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MICRO MOTION ACTUATOR OF COMPLIANT MECHANISM FOR PARAMETER ESTIMATION

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A system model with uncertainty and measurement noise is very difficult to determine. In this paper, a new procedure of parameter estimation would be developed by the principle of maximum likelihood. This modeling procedure is according to an experimental data or a physical system to obtain a hypothetical model, the likelihood function of hypothetical model is determined by Monte Carlo method to adjust these parameter intervals and evaluation; Lastly, the system model of the micro motion actuator would be investigated from the smoothing treatment of likelihood function and parameter estimation of Least-Squares method.

Keywords: Parameter estimation; Least-Squares method; maximum likelihood; Monte Carlo method.

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1. INTRODUCTION

The methods of parameter estimation have been developed and widely spread over various engineering domains for a long time. For instance, design parameters of airplanes and semiconductor device fabrication are determined and adjusted by these estimation techniques. In general, the Least-Squares method is usually applied to estimate the linear system without noise or high signal-noise ratio, but the system with lower signal-noise ratio can not be acquired the satisfied result of the parameter estimation, so, a new research method and a process will be developed.

In the identification task, the modal parameter identification can be classified into two categories by experimental data, the first category is measured data with data processes for parameter identification, and the second category is the direct parameter estimation from measured data. In general, we expect a good parameter estimation that experimental data with noise or influential uncertainty must have the post-treatment that may be smoothing fitting or curve fitting in the process of likelihood function for parameter estimation, there are many relative techniques to do these works and applications, for instance, filter, maximum entropy, polynomial fitting, etc. For the same reason, these techniques are applied to various engineering too, Lee et al. (1993) used cubic fitting to do estimation of curvature from sampled noisy data, while Chen and Rosenfeld (2000) used maximum entropy to develop the smoothing techniques, Luo et al. (2006) remove noise to obtain the smoothed image by coupled anistropic diffusion model. Therefore, the smoothing techniques are very useful for data processes of parameter estimation. In the direct estimation methods of measured data, ARMAX-model and ARX-model are typical methods, but these methods cannot satisfy the more applications and accurate result of parameter estimation, therefore, Musto and Lauderbaugh (1991) combined artificial intelligence and ARMAX models to develop a new parameter estimation method, where artificial intelligence is a heuristic search algorithm that used to search polynomial orders of system model; by ARMAX-model, Ding and Chen (2005) developed two identification algorithms for Hammerstein nonlinear systems, one is an iterative least-squares, the other is a recursive least-squares. Therefore, we can explicitly know that the system with measurement noise and undue assumption would be difficult to obtain the better result from parameter estimation.

In the identification process, the Monte Carlo method is a power method to aid system modeling, this method consists a stochastic process and expected value to resolve the solution of a certain problem (Bauer, 1958; Metropolis and Ulam, 1949); therefore, from a system with modeling error and measurement noise, the parameter estimation can be simulated by Monte Carlo method. In the application of discrete-time system with measurement noise, Pachter and Reynolds (2000) developed a novel generalized minimum variance system for dynamical parameter estimation and used the Monte Carlo method to run the simulation. In an unmanned flight system, a system with combinations of uncertainty can cause influential effects, and Motoda and Miyazawa (2002) use hypothesis test and Monte Carlo simulation to investigate the system model with influential uncertainties.

In this paper, we would use the Monte Carlo method to find a likelihood function that is a smoothing fitting of an experimental data; by the likelihood function, we would investigate the system modeling of a micro motion actuator.

2. GENERAL METHODOLOGY

In the parameter estimation process, we can obtain a set of candidate models from collection of models and find a suitable one with the smallest prediction error; determining the "best" model in the set, but the varied criterion would caused different effects to these "best" parameters, for example, the same mathematical model is solved by Gauss-Newton Method or Least-Squares method, there would be not the same prediction error in the parameter estimation process, also we would test the model that is "good enough" for our purpose, for instance, a micro motion actuator is specific performance that the smallest prediction error is expected less than 1μm by parameter estimation and this work is very difficult from system with lower signal-noise ratio.

In system identification and parameter estimation, deals with the experimental data with noise from observations that could be difficult or unreliable. Suppose, the observations could be described as realization of stochastic variables, the observed value is inserted by a numerical value of a deterministic function that many estimator function are possible, this function is called the likelihood function (Ljung, 1987). Therefore, we would use random jumping method to establish a parameter estimation process of Monte Carlo method and to find likelihood function, this process is that the upper and lower bounds can be adjusted to a suitable range and the mathematical model can be modified by the various conditions (Rao, 1996); the purpose of the estimation process is to find a likelihood function that maximizes the probability of the observed event for experimental data. This procedure contains experimental data, hypothesis model, established parameter interval, numerical simulation and prediction error, and parameter estimation. The flow chart of the parameter estimation is as shown in Figure 1.

