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AGGREGATE PRODUCTION PLANNING WITH FUZZY DEMAND AND VARIABLE SYSTEM CAPACITY BASED ON THEORY OF CONSTRAINTS MEASURES

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Aggregate Production Planning (APP) model with fuzzy demand and variable system capacity is proposed in this research for a practical APP problem. A conventional APP problem assumes crisp market demands and also limited capacity by fixed hardware. In the proposed model, the difficulty in estimation crisp demands is relaxed by using fuzzy demand which also increases the flexibility of estimation and obtains the better production plan that can increase profit. The new approach to handle the fuzzy demand by integrating the possibility level of demand is proposed. Moreover, the limitation of system capacity is resolved by allowing additional investment in small machines and equipment. This investment can increase the necessary production capacity and eliminate the bottleneck of the system. Three performance measures, based on the Theory of Constraints (TOC) concept, which are currently used in many organizations, are used to evaluate performance of the model. It is found that the proposed model can generate higher performance than conventional APP models.

Significance: In medium range capacity planning, market demand is usually uncertain in nature and the limitation of the production capacity is not only limited by workforce level but also limited by hardware or equipment. The fuzzy APP model proposed herein is more flexible and realistic approach to solve APP problems than the conventional models based on TOC measures.

Keywords: Aggregate production planning; theory of constraints; fuzzy demands; trapezoidal fuzzy number; possibility theory.

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1. INTRODUCTION

Aggregate production planning (APP) is a problem of deciding how to vary production capacity, keep stock, and subcontract to satisfy a seasonal demand in the most effective way. It is medium-term planning whereby its planning horizon is usually from 6 to 18 months (Techawiboonwong and Yenradee, 2003). APP links operations with strategies and plays a key role in enterprise resource planning and organizational integration (Singhal and Singhal, 2006). Since Holt, Modigliani, Muth and Simon proposed the HMMS rule in the mid 1950's (Hassmann and Hess, 1960; Deckro and Hebert, 1984), researchers have developed numerous models to solve the APP problem. There are a number of approaches to APP: mathematically optimal procedures (Hassmann and Hess, 1960; Masud and Hwang, 1980; Deckro and Hebert, 1984; Oliff, *et al.*, 1989), simulation and search methods (Goodman, 1973), and heuristic decision rules (Goodman, 1973; Eilon, 1975; Cha and Hwang, 1996). Among the optimal models, Linear Programming (LP) has received the widest acceptance (Fung, *et al.*, 2003). However, most LP models assume that all required input data can be uniquely determined. These models might also have some practical difficulties since a Decision Maker (DM) frequently has insufficient or unobtainable information on how to specify crisp market demands. These data are typically uncertain in nature.

In dealing with uncertainty, stochastic models and fuzzy models are used. Stochastic models can deal with uncertainty but they are hard to solve and statistical estimations prove inefficient because of the lack of statistical observations (Rommelfanger, 1989). They can do little to help decision making in practical scenarios (Fung, *et al.*, 2003). Some other forms of uncertainties are based on fuzzy set theory. Zimmermann (1976) first introduced fuzzy set theory into the conventional LP problem. Since then, fuzzy mathematical programming has developed into several fuzzy optimization methods for solving APP problems (Gen *et al.*, 1992; Lai and Wang, 1996; Wang and Fang, 2000; Wang and Fung, 2001; Wang and Fang, 2004; Wang and Liang, 2005). These methods use grade of membership functions to indicate subjective degrees of satisfaction within given tolerances. Another main approach in dealing with fuzzy models is the possibility theory, which was introduced by L.A. Zadeh in 1978. A grade of possibility is used to indicate the subjective or objective degree of the occurrence of an event (Lai and Hwang, 1996).

Although, there are a number of APP models, most of them assume that the production capacity can be varied by managing workforce level under fixed and limited capacity of hardware. In practice the capacity limited by the hardware or equipment can be relaxed to a certain extent by additional investment in small machines, material handling, equipment, jigs, or other tooling. The consideration of additional investment is very useful for APP decisions to increase capacity instead of accepting lost sales in some periods. Capacity decisions affect product lead times, customer responsiveness, operating costs, and a firm's ability to compete. Inadequate capacity can lose customers and limit growth. Excess capacity can drain a company's resources and prevent investments in more lucrative ventures. When, how much, and in what form to alter capacity are critical decisions. This issue does not exist in conventional APP models.

Revenue, cost or profit are generally used to evaluate performance of APP models. However, it was suggested by Goldratt (1990) that instead of measuring by cost, factories should evaluate their performances by throughput. This suggestion of Goldratt is called the Theory of Constraints (TOC). From the concept of TOC three suitable measures, namely, throughput, inventory and operating expense are proposed to be used as measurements for an organization. These three measures can be tied to the main global measures such as net profit and return on investment. Moreover, they can also be transformed to productivity and inventory turnover. These are very useful measures for organizational management. So, these three measures are incorporated to formulate and evaluate APP models.

2. THREE MEASURES BASED ON TOC PHILOSOPHY

According to TOC philosophy, the throughput, inventory, and operating expense should be used to evaluate the performance of any profitable company.

- Throughput (*TP*): The rate at which the system generates money through sales.
- Inventory (*IN*): The total money invested in purchasing the items the system intends to sell.
- Operating Expense (*OE*): The total money the system spends in turning Inventory into Throughput (Stein, 1997).

Throughput is calculated by subtracting the raw material cost from the sales. Inventory includes any physical inventories such as work in process, finished goods and raw materials, and also includes tools, buildings, capital equipment and furnishings. Note that, conventionally inventory means only work in process, finished goods and raw materials. Operating expense includes expenditures such as direct and indirect labor, supplies, outside contractors and interest payments.

