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An Aggregate Production Planning Strategy Selection Methodology based on Linear Physical Programming

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In this paper, a multi-objective model for aggregate production planning is presented which includes two objectives: (1) minimized cost and (2) minimized effect on the workforce motivation level caused by hire/layoff decisions. Then, six strategies are considered and the most appropriate one is determined to structure the plan. These strategies are set the regular time production quantities in a certain value which is unique for each. A preference based optimization method called Linear Physical Programming (LPP) is used to solve the model. A forecasting phase which chooses the convenient method to forecast the demand for planning horizon is embedded to study in addition to application of LPP to an APP model as another key contribution of this paper.

Significance: This paper presents the application of a relatively new method –Linear Physical Programming - to the Aggregate Production Planning strategy selection process. This method provides a flexibility to decision makers in terms of expressing their preferences for the objectives.

Keywords: Linear Physical Programming; Multi-Objective Optimization; Aggregate Production Planning; Forecasting.

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1. INTRODUCTION

Aggregate Production Planning (APP) is concerned with matching supply and demand output over the medium time range, up to approximately twelve months into the future (Schroeder, 2004). Operation managers try to determine the best way to meet forecasted demand by adjusting production rates, labor levels, inventory levels, overtime work, subcontracting rates, and other controllable variables. Usually, the objective of aggregate planning is to minimize cost over the planning period. However, other strategic issues may be more important then low cost. These strategies may be to smooth employment levels, to drive down inventory levels, or to meet a high level of service (Heizer and Render, 2004). The real world problems that involve a range of planning variables need to take into account not only cost-related objectives but also motivational or service level based objectives simultaneously. Instead of leaving these factors to the managers' experiences, they should be embedded to the model. This situation requires a multi-objective model to obtain more realistic results. Also, another important point is the uncertainties and flexibility factors in objectives and variables for a medium to long term planning period. To struggle with these kinds of problems, objective functions of the model can be expressed with interval numbers as used in fuzzy formulations. Besides, scenario analysis can be manipulated to see the sensitivity of decision parameters under the variation of different circumstances.

When the existing literature is considered, numerous methods are available for planning aggregate production, workforce and inventory levels. Some of the techniques that solve the APP problems can be listed as trial and error (Noori and Radford, 1995), graphical (Stevenson, 1993) and other mathematical techniques such as linear programming, goal programming, mixed integer programming, stochastic models, fuzzy set theory, simulation models, heuristics etc. (Wang and Liang, 2004; Wang and Liang, 2005; Wang and Fang, 2001; Wang and Fang, 2000; Hsu and Lin, 1999). Some of these techniques, which are belong to the large body of APP literature, yield optimum solutions while the others give only near optimal or acceptable solutions. Also the model formulations require varying degrees of sophistication.

In this paper, a multi-objective model for aggregate production planning which includes multiple products and multiple planning periods is presented. This model solved via a relatively new multi-objective decision making methodology named Linear Physical Programming (LPP). LPP method uses crisp numbers but objective functions are piecewise linear which allows to denote desirability degrees of decision makers. In problems, the uncertainty may be expressed with random numbers, fuzzy numbers or interval numbers. Here in LPP the goal values of the objective functions are interval numbers that are determined by the decision maker (manager of the process). The model includes two objectives: (1) minimized cost (2) minimized effect on the workforce motivation level caused by hire/layoff decisions. These two objectives are to be satisfied simultaneously under various constraints including holding inventory, labor level, machine capacity and warehouse space. The main purpose of this model, by considering the different strategies that are based on the regular time production quantity is to determine overtime production quantity, subcontracting volume, backorder level, inventory level and number of workers to be laid off/hired in order to meet the demand while taking cost and motivational factors for a medium range planning horizon into account. The demand values are forecasted with decomposition method by using historical demand values. In this study, LPP's original weight algorithm (Messac et al., 1996) is coded in VBA language via Microsoft Visual Basic v 6.0. The final model with weights obtained from this program is solved via LINGO v 7.0.

The paper is organized as follows: Section 2 presents an outline of the LPP methodology. In section 3, the model design for APP which includes forecasting, LPP formulation and strategy generation phases are given. Section 4 presents an illustrative example of the model. Also, the example includes comments on the effects of the variables and objectives. The paper ends with conclusions in Section 5.

2. LINEAR PHYSICAL PROGRAMMING

LPP is a multi-objective decision making method where the alternatives are various and defined in a continuous space. LPP intends to be a simple and user-friendly optimization method that requires negligible knowledge of optimization. The application of physical programming employs a flexible and natural problem formulation framework. In LPP, the designer does not need to specify optimization weights in the problem formulation phase. Rather, the designer specifies ranges of different degrees of desirability for each design objective. Physical programming also addresses the inherent multi-objective nature of design problems, where multiple conflicting objectives govern the search for the best solution. LPP provides a flexible and more deterministic approach to obtain a solution that satisfies the typically complex texture of a designer's preferences (Messac et al., 2002a).

