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DECIDING OPTIMAL SPECIFICATION LIMITS AND PROCESS ADJUSTMENTS UNDER QUALITY LOSS FUNCTION AND PROCESS CAPABILITY INDICES

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In traditional screw manufacturing, if buyers want to have high quality products, they have to pay the appreciate quality costs to their manufacturers. To satisfy the quality requirements of buyers with minimum loss of quality costs, this study proposes a modified totally expected quality loss model using Taguchi's quality loss function and process capability indices for normal and non-normal distributions. The three-parameter Weibull distribution is used to estimate a skewed process. These models determine the optimal adjusted process mean, adjusted process standard deviation and specification limits based on the minimum costs and process capability requirements of buyers. This study also considers the design of optimal adjusted process parameters and specification limits using two examples: the hexagon head cap screw, and oil seal.

Significance: These proposed models can determine the negotiated optimal specification limits and process adjustments and carry off the concept of design for manufacturability The optimal adjusted process parameters can help suppliers in making production decisions and satisfying buyers' requirements. From the buyer's perspective, they can get high quality products after paying the minimal adjustment costs to suppliers.

Keywords: Normal distribution, process adjustment, process capability indices, quality loss function, Weibull distribution.

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1. INTRODUCTION

Generally for buyers the process of securing their product simply entails giving orders to suppliers that can fulfill their quality requirements. Hence, when yield rates and manufacturing capabilities satisfy the requirements of buyers, suppliers have opportunities to shoot for buyers' orders. However, in the traditional screw industry, it is not usual for buyers to ask whether a process mean or level of manufacturing capability are close to their target. Products are simply accepted when the characteristic values of sampled products fall into specification limits. Therefore, the process mean and manufacturing capability for screw manufacture are often lower than the buyers' requirements. Consequently, when buyers want to have high quality products, they have to pay for these requirements themselves since suppliers transfer the adjusted costs of the process mean and manufacturing capability directly to the buyers. Some buyers, such as those requiring screws for electrical machinery, instruments and motors can specially request a quality level of production from screw suppliers, including the requirements of the process mean and manufacturing capability. Therefore, the issues facing suppliers of how to adjust a process mean and standard deviation to satisfy buyers' specifications under minimal costs are very important. However, there are no papers to date that discuss these issues.

Taguchi's quality loss function has been applied in many areas, such as control charts, process capability indices (PCIs), and specification design. Considering inspection and scrap or rework costs, Kapur and Wang (1987) employed Taguchi's quality loss function to obtain the optimal specification limits and the minimum total expected quality loss per unit (TC) for normal and log–normal distribution. The successive researches following Kapur and Wang's model took account of some conditions, like the different distributions, multiple quality characteristics, and linear quality loss function, to design specification limits (Kapur, 1988; Kapur and Cho, 1994; Kapur and Cho, 1996; Chen, 1999; Wright, 2000; Chen and Chou, 2005). However, none of the proceeding papers addressed the manufacturing capability and adjustment of process mean and standard deviation.

Montgomery (2005, p. 327) indicated that process capability indices are a more helpful tool throughout the product cycle. The PCIs can quantify process variability, analyze this variability relative to product specifications and assist manufacturing to eliminate this variability. Kotz and Lovelace (1998, p. 4) that expressed PCIs may be likened in the manufacturing arena to economic indicators, like the consumer price index in government statistics. Hence, PCIs can be used to analyze the raised costs of a consumer requirement.

This study releases the assumptions of Kapur and Wang's model (1987) that states that the present process cannot be improved (decrease variance) and a process mean equals the middle value within the specification limits. Furthermore, a manufacturing capability is also considered in this study. Therefore, this study develops new models based on Kapur and Wang's method and PCIs to determine minimal total cost, optimal specification limits, appropriate mean and appropriate standard deviation to satisfy customer's requirements for the-nominal-the-best type quality characteristics of the normal distribution and Weibull distribution.

The rest of this paper is organized as follows. The assumptions and notations are listed in Section 2. The proposed models are presented in the Section 3. Section 4 shows the totally expected quality loss models for normal and Weibull distributions. Examples are given in Section 5 and finally conclusions are drawn in Section 6.