The three measures mentioned previously can be linked to the global measures of the production system. Net profit equals Throughput minus Operating Expense ($NP=TP-OE$) and Return on Investment equals Net Profit divided by Inventory ($RI = NP/IN$). Productivity is Net Profit divided by Operating Expense ($PR = NP/OE$). Inventory Turnover are Throughput divided by Inventory ($IT = TP/IN$) (Klusewitz and Rerick, 1996; Woeppel, 2001).

Redefining accounting concepts, Goldratt's approach called for a change in emphasis. Managers should aim to raise throughput while simultaneously reducing inventory and operational expense. However, scope for reducing the latter two is essentially limited, since they must be maintained at some minimal level to avoid a reduction of throughput. These can be considered as constraints in the proposed model. A constraint is the weakest link in the system chain (Dettemer, 1997). There are three types of constraints: policy (paradigm), resource (physical) and material (Woeppel, 2001). In addition, financial constraints are also critical for practical problems (Fung, *et al.*, 2003).

2.1 Notations of Variables, Parameters and Constants

In order to formulate the problem mathematically, the notations below are introduced.

Notations of variables:

$P_{i,t}$: regular time production quantity of product *i* in period *t* (units);

$O_{i,t}$: overtime production quantity of product *i* in period *t* (units);

$S_{i,t}$: subcontracting production quantity of product *i* in period *t* (units);

$I_{i,t}$: inventory quantity of product *i* at the beginning of period *t* (units);

W_t : total workforce level in period *t* (man-day);

H_t: hired workforce at the beginning of period *t* (man-days);

L_t : laid-off workforce at the beginning of period *t* (man-days);

$B_{i,t}$: backorder quantity of product *i* at the beginning of period *t* (units);
E_t : cumulative investment in tools and equipment in period *t* (currency unit);
A_t : additional investment in tools and equipment at the beginning of period *t* (currency unit);
M_t : available machine or system capacity in period *t* (machine-hour);
$D'_{i,t}$: actual demand of product *i* in period *t* which is taken into account of possibility (units);
$d'_{i,t}$: effective demand of product *i* in period *t* (units).

Notations of parameters and constants:
r_i : price of product *i* per unit (currency unit/unit);
c_{mi} : material cost of product *i* per unit (currency unit/unit);
c_{OTi} : overtime labor cost for product *i* (currency unit/man-hour);
c_{Ii}, c_{Bi} : inventory holding cost and backordering cost of product *i* per period per unit (currency unit/unit);
c_{Si} : subcontracting cost of product *i* per unit (currency unit/unit);
c_{Wt}, c_{Ht}, c_{Lt} : regular time wages, hiring cost, and firing cost in period *t* (currency unit/man-days);
a_i : labor time needed for product *i* per unit (man-hour/unit);
b_i : machine time needed for product *i* per unit (machine-hour/unit);
δ : regular time per worker (man-hour/man-day);
ϕ_{Wt} : fraction of regular workforce available for overtime in period *t*;
α : fraction of regular workforce allowed for variation in period *t*;
γ : fraction of increment from initial stage of machine or system capacity per currency unit of investment in tools and equipment;
$W_{t\max}$: maximum available workforce in period *t* (man-days);
$M_{t\max}$: maximum available machine or system capacity in period *t* (machine-hour);
$MC_{t\max}$: maximum possible available machine or system capacity in period *t* if tools and equipment are additionally invested in (machine-hour);
$K_{t\max}$: warehouse capacity (units);
$F_{t\max}$: available financial resource in period *t* (currency unit);
OH_t : fixed overhead cost in period *t* (currency unit);
M_0 : initial available machine or system capacity time (machine-hour);
E_0 : initial investment in tools and equipment (currency unit);
$\widetilde{D}_{i,t} = \left(\underline{D}_{i,t}, \overline{D}_{i,t}, \underline{\beta}_{D_{i,t}}, \overline{\beta}_{D_{i,t}}\right)$ fuzzy demands of product *i* in period *t*;
$D_{i,t}$: crisp demand of product *i* in period *t* which is evaluated to be a real number of interval $\left[\underline{D}_{i,t}, \overline{D}_{i,t}\right]$ (units).

2.2 Three Measures Based on TOC Philosophy

Throughput is total revenue minus purchased material cost. The throughput (*TP*) can be mathematically represented by equation (1).

$$TP = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i D_{i,t} - \sum_{i=1}^{N} r_i B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}\left(P_{i,t} + O_{i,t} + S_{i,t}\right) \quad \ldots \quad (1)$$

The first two terms represent total sales revenue based on total demands and lost sales at the end of the planning horizon. The last term represents the material cost, which includes material cost required by regular time and overtime production, and by subcontracting.

Inventory is defined as "the total money the system invests" including the money invested in raw materials, parts, and all assets (buildings, equipment, fixtures, etc.) (Woeppel, 2001). Inventory in the sense of TOC is different from the conventional meaning, because it includes not only raw materials, WIP, and finished products but also all assets.

Assets that can be considered in the APP, medium-range planning, are small machines, material handling, cutting equipment, jigs, fixtures and other tools. Buildings and large machines is considered in the strategic or long-range planning. Investment in tools and equipment can increase machine or system capacities and reduce bottleneck. Average inventory investment (*IN*) in terms of TOC is represented by equation (2). The first term represents the investment in raw materials while the second term represents the investment in tools and equipment.

$$IN = 1/T\left[\sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi} I_{i,t} + \sum_{t=1}^{T} E_t\right] \quad \ldots \quad (2)$$

In conventional APP models, inventory level of products is normally controlled by constraints. However, investment in tools and equipment has not been included in any APP model. In spite of this investment is one kinds of cost for changing capacity.