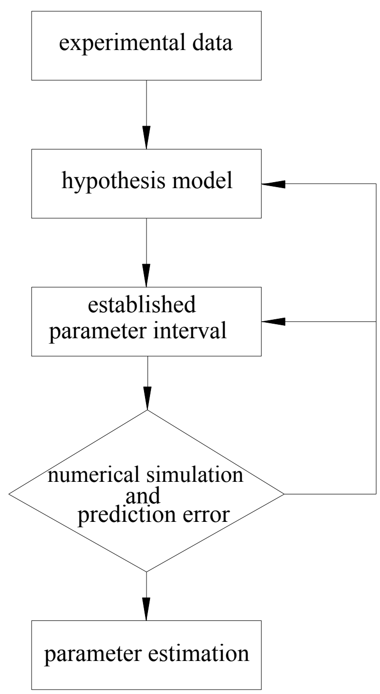

Figure 1. Modeling procedure of parameter estimation.

Experimental data. In general, there is measurement noise in the transient experiment data; therefore, the system parameters cannot be well estimated from these experimental data, directly. In order to obtain the better result, the experimental data must be done the smoothing treatment of the likelihood function.

Hypothesis model. According to experimental data properties or physical phenomena leading to mathematical model that functions to provide transient response data; while the simulation result and experimental data cannot coincide on the design specification, a mathematical model would be repeatedly establishment in the new assuming conditions. In general, contour plots of data properties can provide better information to decide the mathematical model that may be a linear system, a nonlinear system (Motoda and Miyazawa, 2002) or the other mathematical model. Physical phenomena would state the desire to transfer function, state-space representation or describing function in linear system or nonlinear system.

Established parameter interval. A mathematical model is a likelihood function with stochastic variables, these parameters of the mathematical model would be determined by random jumping method, because random jumping method can be used quite conveniently to find the "best" parameter estimation, the random jumping method that belongs to the Monte Carlo method is established the upper and lower bounds for parameter estimations, by generating the random values of variable, we can find the smallest prediction error of a mathematical model. Therefore, these parameters would be determined by sequence of uniform distribution data in the upper and lower bounds of parameter intervals, sometime, these ranges of parameter intervals can be obtained from experimental data; when these parameters with some couple influential effects can cause these simulation data of mathematical model and experimental data not to satisfy the specifications, these ranges of parameter bounds would be adjusted. If the adjusted parameter bounds cannot obtain a better result, the mathematical model would be modified.

Numerical simulation and evaluating the models. Looking for a test by which the varied model's data can be evaluated, we would judge the performance by a prediction error from a certain model, the prediction error between the experimental data and the simulation data of the model's parameter would be given by

$$J(t,\hat{\theta})=\frac{1}{n}\left(\sum_{i=1}^{n}(y(t_i)-\hat{y}(t_i|\hat{\theta}))^2\right) \quad \ldots \quad (1)$$

where $\hat{\theta}$, $y(t_i)$ and $\hat{y}(t_i|\hat{\theta})$ are estimated parameter, experimental data and simulation data of hypothesis model, respectively, the small prediction error can be computed for the model's parameter $\hat{\theta}$ of a "good" model.

$$\hat{\theta}=\arg\min J(t,\theta) \quad \ldots \quad (2)$$

Here arg min denote the minimizing argument of the function, it expresses these parameters and hypothesis model with the best predictor function. Therefore, substituting these parameters into the coefficients of hypothesis model, we can get the small prediction error from this simulation process, and parameter $\hat{\theta}$ of hypothesis model is thus proportion to the likelihood function.

Parameter estimation. When prediction error satisfies the specification, it expresses a minimum error between experimental data and simulation data of likelihood function; these parameters in the parameter interval and hypothesis model can provide the most proper data to system identification. In this paper, we would use the Least-Squares method to estimate these coefficients of the model.

3. EXPERIMENTAL SETUP AND DATA ACQUISITION

In a practical system, data acquisition is usually affected by noisy; the sampled data with noise would make a difficulty to parameter estimation of system identification, therefore, we must be careful in the measurement procedure. In Figure 2, the micro motion actuator was investigated by Wang (Wang, 2005). The structure of the micro motion actuator consists of compliant mechanism, permanent magnet and square hollow coils. Because, there are manufacturing errors in these parts and measurement errors in micro/nano displacement, these factors would innovate noise and uncertainty that lead to decreased efficiency of modeling method, causing parameter estimation of the micro motion actuator become to very difficult. For the experimental test, the overall experimental device can be divided into four units in Figure 3.