LPP was firstly proposed by Messac (1996) to obtain a new approach to decision making problems mostly multi-objective in nature. This study was followed by a comprehensive LPP paper by Messac and his colleagues (Messac et al., 1996). Further, Tappeta et al. (2000) and Messac, et al. (2001), also published regarding the theoretical development of LPP algorithm. LPP has not found opportunity to get embedded into many real world practices yet outside of supply chain management, product family design, production planning etc. For detailed information about LPP, the weight determining algorithm of LPP and mathematical meanings of all components of LPP, see Messac et al. (1996). In this study, Messac, et al. (1996)'s weight algorithm is used and the aggregate objective function (to be minimized) is than constructed as a weighted sum of deviations (d_{is}) over all ranges and criteria.

3. MODEL DEVELOPMENT

In real world problems, the APP decisions generally includes multi-product, multi-period and multi-objective so the obtained solution involves trade-offs. To get the most effective results for the variables such as output rates, worker hiring/layoffs, inventory levels and back order levels etc., some important steps should be considered in designing phase of the model. Firstly, collecting accurate data for demand forecasts and for other parameters has vital importance in APP model. Furthermore, appropriate forecasting methods that will provide minimum error between forecasted values and actual data have to be chosen. Then the LPP based APP model can be formulated by considering the obtained data, assumptions and other statements. At the end, the model can be applied to different APP strategies. Figure 1 illustrates the steps of LPP based APP model development.

3.1. Data Collection and Forecasting

This phase of the model design includes determining required data such as worker payments, unit costs of regular time production, over time production, subcontracting, inventory, back order, hired/laid off workers, time requirements for labor and machine hours per unit product and space requirement for per unit product. Some of the required data for planning periods can be gathered easily, but some of them can not be obtained definitely at the beginning. In this model the demand values are not known and forecasted by using historical demand data.

Since the demand data consist of a sequence of observations over time, it is called as time series. Our concern is trying to estimate how the sequence of demand data will continue into the planning periods. Demand data patterns can be distinguished into four types as horizontal, seasonal, cyclical and trend (Makridakis, et. al., 1998). One or more of these patterns can be seen in a demand data set and especially trend and seasonality can be in a multiplicative or additive manner. According to the characteristics of the data set, various forecasting methods can be applied. The most important measurement in choosing the appropriate method is the forecasting and fitting errors. Detailed information related with the forecasting methods can be found in Koehler et al. (2001) and Weatherford and Kimes (2003).

3.2. Linear Physical Programming Based Aggregate Production Planning

The model presented in this study is mainly based on the model proposed by Wang and Liang (2004). Wang and Liang's model includes three performance criteria: total production cost minimization, carrying and backordering cost minimization and rate of change in labor levels minimization. The present model differs from Wang and Liang's model, in terms of the number and type of performance criteria. Proposed model includes two criteria: total production costs minimization and the effects of hire/layoff decisions on the workforce motivation level minimization.

