}^r)^{-c_{ij}^2-1} (u_{ij}^h - u_{ij}^r), \quad \forall r \in R, \forall (i, j) \in A \quad (96)$$

$$g_{ij}^b \geq c_{ij}^1 (u_{ij}^r)^{-c_{ij}^2} - c_{ij}^1 c_{ij}^2 (u_{ij}^r)^{-c_{ij}^2-1} (u_{ij}^b - u_{ij}^r), \quad \forall r \in R, \forall (i, j) \in A \quad (97)$$

where the collection R represents all the linear constraints employed to approximate the nonlinear constraints (92) and (95). Hence, a mixed-integer linear programming (MILP) model [M3] can be obtained:

$$\min C^{opr} m + \gamma \sum_{i \in P} \sum_{j \in P} (B_{ij}^h + B_{ij}^b) + \sum_{i \in P} \sum_{j \in P} c_j^{wait} (tw_{ij}^h + tw_{ij}^b) + \sum_{i \in I} \bar{P}_i^h + \sum_{i \in I} \bar{P}_i^b \quad (98)$$

subject to constraints (1)–(11), (14), (18)–(32), (37)–(49), (53)–(70), (72)–(81), (83)–(88), (90), (91), (93), (94), (96) and (97).

5. CASE STUDY

This section presents numerical experiments conducted on the real shipping route PBT2, operated by SITC Container Lines, to evaluate the applicability of the model developed in this study. First, we assess the validity of the proposed model to examine its effectiveness in reducing the overall cost. Subsequently, we perform a series of sensitivity analyses to derive managerial insights by varying key input parameters. The programming code is executed using the MATLAB toolbox YALMIP and ILOG CPLEX 12.8.0, and operates on a Windows 11 system equipped with an Intel Core i5-11300H 3.1-GHz CPU and 16 GB of RAM.

5.1. Parameter Setting

The PBT2 service has a total of 7 physical ports and 10 ports of call, and its original port call sequence is Shanghai(SHA)→Yokohama(YOK)→Tokyo(TYO)→Nagoya(NGO)→Dalian(DAL)→Xingang(TXG)→Yantai(YNT)→Nagoya(NGO)→Tokyo(TYO)→Yokohama(YOK)→Shanghai(SHA). The parameters are set as follows: the fixed weekly operating cost C^{opr} = US\$200,000/week, the bunker price γ = US\$300/t, the minimum speed $V_{ij}^{\min} = 5$ knots, the maximum speed $V_{ij}^{\max} = 25$ knots, the maximum number of deployed ships $m^{\max} = 8$, the ship $Cap = 2200$ TEU, the fixed waiting cost for a berth at port j $c_j^{wait} = \text{US\$}100/\text{h}$. The bunker consumption function is specified as $f_{ij}(v_{ij}) = 0.001(v_{ij})^2$, corresponding to $c_{ij}^1 = 0.001$ and $c_{ij}^2 = 2$ in Equation (17) for all sailing legs (i, j) . To improve the model's solving efficiency, the tightest upper bounds of M_1, M_2, M_3, M_4, M_5 and M_8 are taken, respectively: $M_1 = (\max L_{ij})/V_{ij}^{\min}$, $M_2 = M_3 = 168m^{\max}$, $M_4 = 168$, $M_5 = \max D_k$, $M_8 = (\max L_{ij})c_{ij}^1(V_{ij}^{\max})^{c_{ij}^2}$. The penalty factors M_6 and M_7 are both set to US\$300,000/week based on preliminary numerical tests. When smaller values, such as 200,000, were used, some solutions violating the port time windows could still be generated. After increasing the penalty factors to 300,000, no port time window violations were observed in the obtained solutions. Therefore, this setting was adopted to effectively enforce the strict port time window assumption in the computational experiments. The fixed dwell time, the port-to-port distance, and the volume of container shipping demands are generated according to actual data, respectively, as shown in Tables 3, 4, and 5. Three port time window scenarios are set up, as shown in Table 6, where 'F' indicates availability on that day, while a blank space signifies unavailability. In each scenario, the time windows for each port are only one day per week.

Table 3. Fixed dwell time at each port (h).

Port	SHA	YOK	TYO	NGO	DAL	TXG	YNT
Dwell time	18	8	11	9	20	18	22

Table 4. Port-to-port distance matrix (nm).

