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# THE APPLICATION OF MULTINOMIAL CONTROL CHARTS FOR INSPECTION ERROR

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Control charts have become one of the most commonly used tools for monitoring process variations in today's manufacturing environment. The *p* chart plays the important role in controlling the fraction of nonconforming article produced. Instead of simply classifying qualities into conforming and non-conforming, products are classified into several classes of quality in this study. It is named as multinomial control charts. Classic multinomial control charts are built without taking into account the inspection error. However, the inspection through instruments or human observers will ever make mistakes such that the results of control charts are not valid. Therefore, how to examine the influence of inspection error on the multinomial control charts is concerned in this study. In this article, the inspection error influence on the multinomial control charts is examined. Two modified models using statistical approach are proposed to build the corresponding control charts when inspection error exists. In addition, two evaluation indexes including type I error and out-of-control ARL are performed to compare the performance of classic multinomial control charts and two modified models. When type I error is fixed, the out-of-control ARL results show that Model I (adjust true probability distribution) works better than Model II (adjust statistical values). Such approaches can provide more realistic modeling to monitor and identify the production process variations.

**Keywords**： Statistical Process Control, Control Chart, Inspection Error, Multinomial Data

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## 1. INTRODUCTION

In the production processes, due to the machine equipment, material, environment, operator, and so on reasons, it causes that no product is created quite the same as the others. In quality control, variations are divided into chance and assignable variations. Chance variation, also named random variation, is caused under the natural condition, is inevitable, and its influence is small. But assignable variations should not occur in the regular production processes. Once it occurs, it causes the system conspicuous losses. The statistical process control (SPC) identifies whether the production processes have changed or not, or the occurrence of assignable variations, by the analysis of quality characteristic data in order to improve the production processes before the irregular products created. This is the purpose of control charts that is widely used as a tool for SPC. After arranging quality characteristic data and recording data in the control chart, users can determine if the production process is in-control state or not according to the statistical principles. If the production process is not in-control state, the problems caused variations should be removed according to the clues in the control chart.

A control chart is composed of three items: (1) center line (CL), (2) control limits (CLs), and (3) monitoring statistic by sample dots. Based on the inspection or measurement of quality characteristics from the obtained sample, control charts are classified into two types: control charts for variables and for attributes. In other words, if the quality characteristics are measurable and continuous value, such as weight, length and so on, then control charts for variables are used. The most common control charts for variables have the following two kinds: (1) Mean and range chart ($\overline{X}-R$ chart) and (2) Mean and standard deviation chart ($\overline{X}-S$ chart). When quality characteristics can not be

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presented by continuous value, but by certain product attributes or by unqualified number per unit to identify if the product is qualified or not, then control charts for attributes are used. The most common control charts for attributes have the following four kinds: (1) Fraction nonconforming chart (*p* chart), (2) Defective chart (*np* chart), (3) Defect chart (*c* chart), and (4) Defects per unit chart (*u* chart).

Besides standard control charts which are introduced above, there are other statistical process-control technologies. In certain situations they are extremely useful. For example, short run $\overline{X}-R$ chart is suitable for mean of short time production processes, acceptance control chart uses means control chart to control fraction defective or surpasses the specification limits, multivariate control chart simultaneously monitors two or more quality characteristics, and so on. The details of above methods may refer Montgomery (2004). This research aims on the classic multinomial control charts, but instead of simply classifying qualities into conforming and non-conforming, products are classified into several classes of quality. For example: (1) The qualities can be divided into very bad, bad, ordinary, good, and excellence five levels; (2) Carbon resistors can be divided into 1%, 2%, 5%, 10% and greater than 10% five kinds. About the researches of classic multinomial control charts, Marcucci (1985) used three kinds of quality ranks to describe the quality of bricks which are standard, chipped, and cull. He also used Person goodness-of-fit method to build up control charts. Nelson (1987) constructed the chi-square control chart to monitor several classic fractions. Taleb (2009) proposed two procedures to monitor multivariate attribute processes

The inspections of raw materials, or in-process products, or end products are the important parts of quality guarantee. Quality characteristics obtained from inspections are drawn in the control charts in order to monitor and control product process. However, the traditional control chart methods assume that inspection processes have no mistake, but in actually inspection error is very difficult to avoid whatever using visual or mechanical detection (Johnson *et al*., 1991). Montgomery (2004) also pointed out that inspection error was usually caused by the inspectors' error, possible was the poor training of inspectors, and perhaps the inspector equipments' standard scale divisions are not suitable. Also, Latorella and Prabhu (2000) researched the detail human error styles of inspectors. In addition, many researchers have investigated the influences of inspection error on sampling plan and control charts. The influences of inspection error on sampling plans have been considered by several authors, such as Dorris and Foote (1978), Schneider and Tang (1987), Chyu and Wu (2002), Duffuaa and Khan (2002), Ferrell and Chhoker (2002), and Markowski and Markowski (2002). Recently, Wang (2007) extended the inspection/disposition model to consider two types of inspection errors in order to facilitate the adaptation of the economic model to real world applications.