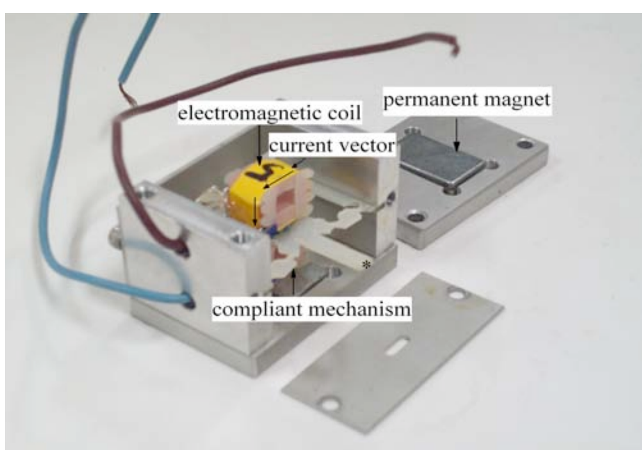

Figure 2. The structure of micro motion actuator.

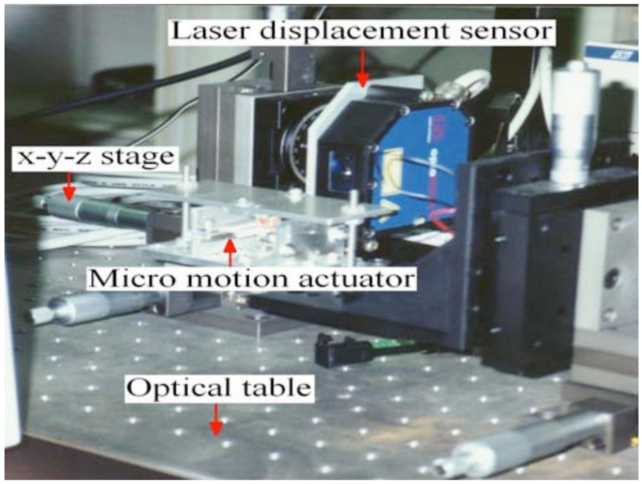

Figure 3. Experimental instruments for the parameter estimation.

Signal sampling. We use the ADLINK PCI-9111HR A/D D/A card to transfer voltage to displacement data.
Position sensor. The measuring instrument is optoNCDT ILD 1800-2 with the resolutions of 0.2 μm and the measuring range of ±1mm, The output signal of the laser position sensor is voltage.
Driver device. Its input/output voltage ratio is 1 and the driver would provide enough current to control the actuator motion.
Adjusted position mechanism. The xyz stage could adjust the orientation of xyz directions for easy experimental operation. In the experimental procedure, the testing conditions are 29.2 □ and 62% relative humidity.

In the dynamic experiment, the step response data of the system is from 1-volt step input and sampling frequency of 4K Hz, and the response is shown as Figure 4. From the experimental data, we can find the output signal with a large influential disturbance, the disturbance cannot be determines to be Gaussian noise by data analysis and testing.

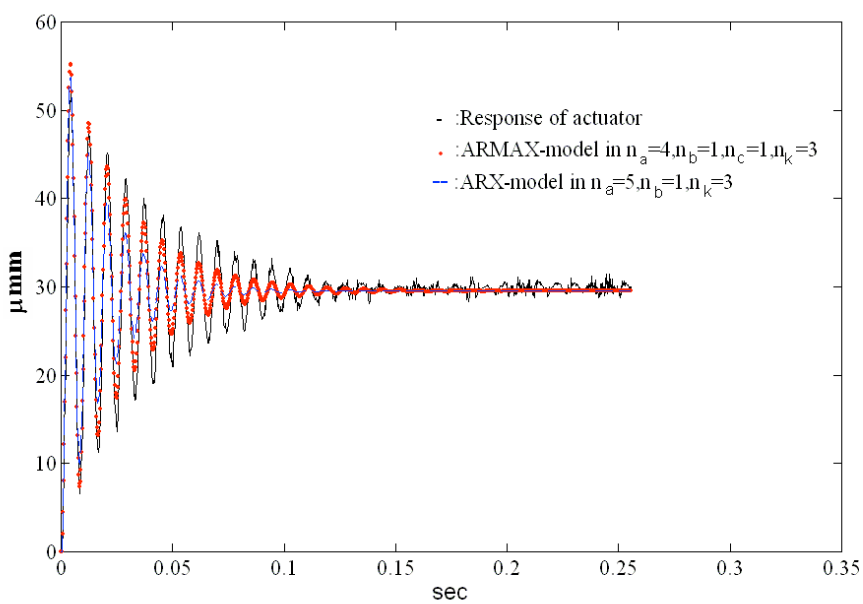

Figure 4. Parameter estimation of ARMAX-model and ARX-model.

4. PARAMETER ESTIMATION OF THE MICRO MOTION ACTUATOR

The experimental data can be directly used to system identification; these methods have been reported in the literature, for example, ARX-model and ARMAX-model. Therefore, ARX-model and ARMAX-model would be collected into a set of candidate models, otherwise, we would use the numerical procedure of Monte Carlo method to find likelihood function that this method uses random jumping method to establish a evaluated parameter process and decide a minimizing argument of the function, lastly, decides a "best" parameter by Least-Squares method.