Operating expense is defined as the total money spent to turn inventory into throughput. It includes all direct and indirect payroll expense, supplies and overheads. All expenses related to time are operating expenses. Operating expense (*OE*) is represented by equation (3). It includes all costs related to labor, overtime, inventory holding, backordering, subcontracting and fixed overheads.

$$OE = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(c_{Wt} W_t + c_{Ht} H_t + c_{Lt} L_t + OH_t\right) \quad \ldots \quad (3)$$

3. FUZZY AGGREGATE PRODUCTION PLANNING (FAPP) MODEL

In this section, the Fuzzy Aggregate Production Planning (FAPP) model for uncertain demands and variable capacity is developed. The DM must determine the best decisions for each product (*i*) of all *N* products according to the adjustable capacity for meeting production requirements for each time period (*t*) of the planning horizon (*T*).

3.1 APP Model with Crisp Demands and Variable Capacity

Conventionally, revenue, cost or profit function is selected to be the objective function of APP problems. Among these objective functions, the profit function is the most preferable (Phruksaphanrat *et al*., 2006). So, the Net Profit (*NP*) is used as the objective function of the proposed APP model. Net Profit (*NP*) is Throughput (*TP*) – Operating Expense (*OE*). Two measures of TOC are included in the profit function. The remaining measure is inventory, which is also necessary to be considered. Inventory in the sense of TOC means all money the system invests including also the investment in tools and equipment. It is included as constraints in the model.

In case that demands can be precisely estimated ($D_{i,t}$) the APP model with variable capacity can be constructed as:

Maximize *NP,*
subject to

$$NP = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i D_{i,t} - \sum_{i=1}^{N} r_i B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}\left(P_{i,t} + O_{i,t} + S_{i,t}\right) - \left[\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(c_{Wt} W_t + c_{Ht} H_t + c_{Lt} L_t + OH_t\right)\right] \quad \ldots \quad (4)$$

$$W_t = W_{t-1} + H_t - L_t, \quad \forall t, \quad \ldots \quad (5)$$

$$W_t \le W_{t\max}, \quad \forall t \quad \ldots \quad (6)$$

$$\sum_{i=1}^{N} a_i P_{i,t} \le \delta W_t, \quad \forall t \quad \ldots \quad (7)$$

$$\sum_{i=1}^{N} a_i O_{i,t} \le \delta \phi_{Wt} W_t, \quad \forall t \quad \ldots \quad (8)$$

$$H_t + L_t \le \alpha W_t, \quad \forall t \quad \ldots \quad (9)$$

$$\sum_{i=1}^{N} I_{i,t} \le K_{t\max}, \quad \forall t, \quad \ldots \quad (10)$$

$$I_{i,t-1} - B_{i,t-1} + P_{i,t} + O_{i,t} + S_{i,t} - I_{i,t} + B_{i,t} = D_{i,t}, \quad \forall i, \forall t, \quad \ldots \quad (11)$$

$$\sum_{i=1}^{N} b_i (P_{i,t} + O_{i,t}) \le M_t, \quad \forall t, \qquad \text{... (12)}$$

$$E_t = E_{t-1} + A_t, \quad \forall t \qquad \text{... (13)}$$

$$M_0 + \gamma(E_t - E_0) = M_t, \quad \forall t \qquad \text{... (14)}$$

$$M_t \le MC_{t\max}, \quad \forall t, \qquad \text{... (15)}$$

$$\sum_{i=1}^{N} \left[c_{mi}(P_{i,t} + O_{i,t} + S_{i,t}) + c_{OTi} a_i O_{i,t} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right] +$$
$$c_{Wt} W_t + c_{Ht} H_t + c_{Lt} L_t + A_t + OH_t \le F_{t\max}, \quad \forall t \qquad \text{... (16)}$$

$$W_t, H_t, L_t, A_t, E_t, M_t, P_{i,t}, O_{i,t}, I_{i,t}, S_{i,t}, B_{i,t} \ge 0, \quad \forall i, \forall t. \qquad \text{... (17)}$$

Constraint (4) represents the net profit (*NP*) which is the Throughput (*TP*) minus the Operating Expense (*OE*). Constraints (5)-(6) are constraints related to workforce level in period *t*. The regular time and overtime production should not be greater than the available labor capacity as shown in equations (7) and (8) (Masud and Hwang, 1980; Gen *et al.*, 1992). The variation of workforce level in each period should not exceed the permitted level based on a company's policy as shown in equation (9). The number of products in the warehouse in each period should not exceed warehouse capacity as shown in equation (10) (Wang and Liang, 2005). The production-inventory balance constraint is shown in equation (11). The total regular time and overtime production should not be greater than the system capacity as shown in equation (12). Conventionally, system capacity in APP models is assumed to be a constant value but it actually can be increased by adding tools and equipment. A cumulative investment in period *t* is equal to cumulative investment in the previous period plus additional investment in period *t*, as shown in equation (13). Furthermore, the system capacity is increased when investment in tools and equipment is increased as shown in equation (14), but cannot exceed the maximum possible capacity, as shown in equation (15). Finally, the required financial resource should not exceed that available as shown in equation (16).

In the proposed model a crisp demand ($D_{i,t}$) is assumed to be an actual demand ($D'_{i,t}$). The model can also be applied to the case where the crisp demand ($D_{i,t}$) is evaluated to be a real number of interval $\left[\underline{D}_{i,t}, \overline{D}_{i,t}\right]$ by adding the following constraint in the model.

$$\underline{D}_{i,t} \ge D_{i,t} \ge \overline{D}_{i,t}, \quad \forall i, \forall t. \qquad \text{... (18)}$$

3.2 FAPP Model with Fuzzy Demands and Variable Capacity

In real industrial situations the demand is not constant. It can vary within a reasonable range with different possibility level, for example, from 95 to 105 units. If a management team believes that the demand is 95 units and sets the forecasted demand as such at 95 units, sales and production sections, having achieved the target of 95 units, will slow down their efforts. As a result, the demand of 95 units will be satisfied. However, if the management team believes that the demand is 105 instead of 95 units, their staffs will make additional efforts to satisfy the target of 105 units. Moreover, at different value of demand, the level of possibility is also different. For example the demand can reduce more to 90 units or increase more to 115 units but the level of possibility is lower than demand quantity between 95 to 105 units. This example shows that the demand is a fuzzy number and the DM can choose the value of demand (within a reasonable range) that is the most beneficial for the organization.