Port	SHA	YOK	TYO	NGO	DAL	TXG	YNT
SHA	0	1036.5	1052.7	925.4	566.8	725.3	536.7
YOK	1036.5	0	21.2	209.4	1170.2	1333.2	1146.4
TYO	1052.7	21.2	0	225.6	1186.4	1349.4	1162.6
NGO	925.4	209.4	225.6	0	1058.1	1221.1	1034.4
DAL	566.8	1170.2	1186.4	1058.1	0	218.7	92.4
TXG	725.3	1333.2	1349.4	1221.1	218.7	0	214
YNT	536.7	1146.4	1162.6	1034.4	92.4	214	0

Table 5. Container shipping demand for each OD port pair (TEU).

Port	SHA	YOK	TYO	NGO	DAL	TXG	YNT
SHA	0	820	780	600	0	0	0
YOK	530	0	0	0	210	310	160
TYO	630	0	0	0	250	310	190
NGO	610	0	0	0	230	250	170
DAL	0	210	250	230	0	0	0
TXG	0	250	230	210	0	0	0
YNT	0	230	270	270	0	0	0

Table 6. Setting of the port time windows scenario.

Scenario	Port	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	SHA			F				
	YOK	F						
	TYO						F	
	NGO							F

Scenario	Port	Sun	Mon	Tue	Wed	Thu	Fri	Sat
2	DAL				F			
	TXG						F	
	YNT	F						
	SHA	F						
	YOK				F			
	TYO					F		
	NGO						F	
	DAL							F
3	TXG				F			
	YNT		F					
	SHA						F	
	YOK		F					
	TYO	F						
	NGO				F			
	DAL	F						
	TXG			F				
	YNT						F	

Note: "F" indicates availability on that day.

5.2. Comparison with the Original Port Rotation

The optimal solution can be found in about 2 hours. In all computational experiments, including the three port time window scenarios and the sensitivity analyses, CPLEX terminated with a final MIP gap of 0.00%, indicating that all reported solutions are optimal under the specified model and parameter settings. The optimal sequence of port calls, ship arrival times, and sailing speeds of the three port time window scenarios are shown in Table 7.

Table 7. Optimal solutions with different port time windows.

Scenario		
1	Port rotation	SHA→TYO→NGO→YOK→DAL→TXG→YNT→TYO→NGO→YOK→SHA
	Arrival time (h)	66.6→144→167.96→188.8→264→302.03→337.68→456→481.57→504→570.6
	Sailing speed(kn)	17.72→17.41→17.68→17.41→12.13→12.13→12.07→15.49→15.59→17.69
2	Port rotation	SHA→YOK→TYO→NGO→YNT→TXG→DAL→YOK→TYO→NGO→SHA
	Arrival time (h)	16.15→96→105.54→132.7→216→264→312→422.4→432→456.4→520.2
	Sailing speed(kn)	16.76→13.77→13.94→13.93→8.23→7.29→12.95→12.97→16.89→16.89
3	Port rotation	SHA→NGO→TYO→YOK→YNT→DAL→TXG→TYO→YOK→NGO→SHA
	Arrival time (h)	144→264→336→360→479.7→510.64→552→696→709.66→744→816
	Sailing speed(kn)	9.07→5→5→10.26→10.34→10.24→10.71→7.98→7.95→14.69

In Scenario 3, the sailing speeds on the NGO–TYO and TYO–YOK legs both reach the lower bound of 5 knots. This is mainly due to the port time windows in this scenario: the available time windows at NGO, TYO, and YOK are Wednesday, Sunday, and Monday, respectively, while the optimized arrival times are 264 h, 336 h, and 360 h. Even at the minimum allowable speed of 5 knots, the vessel still waits 17.88 h before TYO and 8.76 h before YOK. Thus, the low sailing speeds are the cost-minimizing result of model [M3] under the lower speed bound and strict port time windows. In addition, the cycle time in Scenario 3 is 672 h, calculated from the departure time at SHA of 144 h to the return time at SHA of 816 h, which requires 4 ships to maintain a weekly service frequency. This is longer than the typical 3-ship deployment in regular PBT2 operations. The result indicates that, under the strict port time windows specified in Scenario 3, maintaining the original service structure may require additional vessel deployment. In practice, liner companies may further redesign the service network by either adding new ports of call to the service or removing certain existing ports of call and reallocating them to other liner services to obtain a more operationally feasible schedule.