4.1 Parameter Estimation by ARX-model and ARMAX-model

In general, the ARX-model and ARMAX-model are widely applied to the system parameter estimation; in the general *n*th order difference equation, the input-output relationship of ARX-model can be expressed as:

$$y(k)+a_1 y(k-1)+\ \cdots\ +a_{na} y(k-n_a)=b_1 u(k-nk-1)+\cdots+b_{n_b} u(k-nk-n_b+1)+e(k), \quad \ldots \quad (3)$$

where k is the integer time index, $y(k)$, $u(k)$, $e(k)$ and nk are output data, input signal, white noise and the number of delays from input to output, respectively.

The other candidate model is ARMAX-model, the model could be added the equation error as a moving average of white noise, the model is expressed as:

$$y(k)+a_1 y(k-1)+\ \cdots\ +a_{na} y(k-n_a)=b_1 u(k-n_k)+\cdots+b_{nb} u(k-n_k-n_b+1)+e(k)+c_{n1} e(k-1)+\cdots+c_{nc} e(k-n_c) \quad \ldots \quad (4)$$

where k is the integer time index, $y(k)$, $u(k)$, $e(k)$ and n_k are output data, input signal, moving average of white noise and the number of delays from input to output, respectively.

Using the system identification toolbox of MATLAB, we try the varied model structures and the delay; in ARX model, we use n_a=2-5, n_b=1 and n_k =1-3, the results of the estimated parameter are expressed Table 1; in ARMAX model, these verified conditions are n_a=2-5, n_b= n_c =1 and n_k =1-3, the results are Table 2; the best results of ARX model with n_a=5, n_b=1 and n_k =3, and ARMAX model with n_a=4, n_b= n_c =1 and n_k =3 are shown in Figure 4, the small prediction error is more than 1μm, therefore, we would investigate the other model or method.

Table 1. ARX-model errors.

	n_a=2, n_b=1	n_a=3, n_b=1	n_a=4, n_b=1	n_a=5, n_b=1
n_k =1	27.128	17.442	12.911	10.189
n_k =2	27.541	16.490	10.660	7.087
n_k =3	28.008	16.194	9.474	5.370

Table 2. ARMAX-model errors.

	n_a=2, n_b=1, n_c=1	n_a=3, n_b=1, n_c=1	n_a=4, n_b=1, n_c=1	n_a=5, n_b=1, n_c=1
n_k =1	5.818	4.486	5.136	4.952
n_k =2	5.376	2.513	2.546	2.693
n_k =3	5.309	2.281	1.729	1.871

4.2 Smoothing fitting of Monte Carlo method

In the micro motion actuator, we would hope a "best result" from the parameter estimation. Therefore, we would use the smoothing procedure of Monte Carlo method to do maximum likelihood estimation. The procedure is as follows:

Hypothesis model. By the experimental data or physical phenomena leading to a mathematical model, in Figure 5, dynamic analysis of the micro motion actuator contains the compliant mechanism and electromagnetic force circuit. In the compliant mechanism, the mechanism is made of polyethylene which is a viscoelastic material, so, if analysis of the compliant mechanism is used a pseudo-rigid-body model, it would contain a linear model and uncertainty of modeling error (Ward and Hadley, 1993); this electromagnetic force circuit is evaluated by equivalent circuit.

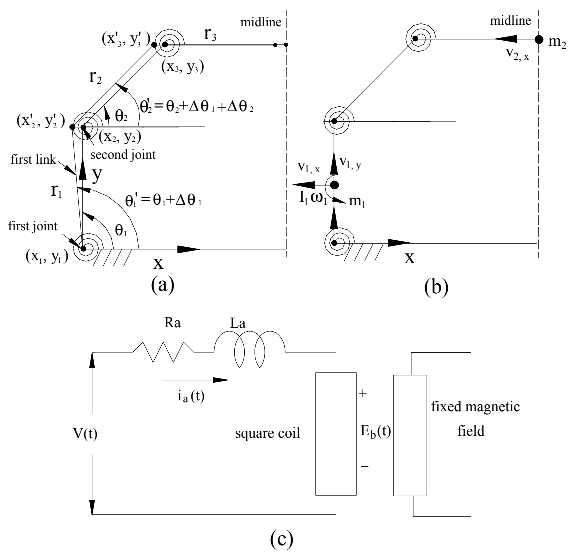

Figure 5. Equivalent pseudo-rigid-body model and equivalent circuit.

In Figure 2, the compliant mechanism with symmetrical property can be analyzed using one-quarter of the mechanism. The third link, r_3, would be constrained motion in the x direction and have zero displacement in the y direction. By these constrained conditions, the start mark, "*" of Figure 2 and point (x_3, y_3) of Figure 5.a have the same displacements. By defining length, r_i, of the *i*th link, angular displacement, $\Delta\theta_i$, of the *i*th joint, the geometric relations of Figure 5.a are derived by Wang (Wang, 2005); therefore, setting $\theta_1 = 90^0$, we have x-component of displacement, $\Delta x_{p,3} = -r_1 \Delta\theta_1$, the y-component of displacement is $\Delta y_{p,3} = 0$, $\Delta\theta_2$ equals $-\Delta\theta_1$. In the kinematic relations of the compliant mechanism, $\Delta x_{p,3}$, $\Delta\dot{x}_{p,3}$, $\Delta\ddot{x}_{p,3}$, $\Delta\theta_1$, ω_1, $\dot{\omega}_1$ are displacement, velocity, acceleration, angular displacement, angular velocity, angular acceleration, respectively.