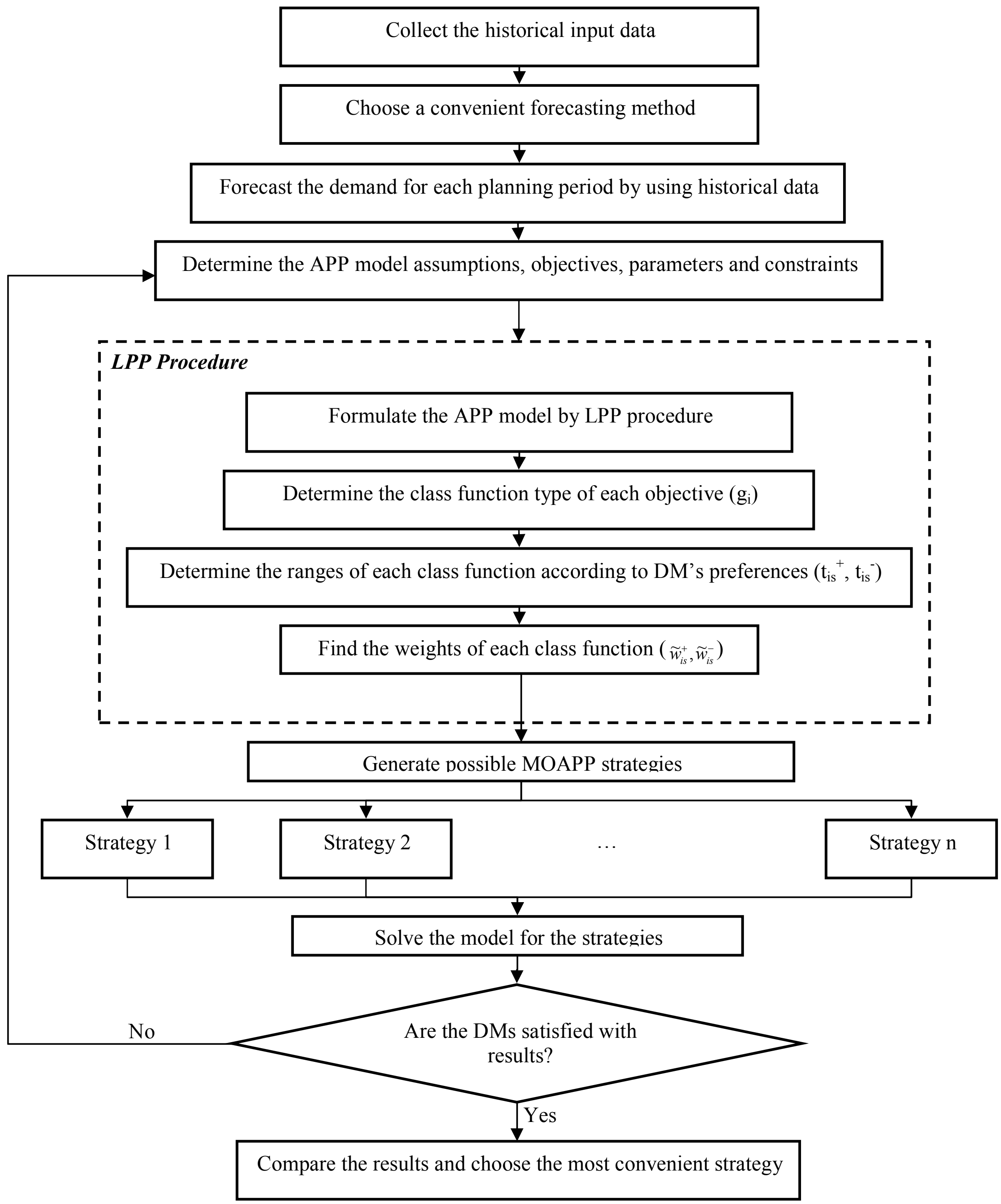

Figure 1. Diagram of the LPP based APP methodology

The total production costs minimization function is modified by inserting worker's payment. Cost of holding workers is embedded to model by the aim of this change. The second performance criterion is also unique, and measures the effects of hire/layoff decisions on the workforce motivation level minimization. In constraints, the total labor time available in each period is separated in a pre-determined fraction for regular time and over time productions. Further, while Wang and Liang's model employs a fuzzy multi-objective linear programming technique to solve the problem, LPP methodology is used in this study. Using LPP for the optimization problems has some significant advantages (Mullur et al., 2003): (i) it is capable of capturing solutions in the non-convex regions of the Pareto frontier, and (ii) there is no need to specify arbitrary weights and ratings. The nomenclature for the proposed model is given as follows.

Decision Variables

p_{nt}^{rt}	Regular time production for n^{th} product in period t (units)
p_{nt}^{ot}	Overtime production for n^{th} product in period t (units)
p_{nt}^{sc}	Subcontracting volume for n^{th} product in period t (units)
p_{nt}^{in}	Inventory level for n^{th} product in period t (units)
p_{nt}^{bo}	Backorder level for n^{th} product in period t (units)
w_t^h	Worker hired in period t (man-hour)
w_t^{lo}	Worker laid off in period t (man-hour)

Parameters

c_{nt}^{rt}	Regular time production cost per unit for n^{th} product in period t ($/unit)
c_{nt}^{ot}	Overtime production cost per unit for n^{th} product in period t ($/unit)
c_{nt}^{sc}	Subcontracting cost per unit for n^{th} product in period t ($/unit)
c_{nt}^{in}	Inventory carrying cost per unit for n^{th} product in period t ($/unit)
c_{nt}^{bo}	Backorder cost per unit for n^{th} product in period t ($/unit)
c_t^w	Worker's payment in period t ($/man-hour)
c_t^h	Cost to hire one worker in period t ($/man-hour)
c_t^{lo}	Cost to layoff one worker in period t ($/man-hour)
p_{nt}^D	Forecasted demand for n^{th} product in period t (units)
p_{nit}^{hisD}	Historical demand for n^{th} product in period t of year i (units)
$p_{nt\min}^{in}$	Minimum inventory level available of n^{th} product in period t (units)
$p_{nt\max}^{bo}$	Maximum backorder level available of n^{th} product in period t (units)
$p_{nt\max}^{sc}$	Maximum subcontracted volume available of n^{th} product in period t (units)
fr^{rt}	Fraction of work force time available for regular time
n_{nt}	Hours of labor per unit of n^{th} product in period t (man-hour/unit)
W_b	Labor level available at the beginning
$W_{t\max}$	Maximum labor level available in period t (man-hour)
r_{nt}	Hours of machine usage per unit of n^{th} product in period t (machine-hour/unit)
$M_{t\max}$	Maximum machine capacity available in period t (machine-hour)
v_{nt}	Warehouse spaces per unit of n^{th} product in period t (m^2/unit)
$V_{t\max}$	Maximum warehouse space available in period t (m^2)
mf^h	Factor denotes the negative effects of hire decisions on the workforce motivation level (0-100).
mf^{lo}	Factor denotes the negative effects of layoff decisions on the workforce motivation level (0-100).