2. ASSUMPTIONS AND NOTATIONS

The study has the following five assumptions: (1) initial mean and standard deviation of a process are known; (2) loss cost, scrap cost, inspection cost, adjusted mean cost and adjusted standard deviation cost per unit are known; (3) process capability index is set between 1.33 and 1.67 from a customer requisition; (4) initial process mean is unequal to the target value within the specification limits; (5) a process can be improved, hence, process mean and process standard deviation can be adjusted.

Before presenting the proposed mathematical models, the study defines and summaries the necessary notations, below.

a	the shape parameter of Weibull distribution;
$f_{t,i}$	the probability density function of the truncated distribution, where i is normal (n) or Weibull distributions (w);
k	the loss cost per unit;
q_i	the probabilities that a random variable falls inside specification limits, where i=n or w;
r	the multiplier of standard deviation;
u	$(y_0 - \mu_n)/\sigma_n + w$;
v	$(y_0 - \mu_n)/\sigma_n - w$;
y_0	the target value within specification limits;
μ_1	the initial process mean;
μ_i	the adjusted process means, where i=n or w;
$\mu_{t,i}$	the means of truncated distributions, where i=n or w;
σ_1	the initial standard deviation;
σ_2	the minimum process standard deviation under the ideal production;
$\sigma_{t,i}$	the standard deviations of truncated distributions, where i=n or w;
$\sigma_{t,i}$	the adjusted process standard deviations, where i=n or w;
C_{pm}	the third generation process capability index;
C_{pmw}	the process capability index based on weighted variance;
L_i	the expected quality loss, where i=n or w;
DC	adjusted standard deviation cost per unit;
IC	inspection cost per unit;
MC	adjusted mean cost per unit;
P_t	the probabilities that a random variable falls below a target;
SC	scrap cost per unit;
TC_i	total quality costs per unit, where i=n or w;
LSL	the lower specification limit;
PCI	process capability index;
USL	the upper specification limit;
$\phi(\cdot)$	the probability density function of the standard normal distribution;
$\Phi(\cdot)$	the cumulative probability density function of the standard normal distribution.
$\Gamma(\bullet, \bullet, \bullet)$	the incomplete gamma function.

3. PROPOSED MODEL

Bilateral tolerances, the plus and minus tolerances, are often employed in this model. Some examples of this type of quality characteristic involve dimensions, voltage, clearance, viscosity of a fluid, diameter of a gear, and shift pressure (Kapur, 1988; Taguchi et al., 1989). When the quality characteristic shifts from the nominal value, the performance of the product deteriorates.

According to Kapur and Wang's model (1987), the minimum total expected quality losses per unit involves three parts, the expected quality loss, the scrap cost and the inspection cost. Kapur and Wang assumed that the present process cannot be improved and a process mean is equal to the midpoint of the specification limits. Thus, Kapur and Wang decreased the expected quality losses based on decreasing the variance in the units shipped to the customer by screening the specification limits.

This study releases Kapur and Wang's assumptions. For traditional screw manufacturing, manufacturers can adjust their present processes based on the requirements of their buyers. Moreover, a process mean (μ) rarely equals its target value (y_0) due to the physical constraints of materials or machines. Hence, expressing USL and LSL as $\mu \pm r\sigma$ will be replaced by $y_0 \pm r\sigma$. Using $y_0 \pm r\sigma$ to determine USL and LSL is more reasonable than $\mu \pm r\sigma$.

The expected quality loss for the–nominal–the–best type quality characteristics is presented as

$$L = E[L(Y_t)] = k\left[(\mu_t - y_0)^2 + \sigma_t^2\right], \quad LSL \le y \le USL,$$

where μ_t and σ_t^2 are the mean and variance of a truncated distribution, respectively.

C_p and C_{pk} currently are frequently used in manufacturing. However, C_p does not consider the process mean shifts, hence, does not measure the real process performance (Kane, 1986; Chan et al., 1988; Chan et al., 1990; Kotz and Lovelace, 1998). C_{pk} adds the concept of measuring the process mean according to its function to improves the shortcomings of C_p, but does not distinguish between whether a process is on–target or off–target (Boyles,1991; Kotz and Lovelace, 1998).

Moreover, C_{pm} is proposed to resolve the shortcomings of the previous two PCIs. The C_{pm} is a loss-based index and involves the variation of production items with respect to the target value and the specification limits preset (Hsiang and Taguchi, 1985; Kotz and Lovelace, 1998). Hence, the study utilizes C_{pm} to present the manufacturing ability of a normal process for the–nominal–the–best type quality characteristics.