The dynamic analysis of the compliant mechanism contains translation, rotation and deformation and even the dynamic analysis of the four-bar compliant mechanism is also very difficult. Also, we would use pseudo-rigid-body model to analyze the compliant mechanism and compliant linkage of mass is assumed at the mass center, pseudo linkage with mass is shown as Figure 5.b. Here, the first pseudo link has mass of m_1, the second pseudo link, the third pseudo link, and one-quarter upper and lower square hollow coils have mass of m_2. By defining β_1 as a fraction of length, r_i, of the *i*th link measured from the mass center to the *i*th center, (x_i, y_i) of compliant joint and ω_i as the angular velocity of the *i*th joint, the geometric relations and constraints of four-bar pseudo-rigid-body model will be derived through the model of Figure 5.b.

In the first link, the mass of m_1 has a velocity that can be expressed as $v_{1,x} = (\beta_1 r_1 \sin\theta_1)\omega_1$ and $v_{1,y} = (\beta_1 r_1 \cos\theta_1)\omega_1$; from constrained motion, the mass of m_2 has a velocity, $v_{2,x} = (r_1 \sin\theta_1)\omega_1 + (r_2 \sin\theta_2)(\omega_1 + \omega_2)$ in the x direction. The geometric relations have $\Delta\theta_1 + \Delta\theta_2 = 0$, $\omega_1 + \omega_2 = 0$ and $\theta_1 = 90^0$. The kinetic energy of pseudo linkage mechanism is written as:

$$K.E. = \frac{1}{2} m_1 v_{1,x}^2 + \frac{1}{2} m_1 v_{1,y}^2 + \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 v_{2,x}^2$$

$$= \frac{1}{2} m_1 (\beta_1 r_1 \sin\theta_1)^2 \omega_1^2 + \frac{1}{2} m_1 (\beta_1 r_1 \cos\theta_1)^2 \omega_1^2 + \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 \{r_1 \sin\theta_1 \omega_1 + r_2 \sin\theta_2 (\omega_1 + \omega_2)\}^2$$

$$=\frac{1}{2}m_1\{(\beta_1 r_1 \sin\theta_1)^2+(\beta_1 r_1 \cos\theta_1)^2+I_1\}\omega_1^2+\frac{1}{2}m_2\{r_1\sin\theta_1\omega_1+r_2\sin\theta_2(\omega_1+\omega_2)\}^2$$

$$=\frac{1}{2}m_1\{(\beta_1 r_1)^2+I_1\}\omega_1^2+\frac{1}{2}m_2\{r_1\omega_1\}^2$$

$$=\frac{1}{2}\{m_1[(\beta_1 r_1)^2+I_1]+m_2 r_1^2\}(-\frac{\Delta\dot{x}_{p,3}}{r_1})^2$$

$$=\frac{1}{2}M\Delta\dot{x}_{p,3}^2 \quad \ldots \quad (5)$$

with

$$M=\left\{\frac{m_1[(\beta_1 r_1)^2+I_1]+m_2 r_1^2}{r_1^2}\right\},$$

where mass moment of inertia of the first link is expressed as I_1.

The system energy is dissipated by the structural damping of the compliant mechanism, therefore, we can assume the structural damping with a center effect at these joint, the structural damping would be written in this form:

$$D.E.=\frac{1}{2}B_1\omega_1^2+\frac{1}{2}B_2(\omega_1+\omega_2)^2$$

$$=\frac{1}{2}B_1\omega_1^2$$

$$=\frac{1}{2}B_1\left(-\frac{\Delta\dot{x}_{p,3}}{r_1}\right)^2$$

$$=\frac{1}{2}B\Delta\dot{x}_{p,3}^2 \quad \ldots \quad (6)$$

where

$$B=\frac{B_1}{r_1^2},$$

the structural damping of the *i*th compliant joint is expressed as B_i.