LPP Formulation for APP is given below.

i. Piecewise linear Archimedian aggregate function can be given as follows:

$$\min J = \sum_{i=1}^{n_{sc}} \sum_{s=2}^{5} \left(\widetilde{w}_{is}^{-} d_{is}^{-} + \widetilde{w}_{is}^{+} d_{is}^{+} \right) \quad \ldots (1)$$

ii. Criteria (Soft Constraints)

The model includes two Class-1S criteria which are subject to minimization. There is no Class 2S, Class 3S and Class 4S criteria in the model. The first objective function is the total cost and is denoted by g_1. This cost function includes regular time production cost (c_{nt}^{rt}), overtime production cost (c_{nt}^{ot}), subcontracting cost (c_{nt}^{sc}), carrying inventory cost (c_{nt}^{in}), backordering cost (c_{nt}^{bo}) worker's payment (c_t^w) and hire and lay off worker costs (c_t^h and c_t^{lo}).

$$g_1 = \sum_{n=1}^{N}\sum_{t=1}^{T}\left(c_{nt}^{rt}p_{nt}^{rt} + c_{nt}^{ot}p_{nt}^{ot} + c_{nt}^{sc}p_{nt}^{sc} + c_{nt}^{in}p_{nt}^{in} + c_{nt}^{bo}p_{nt}^{bo}\right) + \sum_{t=1}^{T}\left(c_t^{w}\left(\sum_{t'=1}^{t}\left(w_{t'}^{h} - w_{t'}^{lo}\right) + W_b\right)\right) + \sum_{t=1}^{T}\left(c_t^{h}w_t^{h} + c_t^{lo}w_t^{lo}\right) \quad \ldots (2)$$

The second objective function, g_2, is a qualitative function measuring the effects of hire/layoff decisions on the workforce motivation level. In this criterion, the number of workers hired/laid off are weighted with motivational impact factor. This factor represents the penalty values which are related with the motivational impacts of hire/lay off decisions on the employees and has a scale that takes value between 0 and 100. If decision maker believes that there is no impact hiring (lay off) one worker on motivation of employees, than "0" will be chosen. If decision maker believes that there is a dreadful impact hiring (lay off) one worker on motivation of employees, than "100" will be chosen. This criterion ensures minimization of workforce level changes taking into account motivational impacts of hire/layoff decisions.

$$g_2 = \sum_{t=1}^{T}\left(mf^{h}w_t^{h} + mf^{lo}w_t^{lo}\right) \quad \ldots (3)$$

iii. Goal Constraints

These constraints try to minimize deviations (d_{is}) from target values. If the final value of the performance criteria is in the ideal range, the total deviation will be zero.

$$g_i - d_{is}^{+} \le t_{i(s-1)}^{+}\ ; \quad g_i \le t_{i5}^{+}\ ; \quad d_{is}^{+} \ge 0 \quad \ldots (4)$$

(for all i in classes 1S, 3S, 4S, $i = 1,\ldots,n_{sc}$; $s = 2,\ldots 5$)

$$g_i + d_{is}^{-} \ge t_{i(s-1)}^{-}\ ; \quad g_i \ge t_{i5}^{-}\ ; \quad d_{is}^{-} \ge 0 \quad \ldots (5)$$

(for all i in classes 2S, 3S, 4S, $i = 1,\ldots,n_{sc}$; $s = 2,\ldots 5$)

As aforementioned, this model includes two 1S performance criteria.

iv. System Constraints (Hard constraints)

These are the hard constraints about the system which considers inventory level, workforce level, machine capacity and warehouse space.

Eq.6 guarantees that demand requirements are satisfied.

$$p_{nt}^{in} - p_{nt}^{bo} = p_{n(t-1)}^{in} - p_{n(t-1)}^{bo} + p_{nt}^{rt} + p_{nt}^{ot} + p_{nt}^{sc} - p_{nt}^{D} \quad \forall n, \forall t \quad \ldots (6)$$

Eq.7 guarantees that minimum inventory level is satisfied.

$$p_{nt}^{in} \ge p_{nt\,\min}^{in} \quad \forall n, \forall t \quad \ldots (7)$$

Eq.8 guarantees that maximum backorder level is not exceeded.

$$p_{nt}^{bo} \le p_{nt\,\max}^{bo} \quad \forall n, \forall t \quad \ldots (8)$$

Eq.9, guarantees that the total of regular time and over time workforce level used for current period can not exceed the total of available work force taking into account the number of workers laid off/hired in the current period.

$$\sum_{t'=1}^{t}\left(w_{t'}^{h} - w_{t'}^{lo}\right) + W_b \ge \sum_{n=1}^{N} n_{nt}\left(p_{nt}^{rt} + p_{nt}^{ot}\right) \quad \forall t \quad \ldots (9)$$