On the contrary, control charts under inspection error have received little attention in recent years, with the exception of three older paper. The influence of inspection error on $\overline{X}$, $R$, CUSUM variables charts has been considered by Abraham (1977). Case (1980) considered the effects of *p* control chart under inspection error and a compensating *p* chart is proposed. Suich (1988) developed the *c* control chart under inspection error and showed adjusted control limits which compensated for inspection error. There has been no similar paper published for control charts in recent years. Therefore, focusing on the classic multinomial control charts problem above, this study has three objects:

1. When inspection error exists, what is its influence on multinomial classic control charts?
2. Using statistical approach to modify correspondent classic multinomial control charts when inspection error exists.
3. To use the evaluation indexes for control chart design to compare the performance of proposed control charts and provide usage suggestion.

## 2. LITERATURE REVIEW

In this section, the inspection error influence on *p* chart, and the situation of classic multinomial control charts without considering the inspection error are reviewed.

### 2.1 The Researches of Inspection Error Influence on p Chart

To classify sampling products into two classes, good and bad, is an important concept for this study. It is a special case of multinomial data for only two classes. The methods are introduced as follows:

(1). The relation between fraction defective and inspection error:

Two kinds of error may be committed when inspection error exists. If the conforming article is classified into nonconforming, then a type I error has occurred. If the nonconforming article is classified into conforming, then a type II error has been made.

To realize the inspection error influence on *p* chart, the concerned events and probabilities are defined as the following:

$E_1$ : The event that conforming article is classified into nonconforming.

$e_1$ : The probability of that a conforming article is classified into nonconforming, which is equal to $P(E_1)$.

$E_2$ : The event that nonconforming article is classified into conforming.

$e_2$ : The probability of that nonconforming article is classified into conforming, which is equal to $P(E_2)$.

$A$ : The event of nonconforming article.

$p$ : The true fraction defective, which is equal to $P(A)$.

$B$ : The event that article is classified into nonconforming after inspection.

$p_e$ : The apparent fraction defective, which is equal to $P(B)$.

Such that, the probability of nonconforming article after sampling inspection can be described as:

$$P(B) = P(A)P(\overline{E_2}) + P(\overline{A})P(E_1) \quad \cdots \quad (1)$$

and the apparent fraction defective can be written as:

$$p_e = p(1-e_2) + (1-p)e_1 \quad \cdots \quad (2)$$

(2). The modification and comparison of the fraction nonconforming charts when inspection error exists:

When inspection error is not considered, true fraction defective (*p*) is used, the center line, upper, and lower limits are presented as the follows:

Center line, $CL_p = p \quad \cdots \quad (3)$

Upper control limit, $UCL_p = p + l\sqrt{\frac{p(1-p)}{n}} \quad \cdots \quad (4)$

Lower control limit, $LCL_p = p - l\sqrt{\frac{p(1-p)}{n}} \quad \cdots \quad (5)$

where *p* is objective value or mean of fraction defective, *n* is sample size, and *l* is control limit width, most of the time, is set as 3.

When inspection error is considered, Case (1980) proposed three modification models of *p* chart. Supposed the sample size *n* is drawn in each sampling. The fraction defective of sample is got after inspection. It is used as a statistic value for monitoring in control charts. The three modified models are summary in Table 1.

**Table 1. Three modified models of *p* chart when inspection error is considered.**

| | Center line | Upper control limit | Lower control limit | Statistical values for calculating center line and control limits |
|---|---|---|---|---|
| Model 1 | $p$ | $p+3\sqrt{\dfrac{p(1-p)}{n}}$ | $p-3\sqrt{\dfrac{p(1-p)}{n}}$ | Objective value $p$ |
| Model 2 | $p_e$ | $p_e+3\sqrt{\dfrac{p_e(1-p_e)}{n}}$ | $p_e-3\sqrt{\dfrac{p_e(1-p_e)}{n}}$ | $p_e$ |
| Model 3 | $CL_p(1-e_2)+(1-CL_p)e_1$ | $UCL_p(1-e_2)+(1-UCL_p)e_1$ | $LCL_p(1-e_2)+(1-LCL_p)e_1$ | $p_e$ and function (2) |

Case (1980) used OC curve to compare the performance of three modified models, the results indicated that the performance of model 3 is better than that of the other two models. He also studied the inspection error influence on *p* chart when the probability of inspection error is not fixed but has a linear function relation with real fraction defective.