The potential energy of the pseudo linkage mechanism is $P.E.=\frac{1}{2}K_1\Delta\theta_1^2+\frac{1}{2}K_2(\Delta\theta_1+\Delta\theta_2)^2$

$$=\frac{1}{2}K_1\Delta\theta_1^2$$

$$=\frac{1}{2}K_1\left(-\frac{\Delta x_{p,3}}{r_1}\right)^2$$

$$=\frac{1}{2}K\Delta x_{p,3}^2\ , \quad \ldots \quad (7)$$

where

$$K=\frac{K_1}{r_1^2},$$

the start mark, ”*” , displacement of the compliant mechanism and angular displacement of the first link relation can be derived form Equations 5 to 7, so, angular displacement, angular velocity, displacement and velocity relations can be established by the Lagrange’s equations:

$$\frac{d\dfrac{\partial K.E.}{\partial\Delta\dot{x}_{p,3}}}{dt}-\frac{\partial K.E.}{\partial\Delta x_{p,3}}+\frac{\partial D.E.}{\partial\Delta\dot{x}_{p,3}}+\frac{\partial P.E.}{\partial\Delta x_{p,3}}=f(t) \quad \ldots \quad (8)$$

where

$$\frac{\partial K.E.}{\partial\Delta\dot{x}_{p,3}}=M\Delta\dot{x}_{p,3},$$

$$\frac{d\frac{\partial K.E.}{\partial \Delta \dot{x}_{p,3}}}{dt} = M\Delta \ddot{x}_{p,3},$$
$$\frac{\partial K.E.}{\partial \Delta x_{p,3}} = 0,$$
$$\frac{\partial D.E.}{\partial \Delta \dot{x}_{p,3}} = B\Delta \dot{x}_{p,3},$$
$$\frac{\partial P.E.}{\partial \Delta x_{p,3}} = K\Delta x_{p,3}.$$

f(t) is an electromagnetic force, and the dynamic equation can be expressed as

$$M\Delta \ddot{x}_{p,3} + B\Delta \dot{x}_{p,3} + K\Delta x_{p,3} = f(t) \quad \text{...} \quad (9)$$

In the equivalent circuit of Figure 5.c, there are input voltage, square coil, induction electromotive force (EMF) and electromagnetic force in the micro motion actuator. The transfer function between the input voltage and displacement of the actuator can be derived as:
Electromagnetic force and current relation is

$$f(t) = K_f i_a(t) \ , \quad \text{...} \quad (10)$$

where $f(t)$, $i_a(t)$ and K_f are electromagnetic force of micro motion actuator, current of the square hollow coil and a force constant, respectively.
Counter EMF and velocity relation is

$$e_b(t) = K_v \Delta \dot{x}_{p,3}(t) \quad \text{...} \quad (11)$$

where K_v is a counter EMF constant.

Taking Laplace transform, assuming zero initial conditions, $I_a(s)$, $X(s)$, $E_b(s)$ and $F(s)$ are transferred from $i_a(t)$, $\Delta x_{p,3}(t)$, $e_b(t)$ and *f*(t), respectively. From these relations, the transfer function of the system is derived as:

$$F(s) = K_f I_a(s) \quad \text{...} \quad (12)$$

$$E_b(s) = K_v s X(s) \quad \text{...} \quad (13)$$

$$V(s) - E_b(s) = (R_a + L_a s) I_a(s) \quad \text{...} \quad (14)$$

where $V(s)$ is input voltage.
Equation 12 is substituted by the Laplace transform of Equation 9 with zero initial conditions to obtain as

$$(Ms^2 + Bs + K)X(s) = K_f I_a(s) \quad \text{...} \quad (15)$$

Equations 13 and 14 are substituted into Equation 15 to obtain as

$$V(s) - K_v s X(s) = (R_a + L_a s)\frac{(Ms^2 + Bs + K)}{K_f} X(s)$$
$$K_f V(s) = K_f K_v s X(s) + (R_a + L_a s)(Ms^2 + Bs + K)X(s)$$
$$K_f V(s) = \{L_a M s^3 + (L_a B + R_a M)s^2 + (R_a B + K L_a + K_f K_v)s + K R_a\} X(s)$$
$$\frac{X(s)}{V(s)} = \frac{K_f}{\{L_a M s^3 + (L_a B + R_a M)s^2 + (R_a B + K L_a + K_f K_v)s + K R_a\}} \quad \text{...} \quad (16.1)$$

In general, the inductance of the square hollow coil is very small, therefore, the inductance, L_a, can be neglected in this derivation. Equation 16.1 can be rewritten as

$$\frac{X(s)}{V(s)} = \frac{K_f}{\{R_a M s^2 + (R_a B + K_f K_v)s + K R_a\}} \quad \text{...} \quad (16.2)$$

Dynamic analysis of Equation 16 can provide important information to select the order type of transfer function in the electromagnetic actuator.

In this dynamic equation derivation of the micro motion actuator, the third order of transfer function can be simplified to a two order system, therefore, the hypothetical model of the Monte Carlo method is assumed as a two order system. While evaluation of prediction error cannot satisfy the specification, we will adjust the parameter interval or the mathematical model.