Generally, demands are imprecise in nature so fuzzy demands are applied to constraints (4) and (11) in the model in section 3.1. Let fuzzy demands be a trapezoidal fuzzy number, denoted by $\tilde{D}_{i,t} = \left(\underline{D}_{i,t}, \overline{D}_{i,t}, \underline{\beta}_{D_{i,t}}, \overline{\beta}_{D_{i,t}}\right)$. $\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}$ is the most pessimistic demand quantity of product *i* in period *t*. Demand quantities between $\underline{D}_{i,t}$ and $\overline{D}_{i,t}$ are the most probable for product *i* in period *t*. $\overline{D}_{i,t} + \overline{\beta}_{D_{i,t}}$ is the optimistic demand quantity of product *i* in period *t*. The membership function $\mu_{\tilde{D}_{i,t}}(D'_{i,t})$ expresses the possibility measurement of fuzzy demands according to the degree of the occurrence of an event (Zadeh, 1987; Wang and Fang, 2001). Given that $D'_{i,t}$ is the decision variable representing the actual demand of product *i* in period *t*. For example, it may be demand quantities between $\underline{D}_{i,t}$ and $\overline{D}_{i,t}$ with the highest possibility, between $\overline{D}_{i,t} + \overline{\beta}_{D_{i,t}}$ and $\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}$ with the lowest possibility, and between $\overline{D}_{i,t} + \overline{\beta}_{D_{i,t}}$ and and between $\underline{D}_{i,t}$ and $\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}$ with a possibility level between zero and one. The membership function $\mu_{\tilde{D}_{i,t}}(D'_{i,t})$ can be represented in equation (19) and shown in Figure 1.

$$\mu_{\tilde{D}_{i,t}}(D'_{i,t}) = \begin{cases} 1-\dfrac{\underline{D}_{i,t}-D'_{i,t}}{\underline{\beta}_{D_{i,t}}}, & \underline{D}_{i,t}-\underline{\beta}_{D_{i,t}} \le D'_{i,t} \le \underline{D}_{i,t} \\ 1, & \underline{D}_{i,t} \le D'_{i,t} \le \overline{D}_{i,t} \\ 1-\dfrac{D'_{i,t}-\overline{D}_{i,t}}{\overline{\beta}_{D_{i,t}}}, & \overline{D}_{i,t} \le D'_{i,t} \le \overline{D}_{i,t}+\overline{\beta}_{D_{i,t}} \\ 0, & \text{else.} \end{cases} \quad \ldots \quad (19)$$

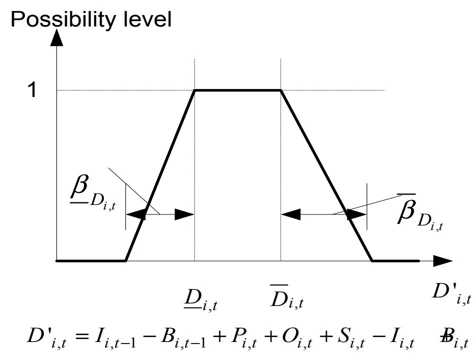

Figure 1. Trapezoidal fuzzy demand $\tilde{D}_{i,t} = \left(\underline{D}_{i,t}, \overline{D}_{i,t}, \underline{\beta}_{D_{i,t}}, \overline{\beta}_{D_{i,t}}\right)$.

There are four types of demands in this paper. Firstly, $\tilde{D}_{i,t}$ is a fuzzy demand represented by a trapezoidal fuzzy number $\left(\underline{D}_{i,t}, \overline{D}_{i,t}, \underline{\beta}_{D_{i,t}}, \overline{\beta}_{D_{i,t}}\right)$. Secondly, $D_{i,t}$ is a crisp demand represented by a range between $\left[\underline{D}_{i,t}, \overline{D}_{i,t}\right]$. Thirdly, $D'_{i,t}$ is the actual demand which is a decision variable where its optimal value will be determined by the proposed models. Finally, $d'_{i,t}$ is defined as an effective demand. It is calculated by multiplying $D'_{i,t}$ to its possibility level as shown in equation (20) and Figure 2.

$$d'_{i,t} = D'_{i,t} \times \mu_{\tilde{D}_{i,t}}(D'_{i,t}), \qquad \forall i, \forall t. \quad \ldots \quad (20)$$

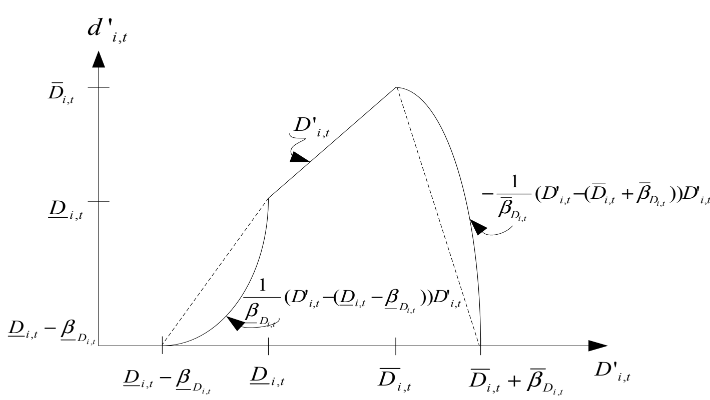

Figure 2. The function of $d'_{i,t}$.