### 2.2 Researches about Classic Multinomial Control Charts without Considering the Inspection Error

When product qualities are not divided into the former dichotomy of good and bad but several classes, *p* chart is not suitable any more. The correlation literatures are explained below.

First supposed the sampling inspection qualities of the products are divided into *k* classes. The probabilities for every class of quality occurred are $\pi_1,\pi_2,\cdots,\pi_k$, where $\sum_{j=1}^{k}\pi_j=1$. Assuming after the inspection for $n_i$ samples in the $i^{th}$ sampling, there are $X_{i,j}$ amounts of samples are classified into class *j*, where $j=1,2,\cdots,k$ and $\sum_{j=1}^{k}X_{i,j}=n_i$. Marcucci[2] suggested using the Person goodness-of-fit method proposed by Duncan (1974). The statistic value for monitoring is:

$$Y_i^2=\sum_{j=1}^{k}\frac{\left(X_{i,j}-n_i\pi_j\right)^2}{n_i\pi_j} \qquad \cdots \qquad (6)$$

When sample size $n_i$ is large enough, Equation (6) is close to $\chi^2$ distribution. Therefore, Marcucci (1985) suggested to monitor the value of $Y_i^2$ in each sampling and use $\chi^2_{1-\alpha}$ as the upper control limit of multinomial control chart. Such that it satisfies $P\left(Y_i^2<\chi^2_{1-\alpha}\right)=1-\alpha$ when production process is under controlled. Here, $\alpha$ is the probability of the type I error.

## 3. PROPOSED METHOD

In this section, first, the research limits and hypothetical assumption are described. Then the relation equation between true and apparent probability distribution is built if inspection error exists. Finally the modified control charts using statistical approach are proposed and designed.

### 3.1 Research Limits and Hypothetical Assumption

The basic hypothesis in this study assumes that the sampling inspection has the following characters:

(a). Samples are independent each other.

(b). There is only one class of inspection result when there are *k* classes.

In addition, when inspection error exists, to realize the inspection error influence on multinomial control charts, the assumptions are:

(a). Inspection error exists with fix error probability.

(b). Probabilities of inspection error between different inspectors are the same.

For convenient explanation, the relative symbols are listed below:

$e_{r,j}$: The probability of error identifying class *r* samples into class *j* when inspection error exists. The error identification occurs when $r \neq j$ and $e_{r,j} \neq 0$.

$e$ : The probability matrix of error identifying. $e = [e_{r,j}]$ is a $k \times k$ matrix. $\sum_{j=1}^{k} e_{r,j} = 1$ when $r = 1,2,\cdots,k$.

$k$ : Class number.

$n_i$ : The sample size in the $i^{th}$ sampling inspection.

$X_{i,j}$: Amount of samples classified into class *j* after inspecting $n_i$ samples in the $i^{th}$ sampling , where $j = 1,2,\cdots,k$ and $\sum_{j=1}^{k} X_{i,j} = n_i$.

$\pi_t$ : The true probability distribution that a class occurs when inspection error does not exist, where $\pi_t = (\pi_{t1}, \pi_{t2}, \cdots, \pi_{tk})$.

$\pi_e$ : The apparent probability distribution that a class occurs when inspection error exists, where $\pi_e = (\pi_{e1}, \pi_{e2}, \cdots, \pi_{ek})$.

### 3.2 Building the Relation Equation Between True and Apparent Probability Distribution (Inspection Error Exists)

Due to the existence of inspection error, the condition that class *r* sample is misclassified into other classes happened. Hence true probability distribution $\pi_t$ changes to apparent probability distribution $\pi_e$. To realize the inspection error influence on multinomial control charts and to further modify original multinomial control charts, the relation equation between true and apparent probability distribution, that is to say the relation between $\pi_t$ and $\pi_e$, is built as follows:

Or $\pi_e = \pi_t \cdot e$ ... (7)

$$\left(\pi_{e1},\pi_{e2},\cdots,\pi_{ek}\right)=\left(\pi_{t1},\pi_{t2},\cdots,\pi_{tk}\right)\cdot\begin{pmatrix} e_{1,1} & e_{1,2} & \cdots & \cdots & e_{1,k} \\ e_{2,1} & e_{2,2} & \cdots & \cdots & e_{2,k} \\ & & \ddots & & \\ e_{k-1,1} & \cdots & \cdots & e_{k-1,k-1} & e_{k-1,k} \\ e_{k,1} & \cdots & \cdots & e_{k,k-1} & e_{k,k} \end{pmatrix} \quad \cdots \quad (8)$$

where '·' is matrix product operator. Some special conditions of error classification are also described below.