Eq.10 and Eq.11 guarantee that the work force level used in regular time and over time can not exceed the pre-determined fractions of the total work force level for the current period

$$\sum_{n=1}^{N} n_{nt} p_{nt}^{rt} \le \left(\sum_{t'=1}^{t} \left(w_{t'}^{h} - w_{t'}^{lo} \right) + W_b \right) fr^{rt} \quad \forall t \qquad \ldots (10)$$

$$\sum_{n=1}^{N} n_{nt} p_{nt}^{ot} \le \left(\sum_{t'=1}^{t} \left(w_{t'}^{h} - w_{t'}^{lo} \right) + W_b \right) \left(1 - fr^{rt}\right) \quad \forall t \qquad \ldots (11)$$

Eq.12 guarantees that maximum workforce volume is not exceeded.

$$\sum_{n=1}^{N} n_{nt} \left(p_{nt}^{rt} + p_{nt}^{ot} \right) \le W_{t\max} \quad \forall t \qquad \ldots (12)$$

Eq.13 guarantees that maximum subcontracting volume is not exceeded.

$$p_{nt}^{sc} \le p_{nt\max}^{sc} \quad \forall n, \forall t \qquad \ldots (13)$$

Eq.14 guarantees that maximum machine capacity is not exceeded.

$$\sum_{n=1}^{N} r_{nt} \left(p_{nt}^{rt} + p_{nt}^{ot} \right) \le M_{t\max} \quad \forall t \qquad \ldots (14)$$

Eq.15 guarantees that maximum warehouse space level is not exceeded.

$$\sum_{n=1}^{N} v_{nt} p_{nt}^{in} \le V_{t\max} \quad \forall t \qquad \ldots (15)$$

Eq.16 guarantees that decision variables are not non-negative.

$$p_{nt}^{rt}, p_{nt}^{ot}, p_{nt}^{sc}, p_{nt}^{in}, p_{nt}^{bo}, w_t^{h}, w_t^{lo} \ge 0 \quad \forall n, \forall t \qquad \ldots (16)$$

3.3. Strategy Generation for Aggregate Production Planning

While generating APP for a medium term period, various legitimate planning strategies can be applied. These strategies include the manipulation of inventory, production rates, labour levels, capacity and other controllable variables. The strategies may depend on capacity options such as changing inventory levels, varying workforce level or production time through overtime or idle time etc. And they also may depend on demand options such as influencing demand or back ordering during high demand periods. Also, by mixing these options, chase or level strategies can be obtained. Chase strategy attempts to make equal the production output rates to demand and level strategy tries maintaining a constant output rate, production rate or workforce level over the planning horizon.

In this study, for the LPP based APP model, six strategies which are related with regular time production quantities are applied in a similar way with Chen and Liao (2003). While using these strategies, the phase of collecting sufficient historical data has vital importance. Even collecting the required data is usually difficult; using correct and a large amount of data is a necessity to get efficient results. The used chase and level based strategies are explained below.

Strategy 1 depends on linear programming results considering with only cost minimization objective function (Eq. 17). The obtained p_{nt}^{LP} value will be equal to p_{nt}^{rt} .

$$p_{nt}^{rt} = p_{nt}^{LP} \quad \forall n, \forall t \qquad \ldots (17)$$

Strategy 2 uses the mean of the demand values. In this chase strategy p_{nt}^{rt} tracks the historical demands for the corresponding product type and production period. p_{nit}^{hisD} is the historical demand for product *n* in period *t* of year *i*. *I* is the number of years considered for historical data.

$$p_{nt}^{rt} = \left(\sum_{i=1}^{I} p_{nit}^{hisD} \right) / I \quad \forall n, \forall t \qquad \ldots (18)$$

Strategy 3 is a level strategy and uses the average of p_{nt}^{rt} values that are obtained in Strategy 2 for all planning periods. In other words, the p_{nt}^{rt} values that will be obtained by strategy 3 is the average of entire historical demand data (p_{nit}^{hisD}) set for n^{th} product.

$$p_{nt}^{rt} = \left(\sum_{i=1}^{I} \sum_{t=1}^{T} p_{nit}^{hisD} \right) / (I * T) \quad \forall n \qquad \ldots (19)$$

Strategy 4 is also a chase strategy and p_{nt}^{rt} values try to satisfy the rate equal to the difference between cumulative historical demand for entire set and cumulative historical production rates (regular time production $p_{nit}^{his\ rt}$, over time production $p_{nit}^{his\ ot}$ and subcontracting rates $p_{nit}^{his\ sc}$) except for the last period. This ensures considering the backorder values before the fact.

$$p_{nt}^{rt} = \sum_{i=1}^{I} p_{nit}^{hisD} - \sum_{i=1}^{I-1} \left(p_{nit}^{his\ rt} + p_{nit}^{his\ ot} + p_{nit}^{his\ sc} \right) \quad \forall n, \forall t \qquad \ldots (20)$$