In the experimental data, except for the same sampling time, the maximum amplitude, natural frequency and damping ratio have uncertain effects from noise, so, according the properties of the experimental data and hypothetical model, the time response of the second order system can be modified as

$$\hat{y}_M(t_i) = A_M \left\{ 1 - r_M e^{-\varsigma_M t_i} \sin(\omega_M t_i + \theta_M) \right\} \quad \cdots \quad (17)$$

Equation 17 is a likelihood function, A_M, r_M, ς_M, ω_M and θ_M are random variables and cannot be correctly estimated from a system with disturbance. Therefore, by Monte Carlo method to find the small predition error from Equation 17, in five ranges of parameter intervals, each interval has a sequence of uniform distribution data; and substituting these parameters into the coefficients of Equation 17, we can get an array of output response data.

In the established the process of these parameter intervals, according to the mathematical model and response characteristics, the experimental data is used to graph the power spectral density, S(w) , there are two peaks in Figure 6. One is from step input at 0 rad/sec, the other is from system response at 760.8545 rad/sec. Therefore, the range of frequency parameter with 20 uniform distribution data is ω_M =[739, 785]. The phase angle, θ_M, is the response from 0 to its final value; and because the final value with noise effect can not stay at the certain value, the estimated parameter range with 10 uniform distribution data is θ_M=[0.48, 1.24]. The other parameters cannot be obtained from the properties of the experimental data, so, we assume r_M=[0, 1] with 10 uniform distribution data, A_M=[28.09, 31.44] with 50 uniform distribution data, and ς_M=[12, 36] with 20 uniform distribution data.

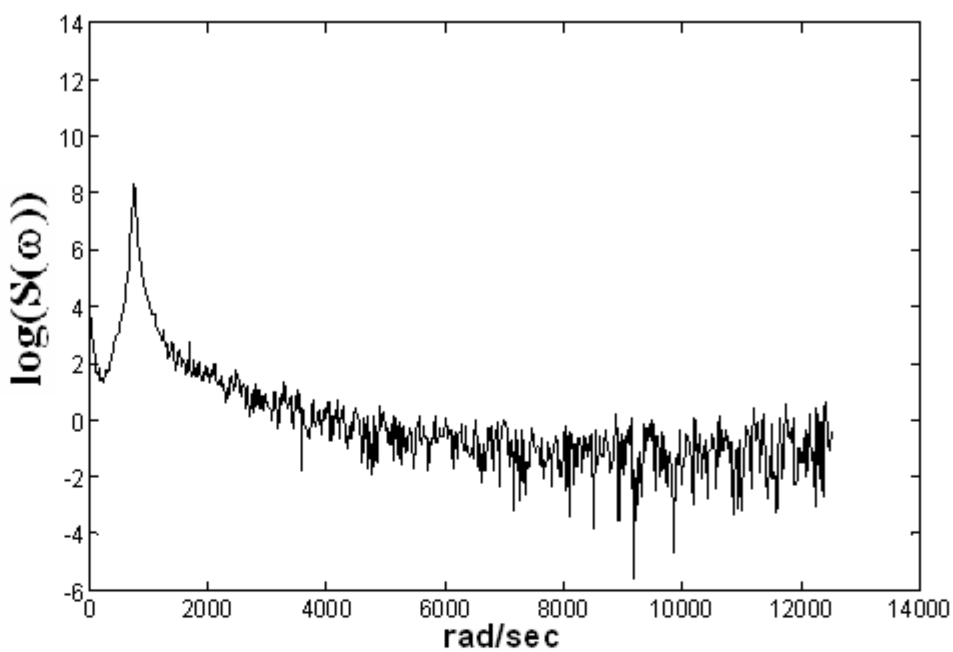

Figure 6. Power spectral density of response.

Substituting these parameters in parameter interval into Equations 17 and 1, we can obtain prediction error, *J*, if the prediction error could not satisfy the specification; we would adjust the range of the parameter interval. By repeating simulation procedure, there is a minimum prediction error in the twenty times evaluation, such that lower error of prediction error indicates that these parameters and mathematical model can provide better curve fitting of new numerical value to experimental data. these parameters are ω_M =767.977 rad/sec, r_M =0.915, θ_M =1.235 rad, A_M =29.539μm and ς_M =25.596 for prediction error, *J*=0.652.

In system parameter estimation, substituting these parameters into the coefficients of Equation 17, we can get an array of smoothing data. By these smoothing data of likelihood function, we would use Least-Squares method to do parameter estimation.

In the general *n*th order difference equation, the Least-Squares method is described as

$$y(k) = -a_{L,1}y(k-1) - \cdots - a_{L,n_a}y(k-n_a) + b_{L,0}u(k) + b_{L,1}u(k-1) + \cdots + b_{L,n_b}u(k-n_b), \quad \ldots \quad (18)$$

where *k* is the integer time index, *y(k)* and *u(k)* are output data and input signal, respectively.