To resolve applications of PCIs in a non-normal process, many PCIs methods have been proposed, such as considering normalizing transformations (Somerville and Montgomery, 1996–1997), an empirical distribution or a three–parameter or four–parameter distribution (Clements, 1989; Shore, 1998), a modification of the standard definition of PCIs (Pearn et al., 1992) and heuristic methods (Bai and Choi, 1997). Most of these methods are complicated and insensitive or need large samples. Bai and Choi proposed PCIs with weighted variance (WV) based on Choobineh and Branting (1986). Their methods are simple, hence, are easily applied by practitioners. Therefore, the study employs C_{pmw} (Bai and Choi, 1997) to measure a process capability with a skewed distribution.

Kotz and Lovelace (1998, p.38) indicated a 1.33 value for C_p to maintain the standard for process capability. Process variance occurs from time–to–time, shift–to–shift and operator–to–operator when $C_p = 1.33$; a higher standard of $C_p = 1.67$ for machine capability includes machine variation only. Kotz and Johnson (1993) suggested C_p equals 1.33 for existing processes and 1.67 for new processes. Montgomery (2005, p.337) presented some recommended minimum values of the process capability ratio. For example, $C_p = 1.33$ for existing processes, $C_p = 1.50$ for new processes and $C_p = 1.67$ for safety, strength, critical parameter and new processes for two-sided specifications. Finley (1992) indicated the required C_{pk} on all critical supplier processes is 1.33 or higher and a C_{pk} value of 1.67 or higher is preferred. According to suggestions of the proceeding studies, the range of process capability indices is set between 1.33 and 1.67 in this study.

The costs of the adjusted mean (MC) of a process involve the expenses incurred in adjusting machines and tools in order to achieve a process mean close to its target. For the costs of the adjusted standard deviation (DC), the expense of adjustment and management of machines, tool wears, production methods and conditions and training of operators should all be taken into account. The variations of adjusted mean and standard deviation costs are assumed to be an exponential function, not linear relation, since a manufacturer needs to take on many different additional costs to reduce the small ranges between the mean and standard deviation and their targets when the mean and standard deviation are very close to their targets. Generally, an adjusted standard deviation is smaller than an initial process standard deviation (σ_1). When an adjusted process mean and standard deviation equal μ_1 and σ_1, respectively, the phenomenon indicates that the process requires no adjustment. The σ_2 is a minimal standard deviation that is produced from ideal production by a manufacturer. However, when, material or person are limited by physics and physiology, the σ_2 cannot be reduced to be 0. Hence, the value of the adjusted standard deviation is larger than or equal to that of σ_2.

Considering these constraints, the model of the minimum TC for the proposed model can be written as

Minimize ... (1)

$$TC = L + (1-q)SC + IC + e^{\left(1-\left|1-\frac{\mu}{y_0}\right|\right)} MC + \left(e^{\left(\frac{\sigma_1-\sigma}{\sigma_1-\sigma_2}\right)} - 1\right)$$

Subject
$$1.33 \le PCI \le 1.67$$
$$\sigma_2 \le \sigma \le \sigma_1$$
.

Currently, the relationship management between manufacturers and suppliers is an important issue (Fawcett et al., 2007). Companies seek supply chain management or concurrent engineering methodology to increase competition in a market. Fawcett et al. thought continuous improvement expectation in quality should be frequently discussed in negotiations among supply chain members. Manufacturing engineers contribute to design programs with design engineers in the development stage of a new product. Designers have to evaluate whether the manufacturers have the manufacturing capability that can product a new product based on the design specification limits. Furthermore, specification limits can also be designed through the negotiations process of manufacturing engineers and design engineers. If specification limits are designed based on negotiation, the Eq.(3) should add $r_1 \le r \le r_2$ into these constraints.

4. TOTALLY EXPECTED QUALITY LOSS MODELS

Totally expected quality loss models are introduced in the section for normal and Weibull processes. The normal distribution is used in the model since it provides a general distribution for statistical theories and real processes; the model employing a Weibull distribution with three parameters has the capacity to model various practical situations because the distribution can present many different non–normal processes by charging its shape parameters. Moreover, the original data from many processes do not come from zero. Hence, the Weibull distribution with a location parameter can express a shift location of a process.