Strategy 5 assumes that the decision makers have perfect information and the regular time production quantity (p_{nt}^{rt}) is equal to the real demand.

$$p_{nt}^{rt} = p_{nt}^{D} \qquad \ldots (21)$$

Strategy 6 attempts to trail demands for as less inventory investment as possible. This is also a chase strategy. p_{nt}^{rt} is obtained by extracting the historical inventory level for product *n* in period *(t-1)* of last year from the results of Strategy 2.

$$p_{nt}^{rt} = \left(\sum_{i=1}^{I} p_{nit}^{hisD} \right) / I - p_{ni_{last}(t-1)}^{his\ in} \quad \forall n, \forall t \qquad \ldots (22)$$

4. AN ILLUSTRATIVE EXAMPLE

An illustrative example is presented to foster better understanding of the model. A pipe clamps firm is considered which produces three main product groups: pipe clamps, anchors and hanging-fixing systems for constructing, heating and electronic sectors. In the illustrative example, pipe clamps product group with three main product types that are standard pipe clamps with rubber profile and welded, standard pipe clamps with rubber profile, standard pipe clamps without rubber profile and heating nut (n=1,2,3) is considered. The firm's historical data includes quarterly demand values of all three products for 6 years. By using these data, demand forecast is realized for next (7^{th}) year's quarters. By using MINITAB Release 13.32 package program, mostly known forecasting methods are run and compared with each other considering minimum MAPE, MAD and MSD statistical measures. Table 1 shows that the appropriate forecasting method is multiplicative decomposition model with trend and seasonal patterns for product 1 and product 2. Also, additive decomposition model with only seasonal pattern is appropriate for product 3.

The forecasted demands for all products and further data for the numerical example is provided in Table 2. The illustrative example has the following conditions and assumptions:

1. There is a four-period planning horizon.
2. On hand inventory consists of 500 units of Product 1, 700 units of Product 2, and 700 units of Product 3 and for each period, an inventory level of at least 500 units of Product 1, 700 units of Product 2, and 600 units of Product 3 has to be ensured.
3. Beginning of the planning horizon backorder level is 300 units of Product 1, 300 units of Product 2, and 0 units of Product 3 and for each period, backorder level can not exceed 600 units of Product 1, 700 units of Product 2, and 600 units of Product 3.
4. For each period and each product, subcontracting volume can not exceed 1000 units.
5. The initial labor level is 16000 man-hours and labor level can not exceed 20000 man-hours for each period.
6. There is no investment plan considering new machines. Machine capacity is constant and can not exceed 30000 machine hours.
7. Worker's payments are \$41, \$36, \$30, and \$36 per worker per hour for periods 1, 2, 3, and 4 respectively.
8. Hiring and layoff costs are \$16 and \$4 per worker per hour, respectively.
9. Maximum available warehouse space can not exceed 80000 m^2 in each period.

10. Motivational impact factor for hiring a new worker (mf^h) is 20 and motivational impact factor for layoff a new worker (mf^{lo}) is 80 for the planning horizon.
11. Time value of money is negligible.

Table 1. Comparison of the forecasting methods

n	Measure	Decomposition				Moving Average	Single Exp. Smoothing	Double Exp. Smoothing	Winters' multiplicative model	Winters' additive model
		Multiplicative		Additive						
		Trend and Seasonal	Only Seasonal	Trend and Seasonal	Only Seasonal					
1	MAPE	3.06	22.00	3.70	22.00	10.42	12.23	10.21	6.03	7.18
	MAD	158.92	1122.00	202.46	1122.00	645.60	701.80	567.60	321.30	377.30
	MSD	40965.00	618930.00	5285.80	631500.00	629253.00	659957.00	488393.00	152996.00	182860.00
2	MAPE	3.76	17.68	3.80	17.67	6.17	5.91	4.74	4.56	4.37
	MAD	186.37	839.30	188.75	838.80	326.50	297.30	235.20	217.25	217.28
	MSD	6022.00	977233.00	47613.60	976750.00	167128.00	122801.00	75977.30	69126.10	65762.80
3	MAPE	3.71	3.57	3.75	3.57	6.80	6.82	6.87	6.57	6.62
	MAD	169.54	163.39	171.28	163.26	319.60	318.70	327.80	299.90	301.20
	MSD	46105.20	6458.30	46236.40	6403.80	151640.00	154723.00	170599.00	134696.00	138659.00