In general the possibility distribution of demand is usually modeled in a possibilistic programming to an APP model (Fung *et al*, 2003; Wang and Liang, 2005). In the previous work of Fung, *et al*. (2003), the fuzzy multiproduct aggregate production planning model is proposed. DM can select the preferred production plan with a common satisfaction level or different combinations of preferred possibility level of market demands and satisfaction level of production capacities. However, if too high possibility level of meeting market demands is set or specified, the model may not have a feasible solution because the range of the demand that can be chosen as a solution is too narrow. At low level of possibility the better solution than at high level of possibility can be found. However, low possibility level means that it is difficult to exist. So, in this research the effective demand ($d'_{i,t}$), which simultaneously considers both fuzzy demand and its possibility levels, is introduced. To use the proposed model, a user is not required to enter or specify the possibility level. It can reduce the problem of setting unsuitable possibility level of market demands.

To simplify the nonlinear function of $d'_{i,t}$, linearization is done and the function of $d'_{i,t}$ can be illustrated as linear piecewise functions as shown in Figure 3 and can be represented by equation (21).

$$d'_{i,t} = \begin{cases} \dfrac{1}{\underline{\beta}_{D_{i,t}}}(D'_{i,t} - (\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}))\underline{D}_{i,t}, & \underline{D}_{i,t} - \underline{\beta}_{D_{i,t}} \le D'_{i,t} \le \underline{D}_{i,t}, \\ D'_{i,t}, & \underline{D}_{i,t} \le D'_{i,t} \le \overline{D}_{i,t}, \\ -\dfrac{1}{\overline{\beta}_{D_{i,t}}}(D'_{i,t} - (\overline{D}_{i,t} + \overline{\beta}_{D_{i,t}}))\overline{D}_{i,t}, & \overline{D}_{i,t} \le D'_{i,t} \le \overline{D}_{i,t} \quad \overline{\beta}_{D_{i,t}} +, \\ 0, & \text{else.} \end{cases} \quad \dots \quad (21)$$

Value of $d'_{i,t}$ is in the range from $\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}$ to $\overline{D}_{i,t}$ as shown in Figure 3. If actual demand ($D'_{i,t}$) is in the range from $\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}$ to $\overline{D}_{i,t}$, $d'_{i,t}$ is equivalent to $D'_{i,t}$. But if $D'_{i,t}$ is greater than $\overline{D}_{i,t}$, $d'_{i,t}$ is lower than $D'_{i,t}$. This is because of the consideration of the possibility level of market demand.

The effective demand ($d'_{i,t}$) plays an important role in the model since it is utilized to calculate the sales revenue in constraint (23) and is used to represent the demand in the product-inventory balance constraint (22).

$$I_{i,t-1} - B_{i,t-1} + P_{i,t} + O_{i,t} + S_{i,t} - I_{i,t} + B_{i,t} = d'_{i,t}, \quad \forall i, \forall t. \quad \dots \quad (22)$$

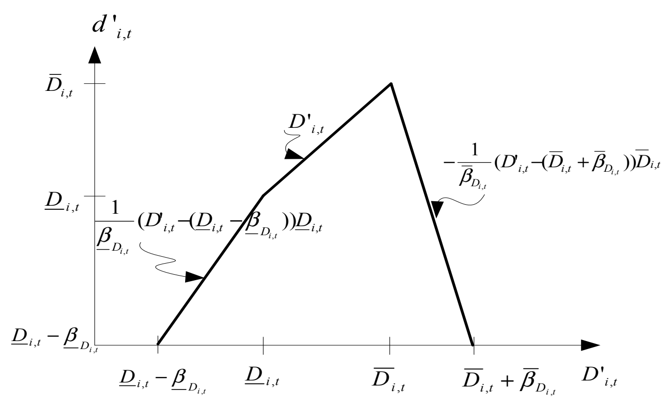

Figure 3. The linearization of $d'_{i,t}$.

Then, APP model with fuzzy demand and variable capacity can be presented as follows.
Maximize *NP*,
subject to

$$NP = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i d'_{i,t} - \sum_{i=1}^{N} r_i B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}\left(P_{i,t} + O_{i,t} + S_{i,t}\right)$$

$$[\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(c_{Wt} W_t + c_{Ht} H_t + c_{Lt} L_t + OH_t\right)], \quad \dots \quad (23)$$

$$d'_{i,t} \le -\frac{1}{\overline{\beta}_{D_{i,t}}}(D'_{i,t} - (\overline{D}_{i,t} + \overline{\beta}_{D_{i,t}}))\overline{D}_{i,t}, \forall i, \forall t, \quad \ldots \quad (24)$$

$$d'_{i,t} \le \frac{1}{\underline{\beta}_{D_{i,t}}}(D'_{i,t} - (\underline{D}_{i,t} - \underline{\beta}_{D_{i,t}}))\underline{D}_{i,t}, \forall i, \forall t, \quad \ldots \quad (25)$$

$$d'_{i,t} \le D'_{i,t}, \forall i, \forall t, \quad \ldots \quad (26)$$

$$d'_{i,t} \ge \underline{D}_{i,t} - \underline{\beta}_{D_{i,t}} \quad \ldots \quad (27)$$

$$W_t, H_t, L_t, A_t, E_t, M_t, P_{i,t}, O_{i,t}, I_{i,t}, S_{i,t}, B_{i,t}, D'_{i,t}, d'_{i,t} \ge 0, \quad \forall i, \forall t. \quad \ldots \quad (28)$$

(5)-(10), (12)-(16), (22).