4.1 Totally expected quality loss model for normal distribution

Assume y_n be a random variable for a normal distribution and $f_n(y_n)$ be a probability density function (pdf) of a normal distribution with mean μ_n and variance σ_n^2. The $f_n(y_n)$ is as follows:

$$f_n(y_n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{\frac{-1}{2}\left(\frac{y_n-\mu_n}{\sigma_n}\right)^2} . \quad -\infty < y_n < \infty, \quad -\infty < \mu_n < \infty, \quad 0 < \sigma_n .$$

Denote $y_{t,n}$ be a random variable for a truncated normal distribution. Here, $f_{t,n}(y_{t,n})$ is a pdf of a truncated normal distribution. Hence, $f_{t,n}(y_{t,n})$ is as follows:

$$f_{t,n}(y_{t,n}) = \frac{1}{q_n} \frac{1}{\sqrt{2\pi}\sigma_n} e^{\frac{-1}{2}\left(\frac{y_n-\mu_n}{\sigma_n}\right)^2}, \quad LSL \le y_w \le USL,$$

where $USL = y_0 + r\sigma_n$ and $LSL = y_0 - r\sigma_n$. The r is a multiple of the standard deviation. The q_n expresses the probability that falls inside specification limits for a normal distribution. Therefore, q_n equals $\Phi(u) - \Phi(v)$ for a normal distribution.

Hence, the functions of $\mu_{t,n}$ and $\sigma_{t,n}^2$ are showed as

$$\mu_{t,n} = \int_{LSL}^{USL} y_n f_{t,n}(y_n) dy_n = \int_{y_0-r\sigma_n}^{y_0+r\sigma_n} y_n f_{t,n}(y_n) dy_n = \mu_n + \frac{\sigma_n}{q_n}[(\phi(v) - \phi(u))],$$

and

$$\sigma_{t,n}^2 = \int_{LSL}^{USL} (y_n - \mu_{t,n})^2 f_{t,n}(y_n) dy_n = \int_{y_0 - r\sigma_n}^{y_0 + r\sigma_n} (y_n - \mu_{t,n})^2 f_{t,n}(y_n) dy_n$$

$$= \sigma_n^2 \left[1 + \frac{1}{q_n} (v\phi(v) - u\phi(u)) - \frac{1}{q_n^2} (\phi(v) - \phi(u))^2 \right].$$

The expected quality loss for the–nominal–the–best type quality characteristics is presented as

$$L_n = E[L(Y_{t,n})] = k[(\mu_{t,n} - y_0)^2 + \sigma_{t,n}^2], \quad LSL \le y_n \le USL,$$

In order to reduce expected quality losses, an adjustment of μ_n and σ_n^2 should be considered. σ_n^2 can be reduced according to an improvement in a present process and decreasing the variance of the units shipped to the customer by adjusting specification limits. For a process mean, the method of improvement is the adjustment of the difference between μ_n and y_0.

This study utilizes C_{pm} to measure the capabilities of a process with normal distribution. The function of C_{pm} is expressed as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma_n^2 + (\mu_n - y_0)^2}} = \frac{r\sigma_n}{3\sqrt{\sigma_n^2 + (\mu_n - y_0)^2}}.$$

According to Eq.(1), the totally expected quality loss model for normal distribution is as follows:

Minimize ... (2)

$$TC_n = L_n + (1 - q_n)SC + IC + e^{\left(1 - \left|1 - \frac{\mu_n}{y_0}\right|\right)} MC + \left(e^{\left(\frac{\sigma_1 - \sigma_n}{\sigma_1 - \sigma_2}\right)} - 1 \right) DC$$

Subject

$$1.33 \le C_{pm} \le 1.67$$
$$\sigma_2 \le \sigma_n \le \sigma_1.$$

If specification limits are designed based on a negotiation of manufacturing and design engineers, then Eq.(2) should add $r_1 \le r \le r_2$ into the constraints.