Table 2. The data for the numerical example

n	t	p_{nt}^{D} (units)	c_{nt}^{rt} ($/unit)	c_{nt}^{ot} ($/unit)	c_{nt}^{sc} ($/unit)	c_{nt}^{in} ($/unit)	c_{nt}^{bo} ($/unit)	n_{nt} (h/unit)	r_{nt} (h/unit)	v_{nt} (m2/unit)
1	1	7423.2	7	14	35	2.333	42	0.7	1.4	4
	2	8110.0	6	12	30	2.000	36	0.7	1.4	4
	3	9149.7	5	10	25	1.667	30	0.7	1.4	4
	4	7235.5	6	12	30	2.000	36	0.7	1.4	4
2	1	6698.1	8	16	40	2.667	48	0.6	1.2	3
	2	6903.4	7	14	35	2.333	42	0.6	1.2	3
	3	6955.3	6	12	30	2.000	36	0.6	1.2	3
	4	7300.9	7	14	35	2.333	42	0.6	1.2	3
3	1	4315.5	7.5	15	37.5	2.500	45	0.5	1.3	3
	2	4695.5	6.5	13	32.5	2.167	39	0.5	1.3	3
	3	5184.3	5.5	11	27.5	1.833	33	0.5	1.3	3
	4	4708.0	6.5	13	32.5	2.167	39	0.5	1.3	3

After the collection of the relevant data, management preferences are obtained in terms of the desirability degrees (Table 3). LPP weighting algorithm is run by using data shown in Table 3, to calculate the normalized weight deviations. Table 4 shows the final weight deviations of performance criteria.

The final model with the weights obtained with weighting algorithm solved via LINGO v 7.0. and results are presented in Table 5 and Table 6.

As shown in Table 6, it is not possible to produce all three products in regular time for all strategies. Furthermore, in all strategies over time working is required and also subcontractor's usage is needed in Strategy 1 and Strategy 6. To increase the benefit provided from the workforce, overtime is preferred to subcontracting. Backorder levels never exceed the zero since the high backordering cost. Also the inventory levels exceed the safety stock levels considering the warehouse space constraint. Three of the strategies (Strategy 1, 3, and 6) do not change the workforce level after decreasing to the required level in the first period. The rest of them include hires and layoffs so they can be considered as the chase strategies in terms of work force level.

The optimal objective values for APP strategies point out differences in terms of desirability degrees. The results of the Strategy 4 and Strategy 5 are not preferred since the first objective function takes undesirable values even the second objective values are ideal. Also Strategy 1, Strategy 2 and Strategy 6 take values around the tolerable and desirable ranges. However, only Strategy 3, which is a level strategy, gives results in desirable range for both of the objectives. This is the best result obtained through the strategies.

Table 3. Management preferences concerned objectives (Target values)

	g_1 Class 1S (Minimization)	g_2 Class 1S (Minimization)
Ideal	<2200000	<250000
Desirable	2200000-2500000	250000-270000
Tolerable	2500000-2800000	270000-290000
Undesirable	2800000-3100000	290000-310000
Highly undesirable	3100000-3400000	310000-330000
Unacceptab le	>3400000	>330000

Table 4. Normalized weight deviations of objectives

	$\widetilde{w}_{12}^{+}$	$\widetilde{w}_{13}^{+}$	$\widetilde{w}_{14}^{+}$	$\widetilde{w}_{15}^{+}$
g_1	0.214	0.237	0.261	0.288
	$\widetilde{w}_{22}^{+}$	$\widetilde{w}_{23}^{+}$	$\widetilde{w}_{24}^{+}$	$\widetilde{w}_{25}^{+}$
g_2	0.215	0.237	0.261	0.287

The most significant reason of preferring the strategy 3 is related with the workforce parameters. LPP based APP tries to minimize the change of worker volumes since hiring/layoff and holding workforce costs are relatively higher. Therefore, the strategies that propose high workforce necessity and variability between periods are not preferred such as Strategy 5 and Strategy 6.

Even not considering a strategy, none of the objective function values can reach to the ideal range while the second one is in desirable range. This situation can be interpreted as the cost minimization objective is not superior to motivational effect objective for the decision makers and the ideal range can not be achieved with the current constraints and degrees of desirability. To improve the level of functions to the better desirability ranges: (1) the decision maker should consider and change the parameters and limitations or try to find cost decreasing techniques; or (2) the decision maker should revise the degrees of desirability.

Table 5. Objective value results of the LPP model

	Cost minimization (g_1)		**The effects of hire/lay off decisions on the workforce motivation level minimization(g_2)**	
	Numerical Results	Preference Degree	Numerical Results	Preference Degree
Strategy 1	2500000.00	Desirable	279152.00	Tolerable
Strategy 2	2581141.00	Tolerable	270000.00	Desirable
Strategy 3	2472611.00	Desirable	267491.70	Desirable
Strategy 4	2902061.00	Undesirable	250000.00	Ideal
Strategy 5	2951396.00	Undesirable	229191.70	Ideal
Strategy 6	2554310.00	Tolerable	288661.20	Tolerable
No Strategy	2450361.00	Desirable	270000.00	Desirable

5. CONCLUSIONS

In this paper, a LPP model for APP has been presented in order to determine the most appropriate plan while achieving total production costs minimization and the effects of hire/layoff decisions on the workforce motivation level minimization goals. A forecasting phase which chooses the convenient method to forecast the demand for planning horizon is embedded to study in addition to application of LPP to an APP model as another key contribution of this paper. Historical data, which are used for forecasting, also required for generating simplified production strategies. Instead of solving the mathematical model directly, six strategies are considered and the most appropriate one is determined to structure the plan.