Equations (24)-(26) are linear piecewise functions of $d'_{i,t}$. Lower bound constraint of $d'_{i,t}$ is represented by equation (27). Other constraints remain the same as in the previous model. This APP model allows the DM to define fuzzy demand for each period and allow investment in some periods to eliminate bottleneck by increasing capacity. The performance of the model is evaluated based on TOC performance measures. These are shown by a case study in the next section.

4. NUMERICAL EXAMPLE

The proposed APP model was applied in a manufacturing company. This company produces electrical appliances for both export and domestic uses. In this case study, two product groups with an 8-period planning horizon are analyzed under the following conditions:

1. The regular time per worker (δ) is 8 hours. Beginning workforce level (W_0) is 3,500 man-days.
2. The cost of regular payroll (c_{Wt}) is \$64 per man-day. The cost of hiring (c_{Ht}) and layoff (c_{Lt}) are \$70 and \$90 per man-day, respectively. The cost of overtime (c_{OTi}) is \$15 per man-hour.
3. Initially, there is no backorder ($B_{i,0}$ = 0) and initial inventory ($I_{i,0}$) of both products is 500 units. Initial investment (E_0) is \$3 million.
4. Subcontracting is not allowed in this case.
5. Faction of regular workforce level available for overtime (ϕ_{W_t}) and variation of workforce level (α) are 0.4 and 0.6, respectively.
6. Operating time and other cost data are presented in Table 1 and resource capacity is shown in Table 2.

Table 1. Operating time and operating cost data.

Product	a_i (man-hr/unit)	b_i (m/c-hr/unit)	c_{mi}	r_i	c_{Ii}	c_{Bi}
1	2	1.5	120	170	10	80
2	3	2.0	145	200	20	100

Table 2. Resource capacity data.

	Period							
	1	2	3	4	5	6	7	8
$W_{t\max}$	5,000	4,000	4,500	4,000	5,000	5,500	4,500	4,000
M_0	32,000	28,400	29,600	20,000	32,000	33,600	29,600	26,400
$MC_{t\max}$	48,000	42,600	44,400	30,000	48,000	50,400	44,400	39,600
$K_{t\max}$	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000
$F_{t\max}$	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
OH_t	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000

The company has encountered some problems. Firstly, they feel uncomfortable to estimate the demand in each period as a constant. So, fuzzy demands with trapezoidal membership functions are used in this case study. The DM can choose an appropriate range of the market demands according to the ability of marketing and sales departments. The fuzzy demand data are estimated by a management team of the company and are shown in Table 3.

Table 3. Fuzzy demand data.

	Period							
	1	2	3	4	5	6	7	8
$\underline{D}_{1,t}$, $\overline{D}_{1,t}$	8,000, 8,100	10,000, 10,100	9,500, 9,600	7,250, 7,350	8,000, 8,100	15,250, 15,350	9,500, 9600	12,000, 12,100
$\underline{D}_{2,t}$, $\overline{D}_{2,t}$	4,500, 4600	13,000, 13,100	8,000, 8,100	5,000, 5,100	7,000, 7,100	7,750, 7,850	7,000, 7,100	10,000, 10,100
$\underline{\beta}_{D_{i,t}}$, $\overline{\beta}_{D_{i,t}}$	500, 500	500, 500	500, 500	500, 500	500, 500	500, 500	500, 500	500, 500

Secondly, the company feels that the production output is not only limited by workforce but also by some small machines and equipment. They also have some allocated budgets for purchasing small machines and additional support tooling to expand the production capacity. It was estimated that an additional investment in tools and equipment of $1 can increase the system capacity by 0.1 machine-hour (γ= 0.1) for each period.

Thirdly, the company's management team plans to use TOC since the company has recently experienced bottleneck problems because of rising sales. TOC measures are set as performance measures for evaluation the system of the company. The main bottleneck in this case study is the system capacity, which was released by investment in tools and equipment. Three TOC performance measures are then used to evaluate advantages of fuzzy demands over crisp demands when additional investment is not allowed, and advantages of APP with and without additional investment in equipment.

4.1 Advantages of Fuzzy Demands over Crisp Demands

In order to determine the effectiveness of fuzzy demands, at first APP models when investment is not allowed are considered. The model with crisp demands [$\underline{D}_{i,t}$, $\overline{D}_{i,t}$] in section 3.1 and the model with fuzzy demands $\tilde{D}_{i,t} = \left(\underline{D}_{i,t}, \overline{D}_{i,t}, \underline{\beta}_{D_{i,t}}, \overline{\beta}_{D_{i,t}}\right)$ in section 3.2 are compared. Note that the additional investment (A_t) is not allowed if A_t is set to zero in both models. The optimal solutions of both cases are shown in Table 4. It was found that when the model with fuzzy demands is used, the throughput is reduced because the actual demands ($D'_{i,t}$) are lower. The average inventory level is increased since there is more stock level. However, the profit, and operating expense are improved since appropriate (in this case, lower) values of actual demands and effective demands are selected. So, there is no backordering in any period while the model with crisp demands has backordering in periods 3, 4, 6 and 8. Thus, the profit and operating expense are improved when demands are selected as fuzzy demands. However, the bottleneck which is the system capacity has not been changed or relaxed.

4.2 Advantages of the Proposed Model with Fuzzy Demands and Variable Capacity over Conventional Model

In a conventional APP model crisp demands when investment is not allowed is considered. This model can be constructed by using the model in section 3.1 and set A_t to zero. To determine the effectiveness of the proposed model with fuzzy demands and variable capacity, the model in section 3.2 is used. The optimal solution for both conventional APP model and proposed model with fuzzy demands and variable capacity is shown in Table 5. It was found that when the additional investment is allowed, the profit, throughput and operating expense are greatly improved since additional investment in tools and equipment can eliminate backordering in each period and lost sales at the end of the planning horizon because the bottleneck of the system has been expanded. Moreover, the production can be increased to satisfy $\overline{D}_{i,t}$ which is the upper bound of the effective demand $d'_{i,t}$. This results in more sales revenue and a lower backordering cost. Thus, the profit, throughput, and operating expense are greatly improved with a little more investment in tools and equipment which belongs to the inventory in the sense of TOC.