Next, under the three port time window scenarios, model [M3] is applied to the original port call sequence of the PBT2 service, referred to as [M3-fixed]. The remaining parameters are consistent with those specified in Section 5.1, and the results are compared with those obtained from model [M3], as shown in Table 8. We can see that, to meet a weekly service frequency, the number of deployed ships by model [M3] is 3, 3, and 4, respectively, while that by model [M3-fixed] is 4, 6, and 5. In

addition, model [M3] reduces the total cost by 13.28%, 32.74%, and 16.26%, respectively, compared to model [M3-fixed]. The reason is that the model [M3-fixed] does not optimize the port call sequence using the original fixed port rotation. Thus, ships spend much time waiting for a berth, increasing the number of vessels deployed on the route. Model [M3] can determine the best port call sequence for the route within the limitations of port time windows, thereby decreasing the berth waiting time and lowering the overall cost. This indicates that the model developed in this study can generate an optimal shipping service for the designed route to reduce the total cost, further demonstrating the proposed model's applicability and flexibility to real-world problems.

Table 8. Comparison of results between [M3] and [M3-fixed].

Scenario	Model	Total cost ($\times 10^5$ US\$)	Fleet size	Total waiting time for a berth (h)
1	[M3]	10.44	3	0
	[M3-fixed]	12.04	4	182.4
2	[M3]	9.28	3	0
	[M3-fixed]	13.80	6	262.1
3	[M3]	9.84	4	26.64
	[M3-fixed]	11.75	5	150.28

5.3. Effect of bunker prices

First, we investigate the impact of bunker prices on the solution. We vary the bunker price from US\$200 per ton to US\$600 per ton in increments of US\$100, use the port time window scenario 1 from Table 6, and keep the other parameters consistent with those specified in Section 5.1. Table 9 displays the results. As the bunker price increases, the number of vessels deployed rises from 3 to 5, while the average sailing speed declines from 15.47 knots to 7.66 knots. We can also see that the port call sequence changes when the unit bunker price is between US\$400 and US\$600. This indicates that the optimal port call sequence of the designed shipping route changes with the increase in bunker price, which is inconsistent with the results of the circular service route model in Jiang, Mao and Zhang (2020).

Table 9. Optimal solutions with different bunker prices.

Bunker price (US\$/t)	Fleet size	Fuel cost ($\times 10^5$ US\$)	Average sailing speed (kn)	Total cost ($\times 10^5$ US\$)
200	3	2.96	15.47	8.96
	Route: SHA $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
300	3	4.44	15.47	10.44
	Route: SHA $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
400	4	3.40	10.05	11.49
	Route: SHA $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
500	4	4.25	10.05	12.34
	Route: SHA $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
600	5	2.84	7.66	12.93
	Route: SHA $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			

5.4. Effect of port time windows

We discuss how the relaxation of the time window influences the solution. We use Scenario 1 from Table 6 as the benchmark. In Scenario 4, we extend each port's time window by adding available days. Scenario 5 further builds on Scenario 4 by implementing additional changes, as detailed in Table 10. The results presented in Table 11 show that while the number of deployed ships remains unchanged in Scenarios 4 and 5 compared to Scenario 1, both fuel and total costs are reduced. This decrease is due to the relaxation of port time window constraints, which allows for more efficient port call sequences and sailing speeds, thereby lowering costs.

Table 10. Setting of Scenarios 4 and 5.

Port	Scenario	Sun	Mon	Tue	Wed	Thu	Fri	Sat
SHA	1			F *				
	4			F		F		
	5			F		F		F
YOK	1	F						
	4	F		F				
	5	F		F		F		
TYO	1						F	
	4	F					F	
	5	F		F			F	
NGO	1							F
	4		F					F
	5		F		F			F
DAL	1				F			
	4				F		F	
	5		F		F		F	
TXG	1						F	
	4	F					F	
	5	F		F			F	
YNT	1	F						
	4	F		F				
	5	F		F		F		

* 'F' indicates availability on that day.

Table 11. Optimal solutions with different relaxations of the time windows.

Scenario	Fleet size	Fuel cost ($\times 10^5$ US\$)	Average sailing speed (kn)	Total cost ($\times 10^5$ US\$)
1	3	4.44	15.47	10.44
	Route: SHA $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
4	3	2.96	13.76	8.96
	Route: SHA $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ SHA			
5	3	2.93	13.76	8.93
	Route: SHA $\rightarrow$ TYO $\rightarrow$ YOK $\rightarrow$ NGO $\rightarrow$ YNT $\rightarrow$ TXG $\rightarrow$ DAL $\rightarrow$ YOK $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ SHA			