**Case 1: Special case – no inspection error**

In this case, for every *r*, when $e_{r,r}=1$ and $e_{r,j}=0$ where $r \neq j$, the relation equation can be written as (9) according Equation (8),

$$\left(\pi_{e1},\pi_{e2},\cdots,\pi_{ek}\right)=\left(\pi_{t1},\pi_{t2},\cdots,\pi_{tk}\right)\begin{pmatrix} 1 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & \cdots & 0 & 1 \end{pmatrix}=\left(\pi_{t1},\pi_{t2},\cdots,\pi_{tk}\right) \quad \cdots \quad (9)$$

**Case 2: Inspection error distributes in every class averagely.**

In this case, $e_{r,j}=1/k$ for every *r* and *j*, the relation equation can be written as (10) according Equation (8),

$$\left(\pi_{e1},\pi_{e2},\cdots,\pi_{ek}\right)=\left(\pi_{t1},\pi_{t2},\cdots,\pi_{tk}\right)\begin{pmatrix} 1/k & \cdots & 1/k & 1/k \\ 1/k & \ddots & 1/k & 1/k \\ \cdots & \cdots & \ddots & \cdots \\ 1/k & \cdots & 1/k & 1/k \end{pmatrix}=\left(1/k,1/k,\cdots,1/k\right) \quad \cdots \quad (10)$$

**Case 3: Inspection error occurres only in the adjacent classes.**

In this case, Equation (8) can be presented as:

$$\left(\pi_{e1},\pi_{e2},\cdots,\pi_{ek}\right) \quad \cdots \quad (11)$$
$$=\left(\pi_{t1},\pi_{t2},\cdots,\pi_{tk}\right)\begin{pmatrix} e_{1,1} & e_{1,2} & 0 & & & & \\ e_{2,1} & e_{2,2} & e_{2,3} & & & 0 & \\ & \ddots & \ddots & \ddots & & & \\ & & e_{r,r-1} & e_{r,r} & e_{r,r+1} & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & e_{k-1,k-2} & e_{k-1,k-1} & e_{k-1,k} \\ & & & & & e_{k,k-1} & e_{k,k} \end{pmatrix}$$

### 3.3 Modified Control Chart Design

When inspection error exists, it needs to modify classic multinomial control charts that do not consider inspection error. Three models are discussed: one non-modified ordinary control chart and two modified models. They are explained below:

(1) Non-modified ordinary control chart

In this model, the control limits derived from true probability distribution $\pi_t$ that a class occurs are used without considering inspection error. The statistic value for monitoring is $Y_i^2 = \sum_{j=1}^{k} \frac{\left(X_{i,j} - n_i \pi_{tj}\right)^2}{n_i \pi_{tj}}$ and $\chi^2_{1-\alpha}$ is used as the control limit. However, the true amounts $(X_{i,1}, X_{i,2}, \cdots, X_{i,k})$ are influenced by inspection error.

(2) The modified model I (adjust true probability distribution $\pi_t$)

In this model, the statistical values $(X_{i,1}, X_{i,2}, \cdots, X_{i,k})$ that have been influenced by inspection error are monitored. Since true probability distribution $\pi_t$ has been changed to apparent probability distribution $\pi_e$. The monitored statistic $Y_i^2 = \sum_{j=1}^{k} \frac{\left(X_{i,j} - n_i \pi_{tj}\right)^2}{n_i \pi_{tj}}$ is adjusted to $\sum_{j=1}^{k} \frac{\left(X_{i,j} - n_i \pi_{ej}\right)^2}{n_i \pi_{ej}}$ and $\chi^2_{1-\alpha}$ is used as the control limit. The apparent probability $\pi_e = \left(\pi_{e1}, \pi_{e2}, \cdots, \pi_{ek}\right)$ can be obtained from Equation (8).