Table 4. Optimal solutions of the APP model with crisp demands and without additional investment (Conventional model) and the FAPP model with fuzzy demands and without additional investment.

	Period							
	1	2	3	4	5	6	7	8
	The APP model with crisp demands and without additional investment (A_t =0): *TP* =7,245,250, *NP* = 5,941,960, *IN* = 3,187,250, *OE* = 1,303,290							
$P_{1,t}$, $P_{2,t}$ (x100)	16,000, 0	1,600, 9,600	9,400, 4,400	7,250, 4,563	11,083, 3,887	5,401, 7,675	10,300, 4,409	11,200, 3,200
$O_{1,t}$,$O_{2,t}$ (x100)	0, 4,000	0, 3,400	0, 3,300	0, 0	0, 3,800	6,765, 0	0, 2,666	0, 1,600
$I_{1,t}$, $I_{2,t}$ (x100)	8,500, 0	100, 0	0, 0	0, 0	3,083, 0	0,0	800, 0	0, 0
A_t	0	0	0	0	0	0	0	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 250	0, 688	0, 0	0, 75	0, 0	0, 5,200
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	4,000	4,000	4,000	4,000	4,229	4,229	4,229	4,000
M_t	32,000	28,400	29,600	20,000	32,000	33,600	29,600	26,400
$D'_{1,t}$, $D'_{2,t}$	8,000, 4,500	10,000, 13,000	9,500, 8,000	7,250, 5,000	8,000, 7,000	15,250, 7,750	9,500, 7,000	12,000, 10,000
	The FAPP model with fuzzy demands and without additional investment (A_t =0): *TP* =7,220,750 *NP* = 6,382,230 *IN* = 3,420,000 *OE* = 838,524							
$P_{1,t}$, $P_{2,t}$ (x100)	15,867, 89	1,867, 9,422	4,750, 7,500	7,333, 4,500	12,667, 2,798	5,988, 7,250	11,067, 3,864	4,933, 7,378
$O_{1,t}$,$O_{2,t}$ (x100)	0, 4,011	0, 3,378	4,983, 0	0, 0	0, 3,702	6,745, 0	0, 2,636	0, 2,122
$I_{1,t}$, $I_{2,t}$ (x100)	8,267, 0	33, 0	767, 0	1,350, 0	6,517, 0	4500,0	6,567, 0	0, 0
A_t	0	0	0	0	0	0	0	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	4,000	4,000	4,000	4,000	4,216	4,216	4,216	4,000
M_t	32,000	28,400	29,600	20,000	32,000	33,600	29,600	26,400
$D'_{1,t}$, $D'_{2,t}$	8,100, 4,600	10,100, 12,800	9,000, 7,500	6,750, 4,500	7,500, 6,500	14,750, 7,250	9,000, 6,500	11,500, 9,500

5. CONCLUSIONS

This research developed the FAPP model with fuzzy demand and variable system capacity for a practical APP problem. The model can alleviate problems related to estimation of crisp demands and limited capacity of a conventional APP model. Firstly, the estimation of the demand by a fuzzy demand in each period is more flexible and realistic than a crisp demand. The new approach to model fuzzy demand that the possibility level of demand is integrated is proposed. It makes the model more easy to use than conventional approaches. Then, the model generates the best plan which tells the DM how to use resources in each period in order to get the highest profit which is better than the model with crisp demands. Secondly, investment in tools and equipment can eliminate the bottleneck of the system and makes the production system more effective and profitable. Additional investment can greatly improve all performance measures by reducing backordering and lost sales quantities. This consideration of additional investment does not exist in conventional APP models but it is very useful for APP decisions. Finally, three performance measures based on the TOC concept, effective performance suggested by many factories, are used to evaluate performance of the model.

The FAPP model may be extended to cover the case where the production output is limited by an availability of some major raw materials. This situation is prevailing in companies producing agricultural products, e.g., sugar, vegetable oils, canned fruit, fresh milk, juices and wines. These companies encounter limited and uncertain supply of raw materials that significantly affects the production output and profit. The supply of raw materials is uncertain in terms of quantity, quality and price. An FAPP model with a fuzzy supply may be developed to circumvent these limitations.

Table 5. Optimal solutions of the APP model with crisp demands and without additional investment (Conventional model) and the APP model with fuzzy demands and with additional investment (Proposed model).