4.2 Totally expected quality loss model for Weibull distribution

Assume y_w be a random variable for a Weibull distribution. Then $f_w(y_w)$ is a pdf of a Weibull distribution with shape parameter (a), scale parameter (λ) and location parameter (ξ). The $f_w(y_w)$ is as follows:

$$f_w(y_w) = \frac{a}{\lambda} \left(\frac{y_w - \xi}{\lambda} \right)^{a-1} e^{-\left(\frac{y_w - \xi}{\lambda}\right)^a}, \quad \xi \le y_w < \infty, \quad 0 < a, \quad 0 < \lambda.$$

According to Wyckoff, Bain and Engelhardt (1980), $y_{w,1}$ is used to estimate ξ, where $y_{w,1}$ is the first-order statistic in a Weibull distribution of random sample size n. The $\hat{a}$ and $\hat{\lambda}$ are estimators of a and λ, respectively and are follows as:

$$\hat{a} = \frac{2.989}{\log(y_{w,k} - y_{w,1}) - \log(y_{w,h} - y_{w,1})} \quad \cdots \quad (3)$$

and

$$\hat{\lambda} = \frac{\bar{y}_w - y_{w,1}}{\Gamma(1 + 1/\hat{a})},$$

where $\log(\cdot)$ is natural logarithmic function and $y_{w,k}$ and $y_{w,h}$ are the 94th and 17th sample percentiles, respectively.

Let $y_{t,w}$ denote a random variable for a truncated Weibull distribution and $f_{t,w}(y_{t,w})$ be a pdf of a truncated Weibull distribution. Hence, $f_{t,w}(y_{t,w})$ is as follows:

$$f_{t,w}(y_{t,w}) = \frac{1}{q_w}\frac{a}{\lambda}\left(\frac{y_{t,w}-\xi}{\lambda}\right)^{a-1} e^{-\left(\frac{y_{t,w}-\xi}{\lambda}\right)^a}, \quad LSL \le y_w \le USL, \quad 0 < a, \quad 0 < \lambda,$$

where, $USL = y_0 + w\sigma_w$ and $LSL = y_0 - w\sigma_w$ ($\ge \xi$). The w is a multiple of the standard deviation. The q_w expresses the probability that falls inside specification limits for a Weibull distribution. Therefore, q_w equals $e^{-\left(\frac{LSL-\xi}{\lambda}\right)^a} - e^{-\left(\frac{USL-\xi}{\lambda}\right)^a}$ for a Weibull distribution.

The functions of the mean ($\mu_{t,w}$) and variance ($\sigma_{t,w}^2$) of the truncated Weibull distribution are shown as

$$\mu_{t,w} = \int_{LSL}^{USL} y_w f_{t,w}(y_w)dy_w = \xi + \frac{\lambda}{q_w}\Gamma\left(1+\frac{1}{a},\left(\frac{LSL-\xi}{\lambda}\right)^a,\left(\frac{USL-\xi}{\lambda}\right)^a\right),$$

and

$$\begin{aligned}\sigma_{t,w}^2 &= \int_{LSL}^{USL}(y_w-\mu_{t,w})^2 f_{t,w}(y)dy_w \\ &= \xi^2 + 2\xi\frac{\lambda}{q_w}\Gamma\left(1+\frac{1}{a},\left(\frac{LSL-\xi}{\lambda}\right)^a,\left(\frac{USL-\xi}{\lambda}\right)^a\right) \\ &+ 2\xi\lambda\frac{\lambda^2}{q_w}\Gamma\left(1+\frac{2}{a},\left(\frac{LSL-\xi}{\lambda}\right)^a,\left(\frac{USL-\xi}{\lambda}\right)^a\right) - \mu_{t,w}^2,\end{aligned}$$

where $\Gamma(\bullet,\bullet,\bullet)$ represents an incomplete gamma function.

The expected quality loss for the Weibull distribution is presented as

$$L_w = E\left[L(Y_{t,w})\right] = k\left[(\mu_{t,w}-y_0)^2 + \sigma_{t,w}^2\right], \quad LSL \le y_{t,w} \le USL,$$

This study utilizes C_{pmw} to measure capabilities of a process with Weibull distribution. The function of this C_{pmw} is based on the expression of Bai and Choi (1997) and is shown as follows:

$$C_{pmw} = \frac{USL-LSL}{6\sqrt{\sigma_w^2+(\mu_w-y_0)^2}}\frac{1}{\sqrt{1+|1-2Pw|}} = \frac{r\sigma_w}{3\sqrt{\sigma_w^2+(\mu_w-y_0)^2}}\frac{1}{\sqrt{1+|1-2Pw|}},$$

where Pw is the probability that a random variable of Weibull distribution falls below a target. Therefore, $Pw = P\{y_w \le y_0\}$.