5.5. Effect of container shipping demands

This section examines how variations in container shipping demand impact the solution. With the port time window set to Scenario 1 from Table 6 and using the benchmark container shipping demand values for each OD port pair from Table 5, we analyze demand reductions of 25% and 50%. The optimal solutions are presented in Table 12. The results show that as container shipping demand decreases, the optimal route reduces the number of repeated port calls, thereby lowering the total operating cost. It should be noted that this adjustment does not imply that customer demand at these ports is abandoned. In the benchmark route, some ports, especially the Japanese ports, are called twice within one service cycle. When demand decreases by 50% from the benchmark level, the route is simplified from a pendulum-type structure into a single-loop structure, with each port still being visited once. Therefore, the cargo demands originating from these ports, as well as the demands destined for these ports from other ports, can still be satisfied through the optimized route. Specifically, after the demand decreases, cargo loaded at the three Bohai Bay ports and destined for the Japanese ports can remain on board as the same vessel continues via Shanghai and then proceeds to the Japanese ports in the next service cycle. Therefore, the simplified single-loop route still satisfies the relevant cargo demands rather than abandoning them.

Table 12. Optimal solutions with different container shipping demand.

Shipping demand (%)	Fleet size	Fuel cost ($\times 10^5$ US\$)	Average sailing speed (kn)	Total cost ($\times 10^5$ US\$)
0	3	4.44	15.47	10.44
	Route: SHA $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
-25	3	3.60	14.01	9.61
	Route: SHA $\rightarrow$ TYO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ SHA			
-50	2	2.76	15.77	6.76
	Route: SHA $\rightarrow$ TYO $\rightarrow$ NGO $\rightarrow$ YOK $\rightarrow$ DAL $\rightarrow$ TXG $\rightarrow$ YNT $\rightarrow$ SHA			

6. CONCLUSIONS AND FUTURE WORK

This paper addresses a liner shipping route design problem incorporating port time windows to simultaneously determine the port rotation, the shipping schedule at each port call, the number of vessels deployed, and the sailing speed for each shipping leg, all while adhering to port time window constraints. We formulate this problem as a MINLP model and convert it into a MILP model using integer programming techniques, the penalty function method, and outer linear approximation. To evaluate the efficiency and applicability of the proposed model, we conduct a series of numerical experiments using the PBT2 near-sea route operated by SITC Container Lines as a case study. The results demonstrate that state-of-the-art MILP solvers can effectively solve the proposed model and identify an optimal shipping route that minimizes the total cost. Furthermore, we conduct sensitivity analyses to examine the impact of variations in bunker price, port time windows, and shipping demand volume. Based on these analyses, we summarize the following valuable insights and recommendations:

(1) Port time windows are a key factor influencing the design and scheduling of shipping routes by container shipping companies, directly impacting voyage profitability. When designing shipping routes, container shipping companies should fully consider port time windows as a crucial factor and dynamically adjust the port call sequence and sailing speed when time windows change to enhance voyages' economic efficiency and profitability.

(2) Under the constraints of port time windows, the strategies commonly adopted by container shipping companies—such as deploying more vessels and reducing sailing speeds—remain effective in controlling operational costs when facing rising fuel prices. In addition, adjusting the port call sequence is also a feasible method when dealing with rising fuel prices.

(3) When shipping market demand declines, container shipping companies can reduce repeated port calls and fleet size to lower operational costs while maintaining per-voyage profitability. In this study, such an adjustment does not mean abandoning customer demand at the affected ports. Instead, it reflects a route-structure adjustment in which some ports that were originally called twice in a pendulum-type route may be called once in a single-loop route, while their inbound and outbound cargo demands are still satisfied. Nevertheless, in practical operations, liner companies should carefully balance the cost-saving benefits of reducing repeated port calls against possible impacts on service frequency, customer satisfaction, and service reliability.

This study has several limitations that provide opportunities for future research. The proposed RSDPTW model adopts a deterministic framework in which the waiting time, port dwell time, and shipping demand are treated as known parameters or decision variables. In practice, however, berth waiting times may be affected by port congestion, labor availability, weather conditions, tide, and berth availability, while port dwell time may vary with loading and unloading volumes, quay crane availability, handling productivity, and terminal congestion. In addition, shipping demand may fluctuate due to seasonal patterns, freight market volatility, and supply chain disruptions. These simplifications help maintain the tractability of the proposed model, but may underestimate operational uncertainty and schedule variability in practical liner shipping services. Future research could extend the proposed framework by incorporating stochastic berth waiting times and uncertain berth availability, as suggested in recent studies on stochastic and low-carbon berth allocation and uncertainty-oriented port operations (Ma and Pu, 2025; Dragović *et al.*, 2025; Zhao *et al.*, 2026), and by further considering demand uncertainty, cargo-volume-dependent port dwell times, environmental regulations, carbon-emission constraints, and port disruptions to improve the robustness, sustainability, and resilience of liner shipping route schedule design.

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