	Period							
	1	2	3	4	5	6	7	8
	The FAPP model with crisp demands and without additional investment(A_t =0): *TP* =7,245,250, *NP* = 5,941,960, *IN* = 3,187,250, *OE* = 1,303,290							
$P_{1,t}$, $P_{2,t}$ (x100)	16,000, 0	1,600, 9,600	9,400, 4,400	7,250, 4,563	11,083, 3,887	5,401, 7,675	10,300, 4,409	11,200, 3,200
$O_{1,t}$,$O_{2,t}$ (x100)	0, 4,000	0, 3,400	0, 3,300	0, 0	0, 3,800	6,765, 0	0, 2,666	0, 1,600
$I_{1,t}$, $I_{2,t}$ (x100)	8,500, 0	100, 0	0, 0	0, 0	3,083, 0	0,0	800, 0	0, 0
A_t	0	0	0	0	0	0	0	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 250	0, 688	0, 0	0, 75	0, 0	0, 5,200
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	4,000	4,000	4,000	4,000	4,229	4,229	4,229	4,000
M_t	32,000	28,400	29,600	20,000	32,000	33,600	29,600	26,400
$D'_{1,t}$, $D'_{2,t}$	8,000, 4,500	10,000, 13,000	9,500, 8,000	7,250, 5,000	8,000, 7,000	15,250, 7,750	9,500, 7,000	12,000, 10,000
	The FAPP model with fuzzy demands and additional investment: *TP* =7,615,250, *NP* = 6,891,310, *IN* = 3,256,720, *OE* = 723,938							
$P_{1,t}$, $P_{2,t}$ (x100)	14,950, 700	0, 10,667	9,600, 4267	7350, 5100	8100, 7100	15,350, 7,850	9,600, 7,100	12,100, 10100
$O_{1,t}$,$O_{2,t}$ (x100)	0, 3,400	2,750, 2,433	0, 3,833	0, 0	0, 0	7,750, 0	0, 4,733	6,400, 0
$I_{1,t}$, $I_{2,t}$ (x100)	7,350, 0	0, 0	0, 0	0, 0	0, 0	0, 0	4,850, 0	0, 0
A_t	19,250	0	0	80,750	0	0	0	32,000
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
W_t	4,000	4,000	4,000	4,000	4,688	4,844	4,000	4,000
M_t	33,925	30,325	31,525	30,000	42,000	43,600	39,600	39,600
$D'_{1,t}$, $D'_{2,t}$	8,100, 4,600	10,100, 13,100	9,600, 8,100	7,350, 5,100	8,100, 7,100	15,350, 7,850	9,600, 7,100	12,100, 10,100

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7. REFERENCES

1. Cha, C.N. and Hwang, H. (1996). Experimental comparison of the switching heuristics for aggregate production planning problem. Computers and Industrial Engineering, 31 (3/4): 625- 630.
2. Dettmer, W.H. (1997). Goldratt's theory of constraint: A systems approach to continuous improvement. Quality press, USA.
3. Deckro, R. F. and Hebert, J. E. (1984). Goal programming approaches to solving linear decision rule based aggregate production planning models. IIE Transactions, 16: 308-315.
4. Eilon, S. (1975). Five approaches to aggregate production planning. AIIE Transaction, 7: 118-131.
5. Fung, R.Y.K, Tang, F. and Wang D. (2003). Multiproduct aggregate production planning with fuzzy demands and fuzzy capacities. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 33:302-313.
6. Gen, M., Tsujimura, Y. and Ida K. (1992). Method for solving multiobjective aggregate production planning problem with fuzzy parameters. Computers and Industrial Engineering, 23: 117-120.
7. Goldratt, E. M. (1990). Theory of constraints. North River Press, Massachusetts.
8. Goodman, D.A. (1973). A New Approach to scheduling aggregate production and workforce. AIIE Transaction, 5: 135-141.
9. Hanssmann, F. and Hess, S. W. (1960). A linear programming approach to production and employment scheduling. Management Technology, 1: 46-51.
10. Klusewitz, G. and Rerick, R. (1996). Constraint management through the drum-buffer-rope system. IEEE/SEMI Advanced semiconductor manufacturing conference. USA: ASMC, pp. 7-12.

11. Lai, Y.J and Hwang, C.L. (1996). Lecture notes in economics and mathematical systems, fuzzy multiple objective decision making. Spinger-Verlang, Great Britain.
12. Masud, A.S.M. and Hwang, C.L. (1980). Aggregate production planning model and application of three objective decision methods. International Journal of Operations Research, 18: 741-752.
13. Oliff, M.D., Lewis, H.S. and Markland R.E. (1989). Aggregate planning in crew-loaded production environments, Computers Operations Research. 16: 13-25.
14. Phruksaphanrat, B., Ohsato, A. and Yenradee, P. (2006). A comment on the formulation of an aggregate production planning problem. The 2nd IEEE International Conference on Cybernetics and Intelligent Systems, pp. 292-297.
15. Rommelfanger, H. (1989). Interactive Decision Making in Fuzzy Linear Optimization Problems, European Journal of Operations Research, 41: 210-217.
16. Singhal, J. and Singhal, K. (2006). Holt, Modgliani, Muth, and Simon's work and its role in the renaissance and evolution of operations management. Journal of Operations Management, 25, 300-309.
17. Stein R.E. (1997). Theory of constraints: Applications in quality and manufacturing. 2nd ed. Marcel Dekker, Inc., New York.
18. Techawiboonwong, A. and Yenradee, P. (2003). Aggregate production planning with workforce transferring plan for multiple product types. Production Planning and Control, 14, 5: 447-458.
19. Wang, R-C and Fang H-H. (2000). Aggregate production planning with in a fuzzy environment. International Journal of Industrial Engineering – Theory, Applications and Practice, 7: 5-14.
20. Wang, R-C and Fang, H-H. (2001). Aggregate production planning with multiple objectives in a fuzzy environment. European Journal of Operation Research, 133: 521-536.
21. Wang, R-C. and Fang, H-H. (2004). Application of fuzzy multi-objective linear programming to aggregate production planning. Computer & Industrial Engineering, 46: 17-41.
22. Wang, R-C. and Liang, T.F. (2005). Applying possibilistic linear programming to aggregate production planning. International Journal of Production Economics, 98, 328-341.
23. Woeppel, M.J. (2001). Manufacturer's guide to implementing the theory of constraints. St. Lucie Press, New York.
24. Zadeh, L.A., (1987). Fuzzy sets and a basis for a theory of possibility, fuzzy sets and applications. In Selected Papers by L.A. Zadeh. (Ed.: Zadeh L.A.). John Wiley & Sons, USA, pp. 193-218.
25. Zimmermann, H.-J. (1976). Description and optimization of fuzzy systems. International Journal of General Systems. 2: 209-215.
26. Zimmermann, H.-J. (2001). Fuzzy set theory and its applications. 4th ed. Kluwer Academic Publishers, USA.

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