According to Eq.(1), the totally expected quality loss model for normal distribution is as follows:

Minimize ... (4)

$$TC_w = L_w + (1-q_w)SC + IC + e^{\left(1-\left|1-\frac{\mu_w}{y_0}\right|\right)}MC + \left(e^{\left(\frac{\sigma_1-\sigma_w}{\sigma_1-\sigma_2}\right)}-1\right)DC$$

Subject $\begin{array}{l}1.33 \le C_{pmw} \le 1.67 \\ \sigma_2 \le \sigma_w \le \sigma_1\end{array}$.

If specification limits are designed based on a negotiation between manufacturing and design engineers, the Eq.(4) should add $r_1 \le r \le r_2$ into its constraints.

The Eq.(2) and Eq.(4) are a nonlinear optimization problem. This study calculates constrained nonlinear optimizations using Mathematica 6.0.

5. INDUSTRY APPLICATION

Two examples are presented to illustrate the minimum totally expected quality losses considering process capability indices and making adjustments of the process mean and standard deviation. The first example is a real case of hexagon head cap screws, and the second is adopted from Chang and Lu (1994).

5.1 A hexagon head cap screw

This example herein considers the lengths of a hexagon head cap screw. The product is DIN933 $M5 \times 20$, where

DIN933 is a German Industry Standard, *M* is a metric system, 5 is that a shaft diameter of 5 millimeters (mm) and 20 is the length of 20 mm. Figure 1 demonstrates a picture of a hexagon head cap screw. Figure 2 depicts the bilateral tolerances of the diameter and length of a hexagon head cap screw. The material used to manufacture the screw is carbon steel. Data in this example are obtained from a world-class company in Taiwan that manufactures bolts, nuts and tapping screws and sells them to electric machinery, instrument and motor industries.

Figure 1. A picture of a hexagon head cap screw

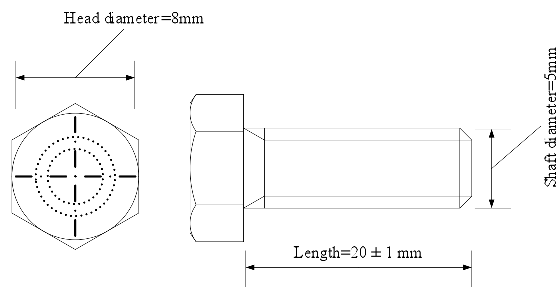

Figure 2. A diagram of bilateral tolerances for a hexagon head cap screw

The processes used by the screw manufacturing industry involves cutting line material, heading, head trimming, slot cutting, thread rolling and read made good. The hexagon head cap screw is used generally in motor industries for applications, such as a fastening the doors of vehicles. Hence, a motor manufacturer expects security of the mechanism and requests the screw supplier in Taiwan to provide high quality products. The specification limits of the screw length are target 20 +/− 1.0 mm. Furthermore, to retain the quality of the screw, the value of the process capability index must fall between 1.33 and 1.67. The supplier can apply to the motor manufacturer for the adjusted costs of the process after the screws are accepted as meeting the requirements. To estimate the mean and standard deviation of the screw length, 120 data values are selected at 8-hours intervals by a vernier caliper. Table 1 lists the lengths of the screw data.

Table 1. A set of hexagon head cap screw data

20.1	18.6	18.0	19.5	19.7	19.4	18.1	19.5	19.2	18.4
19.2	19.0	19.6	19.0	18.9	20.0	19.1	19.6	20.2	17.8
20.2	19.4	18.4	19.1	20.1	19.4	20.1	19.5	19.0	19.4
18.8	18.0	19.8	18.5	19.1	19.4	18.4	19.4	19.1	19.1
17.3	17.1	19.3	19.1	18.0	18.4	19.6	19.9	18.8	17.8
19.2	19.1	19.6	18.5	19.7	18.4	19.5	19.1	20.1	18.3
18.2	19.0	17.6	19.4	19.2	19.8	18.4	19.4	18.9	20.9
20.0	18.9	19.2	18.4	19.8	18.6	18.8	18.7	18.4	19.6
19.9	19.3	19.2	19.0	19.7	17.3	18.7	19.6	18.9	19.5
19.3	17.0	18.8	19.1	18.3	19.0	18.2	19.5	19.4	17.8
18.1	18.1	20.2	19.2	19.5	19.6	19.3	18.6	19.5	20.6
18.7	19.6	19.5	18.4	18.9	18.8	18.3	19.2	18.3	19.2