These strategies, that set the regular time production quantities in a certain value which is unique for each, require less computation and may receive more acceptances by industry. Then, the model is applied to an illustrative example for each strategy. In order to show its practicability and benefits, the results of the strategies are compared and an optimally efficient strategy is preferred.

Table 6. Results of the LPP model

		No Strategy			1			2			3			4			5			6		
	t \ n	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Regular time values p_{nt}^{rt} **- units**	t=1	4107	6998	4216	3160	6998	4216	5027	4735	4353	5487	5001	4726	6470	4980	3650	7423	6698	4316	4527	4035	3753
	t=2	3846	6903	4696	2898	6903	4696	5535	4888	4715	5487	5001	4726	7850	6770	5570	8110	6903	4696	5035	4188	4115
	t=3	3452	6955	5184	3235	6103	5184	6367	5022	5225	5487	5001	4726	8040	6390	5530	9150	6955	5184	5867	4322	4625
	t=4	3496	7301	4708	1911	7301	4708	5020	5360	4610	5487	5001	4726	5910	7210	4470	7236	7301	4708	4520	4660	3910
Over time values p_{nt}^{ot} **- units**	t=1	3740	0	0	4874	0	0	2697	2263	0	2236	1997	0	1253	2018	566	300	300	0	2197	2685	462
	t=2	4919	0	0	4874	0	0	2575	2015	0	3029	1902	0	260	133	0	0	0	0	2086	2715	581
	t=3	4919	0	0	4874	0	0	3744	1934	0	3256	1954	0	2292	565	0	0	0	0	2283	2503	559
	t=4	3739	0	0	4874	0	0	1255	1941	0	1748	2300	0	143	91	0	0	0	0	1994	2641	798
Backorder level p_{nt}^{bo} **- units**	t=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	t=2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	t=3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	t=4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Subcontracting volume p_{nt}^{sc} **- units**	t=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000	278	0
	t=2	0	0	0	68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	989	0	0
	t=3	0	0	0	1000	852	0	0	0	0	0	0	0	0	0	0	0	0	0	1000	131	0
	t=4	0	0	0	450	0	0	0	0	0	0	0	0	0	0	0	0	0	0	721	0	0
Inventory level p_{nt}^{in} **- units**	t=1	624	700	600	811	700	600	500	700	738	500	700	1110	500	700	600	500	700	700	500	700	600
	t=2	1279	700	600	541	700	600	500	700	757	906	700	1141	500	700	1475	500	700	700	500	700	600
	t=3	500	700	600	500	700	600	1461	700	798	500	700	682	1682	700	1820	500	700	700	500	700	600
	t=4	500	700	600	500	700	600	500	700	700	500	700	700	500	700	1582	500	700	700	500	700	600

Number of workers hired/laid off w_t^h, w_t^{lo} **- man-hours**		No Strategy		1		2		3		4		5		6	
		w_t^h	w_t^{lo}	w_t^h	w_t^{lo}	w_t^h	w_t^{lo}	w_t^h	w_t^{lo}	w_t^h	w_t^{lo}	w_t^h	w_t^{lo}	w_t^h	w_t^{lo}
	t=1	0	3125	0	3489	0	3127	0	3344	0	498	0	362	0	3375
	t=2	0	0	0	0	0	0	0	0	1468	0	1092	0	0	0
	t=3	0	0	0	0	990	0	0	0	0	158	1380	0	0	0
	t=4	0	0	0	0	0	0	0	0	0	2102	0	1885	0	0

Also the results support that LPP is an effective method for use in aggregate production planning applications which are mostly multi-objective in nature. By employing this approach, typically complex texture of a decision maker's preferences can be satisfied. LPP measures the weights of the decision-maker's preference levels automatically for each performance criteria taking them individually under consideration and forms aggregate objective function. Thus, it removes the necessity to choose weights required by some decision support tools such as analytic hierarchy process and prevents the decision maker from determining inappropriate weight settings. Consequently, the illustrative example indicated the effectiveness of the proposed model raises the value of this research from the point of view of practicability and supports aspect being scientific of the research in the future.

Future researches might consider improving the proposed model by adding new performance criteria and constraints considering late orders and legal limitations, etc. For instance, while legal limitations can be added to the model as the hard constraints which are to be satisfied, late order minimization can be added to the model as a soft constraint which can be stated as degrees of desirability.

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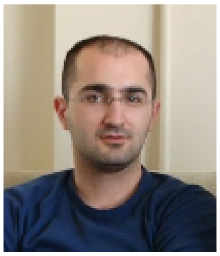

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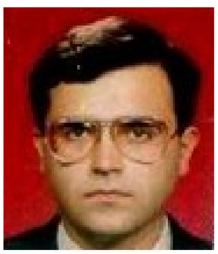

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