By the Least-Squares estimation, the estimated parameter can be written as:

$$[\hat{\theta}_L] = \left([X_L]^T [X_L]\right)^{-1} [X_L]^T [Y_L] \quad \ldots \quad (19)$$

where $[X_L] = \begin{bmatrix} -y(k-1) & \cdots & -y(k-n_a) & u(k) & u(k-1) & \cdots & u(k-n_b) \\ -y(k) & \cdots & -y(k-n_a+1) & u(k+1) & u(k) & \cdots & u(k-n_b+1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -y(k+n-1) & \cdots & -y(k+n-n_a) & u(k+n) & u(k+n-1) & \cdots & u(k+n-n_b) \end{bmatrix}$,

$[Y_L] = \left[y(k) \quad y(k+1) \quad \cdots \quad y(k+n)\right]^T$,

$[\hat{\theta}_L] = \left[a_{L,1} \quad \cdots \quad a_{L,n_a} \quad b_{L,0} \quad b_{L,1} \quad \cdots \quad b_{L,n_b}\right]^T$.

From the second-order models, the experimental data and these smoothing data of likelihood function can be directly used to the Least-Squares estimation, the experimental data would be directly evaluated to second and third order system by Least-Squares method and parameter estimation of second order system with smoothing fitting can be obtained the result in Table 3, the experimental data without data process and the smoothing data of likelihood function can be expressed D-E method and S-E method, respectively, in Table 3. the parameter estimation of the likelihood function with the small prediction error, *J*= 0.652 and parameter estimation without data process are shown as in Figure 7. The Z-transform parameters of the transfer function are b_0= 4.030, b_1= -8.919, b_2= 5.969, a_1= -1.951, a_2= 0.987 from parameter estimation of the likelihood function. In the ***S*** domain, the transfer function can be rewritten as $\frac{4.03s^2 - 5970.61s + 17441286.92}{s^2 + 51.19s + 590444.53}$, and two poles are -25.6 + 767.98*i* and -25.6 – 767.98*i*; there are not unstable poles in the system.

In the parameter estimation procedure for a micro motion actuator, the Least-Squares parameter estimation can obtain the best results by the smoothing treatment of likelihood function.

Table 3. Smoothing fitting of response errors.

	$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$	$\frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$	$\frac{b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$	$\frac{b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$	$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$	$\frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$	$\frac{b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$
D-E Method	16.241	16.281	16.549	16.959	27.539	27.716	27.891
S-E Method	x	x	x	x	0.652	13.614	9.925

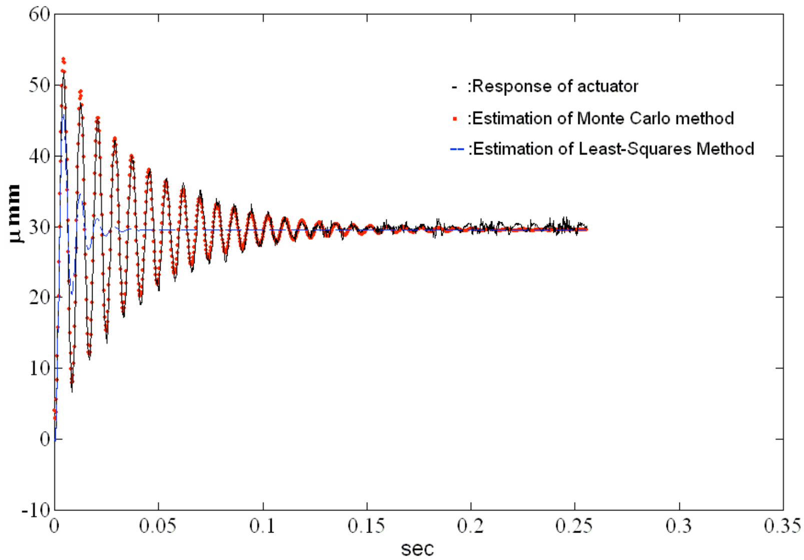

Figure 7. Parameter estimation of smoothing method

5. DISCUSSION AND CONCLUSION

In a set of candidate models, ARX model and ARMAX model, the better result of parameter estimation can be find the higher-order model than the lower-order model, but the higher-order model would innovate more desired poles that may make system instability or uncertainty; otherwise, ARMAX model is less prediction error than ARX model in the experimental data with disturbance, Another candidate model, parameter estimation of likelihood function is through hypothesis model, established parameter interval and evaluated prediction error to do the smoothing treatment of the experimental data with noise, the advantages of the smoothing data can provide a better post treatment for parameter estimation. By this method, we do not need a complex theory to design a filter that reduces the effect from disturbance or difficult theory to estimate the parameter.

In the parameter estimation process, we find the experimental data with a larger influential disturbance that could not be tested which types in the micro motion actuator, but, Smoothing fitting of Monte Carlo method has the more accurate parameter estimation, by pseudo-rigid-body model and equivalent circuit to find dynamic model, properties of experimental data and power spectral density are used to obtain frequency and phase angle in system response, the unknown parameter ranges would be reduced. By search process of Monte Carlo method, we could get the smoothing fitting of likelihood function and a good result of parameter estimation from Least-Squares estimation; the prediction error is “enough small” and satisfied for our purpose.

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BIOGRAPHICAL SKETCH

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