Figure 3 demonstrates that the original data in the histogram are almost normally distributed, and they satisfy the normal assumption according to the Kolmogorov-Smirnov test at a 5% level of significance. This study estimates that original population mean (μ_1) and standard deviation (σ_1) by the sample mean ($\bar{y}_n$) and sample standard deviation (s_n) respectively. Hence, the current $\bar{y}_n = 19.0350$ and $s_n = 0.7256$. The current process mean is smaller than the target within the specification limits. This phenomenon indicates that the process mean does not meet the target. The current process capability is 0.2761 based on C_{pm}, a value cannot satisfy the requirements of the motor industry. Therefore, the process of the screw should be adjusted and the screws screened.

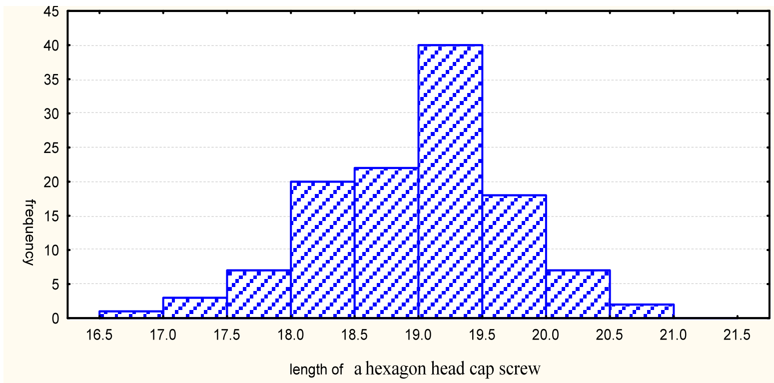

Figure 3. A histogram of the original data of a hexagon head cap screw

According to loadings of the machines and persons, the supplier examined the ideal σ_2 that is 0.05. The supplier also knows *k*=8.0, DC=1.0, IC=0.1, MC=1.0 and SC=2.0. Table 2 lists the values of the optimal *r*, μ_n, σ_n, C_{pm}, minimum TC, and specification limits based on the calculation of Eq.(2). The optimal *w*=3.9936, $\mu_n = 20.0085$, $\sigma_n = 0.2013$, $C_{pm} = 1.33$, LSL=19.1963, USL=20.8037 and the minimum TC= 4.3145. When μ_n and σ_n are adjusted to be 20.0085 and 0.2013, respectively, this process has minimum total losses and the process capability and the optimal specification limits also fulfill the requirements of the motor manufacturer. The adjustment costs (DC+MC) are 3.89, hence, the motor manufacturer must pay the compensation to the screw supplier. The minimum total losses represent wins on both the parts of the supplier and buyer since the motor manufacturer just pays the minimal costs for getting high quality screws and the screw supplier knows how to adjust the mean and standard deviation of the process under the minimal costs to improve the quality of the screw.

When specification limits are designed based on a negotiation between the supplier's manufacturing engineers and motor manufacturer designers, both engineers agree $4.2 \le r \le 4.5$. Hence, the optimal *r*=4.2, $\mu_n = 20.0085$, $\sigma_n = 0.2012$, $C_{pm} = 1.3988$, LSL=19.1551, USL=20.8449 and the minimum TC= 4.3146. The Eq.(2) can be used in the design stage of a new product since by considering the negotiation between designers and manufacturing engineers. Consequently, the model can carry out the basic conception of concurrent engineering (design for manufacturability).

Table 2. The optimal parameters, C_{pm}, TC and specifications for length of a hexagon head cap screw. Upper entry is for global optimum; lower entry is for negotiation

r	μ_n	σ_n	C_{pm}	TC	Specification limits
3.9936	20.0085	0.2013	1.3300	4.3145	[19.1963, 20.8037]
4.2000	20.0085	0.2012	1.3988	4.3146	[19.1551, 20.8449]

5.2 Thickness of an oil seal

This example is adopted from Chang and Lu (1994), who collected 65 data values for the thickness of an oil seal. Chang and Lu (1994) indicated that the distribution of the data was unknown and non-normal. Figure 4 demonstrates the original data of thickness of an oil seal in the histogram. The normal assumption was rejected according to the Kolmogorov-Smirnov test. This study employs a Weibull distribution with three parameters to estimate this skewed distribution. The current mean and standard deviation of the skewed distribution were 2.0215 and 0.2190. According to

Eq.(3), the $y_{w,1}$, $\hat{a}$ and $\hat{\lambda}$ are 1.6, 2.1561 and 0.4759, respectively. The current process mean is larger than the target within specification limits, hence, the process mean is unequal with the target. The current process capability is 0.7098 based on C_{pmw} with P*w*=0.5692 that does not fall between 1.33 and 1.67. Hence, the process should be improved in order to satisfy high process capability.