(3) The modified model II (adjust statistical values $(X_{i,1}, X_{i,2}, \cdots, X_{i,k})$)

In this model, the statistical values $(X_{i,1}, X_{i,2}, \cdots, X_{i,k})$ that have been influenced by inspection error are monitored. Since true probability distribution $\pi_t$ has been changed to apparent probability distribution $\pi_e$. The statistical values $(X_{i,1}, X_{i,2}, \cdots, X_{i,k})$ can been adjusted depending on Equation (12). Hence the monitored statistic $Y_i^2 = \sum_{j=1}^{k} \frac{\left(X_{i,j} - n_i \pi_{tj}\right)^2}{n_i \pi_{tj}}$ is adjusted to $\sum_{j=1}^{k} \frac{\left(X_{i,j}' - n_i \pi_{tj}\right)^2}{n_i \pi_{tj}}$ and $\chi^2_{1-\alpha}$ is used as the control limit.

$$(X_{i,1}', X_{i,2}', \cdots, X_{i,k}') = (X_{i,1}, X_{i,2}, \cdots, X_{i,k}) \cdot e^{-1} \qquad \cdots \qquad (12)$$

### 3.4 Inspection Error Influence and Sensitivity Analysis

When inspection error exists, the below two indexes are used to examine the inspection error influence on the proposed control charts and compare the goodness of the modified control chart design.

(1) Type I error

When the true probability distribution $\pi_t$ is not changing, the probability of the samples' statistical values out of control limits is type I error. It is better when its value is closer to $\alpha$.

(2) Average run length (ARL)

Out-of-control ARL is the sample amount that the control charts need to take to get out-of-control signal when the true probability distribution $\pi_t$ is changing by certain reasons. The out-of-control signal is caused by the statistical values' out of control limits. When type I error is fixed, the small out-of-control ARL is, the better control chart is.

## 4. INDUSTRY APPLICATION

In this section, a real industry application example is used to calculate and to explain above procedures. Assumed sample size 300 in each sampling is considered. The product quality is classified into three classes (T1)Perfect Translucence, (T2)Good Translucence and (T3)Not Translucence following the quality characteristic "Translucence", true probabilities of each class occurs are 0.1, 0.8, and 0.1 respectively. When inspection error exists, the probability matrix of error occurred only in the adjacent class identifying, that is Equation (12), is considered:

$$e = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.05 & 0.9 & 0.05 \\ 0 & 0.1 & 0.9 \end{pmatrix} \quad \cdots \quad (12)$$

To realize the inspection error influence on classic control chart, three actual type I error $\alpha$ values are set to compare. The results of testing type I error using three methods are shown in Table 2. They indicate that the value of modified model I is the closest one to the actual type I error value $\alpha$, and the worst is the non-modified method. Hence the inspection error has very large influence on Non-modified ordinary multinomial control charts.

**Table 2. The results of testing type I error value comparing three methods.**

| Method | $\alpha = 0.1$ | $\alpha = 0.05$ | $\alpha = 0.01$ |
|---|---|---|---|
| Non-modified ordinary control chart | 0.957 | 0.651 | 0.428 |
| The modified model I | 0.088 | 0.044 | 0.015 |
| The modified model II | 0.276 | 0.187 | 0.064 |

Furthermore, out-of-control ARL is used to compare the two modified models. When type I error is fixed at 0.05 and true probabilities of each class occurs are changing from 0.1, 0.8, and 0.1 to (0.1+*k,* 0.8-*k*/2, 0.1-*k*/2) respectively for some reasons, the out-of-control ARL results are listed in Table 3 for *k*=0.01, *k*=0.03, and *k*=0.05. Results show that out-of-control ARL of the modified model I is smaller than that of model II. Such that, in this example, the modified model I is suitable for classic control chart when inspection error occurs.

**Table 3. The out-of-control ARL results when $\pi_t$ = (0.1, 0.8, 0.1) has been changed to $\pi_e$=(0.1+*k*, 0.8-*k*/2, 0.1-*k*/2).**

| Method | *k*=0.01 | *k*=0.03 | *k*=0.05 |
|---|---|---|---|
| The modified model I | 12.346 | 3.704 | 1.733 |
| The modified model II | 12.699 | 3.953 | 1.828 |

## 5. CONCLUSION

In this paper, we have seen that inspection error seriously influence the performance of classic multinomial control charts. Since true probability $\pi_t$ has been changed to apparent probability $\pi_e$ when inspection error exists, two models have been presented to modify classic multinomial control charts. When type I error is fixed, the out-of-control ARL results show that Model I (adjust true probability $\pi_t$) works better than Model II (adjust statistical values). Hence Model I is recommended to modify the classic multinomial control charts when inspection error actually exists.

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