Assume *k*=5.0, DC=3.0, IC=0.1, MC=2.0, SC=3.0 and $\sigma_2 = 0.01$. Table 3 lists the values of the optimal parameters, minimum TC, and specification limits based on the calculation of Eq.(4). The optimal *r*=4.0001, *a*=4.5, $\lambda = 0.4345$, $\mu_w = 1.9965$, $\sigma_w = 0.1000$, $C_{pmw} = 1.3301$, LSL=1.6, USL=2.4 and the minimum TC= 7.8793. The adjustment costs (DC+MC) are 7.7293. To get the minimal TC and satisfy the requirements of process capability, the current process mean and standard deviation should be adjusted to be 1.9965 and 0.1, respectively. Hence, the skewed process is the Weibull distribution with *a*=4.5, $\lambda = 0.4345$ and $\xi = 1.6$.

When specification limits are designed based on a negotiation between manufacturing engineers and designers or consumers, the optimal *a*=4.9495, $\lambda = 0.4190$, $\mu_w = 1.9845$, $\sigma_w = 0.0889$, $C_{pmw} = 1.4111$, LSL=1.6, USL=2.4 and the minimum TC= 8.1263 based on *r*=4.5.

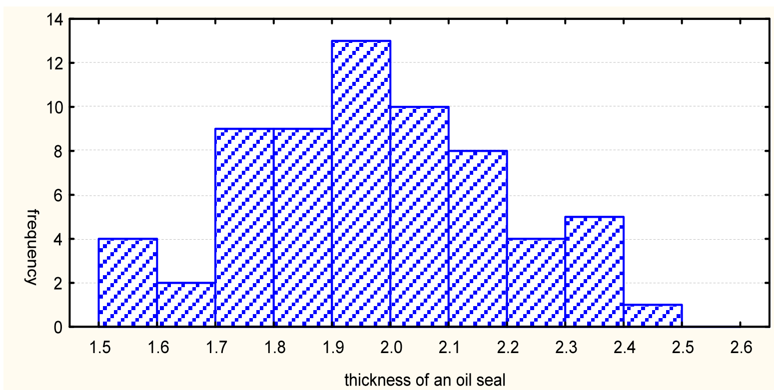

Figure 4. A histogram of the original data of thickness of an oil seal

Table 3. The optimal parameters, C_{pmw}, TC and specifications for thickness of an oil seal. Upper entry is for global optimum; lower entry is for negotiation

r	a	λ	μ_w	σ_w	C_{pmw}	TC	Specification limits
4.0001	4.5000	0.4345	1.9965	0.1000	1.3301	7.8793	[1.600, 2.400]
4.5000	4.9495	0.4190	1.9845	0.0889	1.4111	8.1263	[1.600, 2.400]

6. CONCLUSIONS

Considering normal and skewed processes, this study proposes modified totally expected quality loss model using process capability indices for normal and Weibull distributions. The modified model for a normal distribution utilizes C_{pm} to evaluate the process capabilities and discriminate between on-target and off-target processes. In the real world, many processes have non-normal distribution. The three–parameter Weibull distribution can assist practitioners to estimate a skewed process. The modified model for the Weibull distribution employs C_{pmw} to measure process capabilities of a skewed process. In minimal totally expected quality losses, these models decide the optimal adjusted process mean, adjusted process standard deviation and specification limits to satisfy the requirement of buyers or consumers. For suppliers, these models show valuable information of the optimal adjustment of a process mean and a process standard deviation to help them make production decisions. Furthermore, for buyers or consumers, the models also guarantee that they can get high quality products based on the payment of minimal adjustment costs. These models also determine the negotiated optimal specification limits and process adjustments and carry off the concept of design for manufacturability.

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BIOGRAPHICAL SKETCH

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