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WEIGHTED RANK REGRESSION WITH DUMMY VARIABLES FOR ANALYZING ACCELERATED LIFE TESTING DATA

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In this article, we propose a new rank regression model to extrapolate the product lifetimes at normal operation environment from accelerated testing data. Weighted least squares method is used to compensate for nonconstant error variance in the regression model. A group of dummy variables is incorporated to check model adequacy. We also developed customizing software for quick-and-easy implementation of the method so that reliability engineers can easily exploit it. Simulation studies show that, under light censoring, the proposed method performs comparatively well in predicting the lifetimes even with small sample sizes. With its computational ease and graphical presentation, the proposed method is expected to be more popular among reliability engineers.

Key Words: dummy variable technique; weighted least squares; rank regression method; probability plot; accelerated life test; stress-life relationship

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1. INTRODUCTION

Development of reliable products is crucial for manufacturers to survive in increasingly competitive environment. In general, the process to evaluate product's reliability is time-consuming, hence it hinders manufacturers from meeting market requirements within time period given for product development. Accelerated life tests (ALTs) have been widely used in many applications to shorten the evaluation time. In the ALT, products are exposed to a harsher environment than normal operating condition in order to expedite their failure mechanism, thereby reducing required test time. Based on accelerated testing results, product lifetimes at normal condition are extrapolated via known stress-life relationship. For more general discussion and detailed information on the ALT, see Nelson[1] and Meeker and Escobar[2].

General way to analyze life testing data is to firstly assume a probability distribution to represent the nature of failure-times of the interest. Then, a chosen model is statistically fitted to sample data in order to estimate unknown model parameters. Among commonly methods used to estimate unknown parameters there are rank regression (RR) method (often referred as "graphical method" or "probability plot") and maximum likelihood (ML) method[1-2]. The ML method has been generally accepted as the most efficient method in many applications. However, it may produce biases in estimating the distribution parameters for small data sets or highly censored data[3-6]. Alternatively, the RR method has been primarily favored by practitioners because of its computational convenience and graphical presentation. Indeed, it does not require any iterative procedure for parameter estimation, such as the Newton-Raphson algorithm in the ML method, which is potentially sensitive to initial value[7].

This article proposes an extended rank regression method to the ALT data. We introduce weighted least squares to deal with heteroscedasticity of error variance in the rank regression model. Dummy variables are newly introduced to check the validity of stress-life relationship all over test range. Finally, we develop customizing software with graphical user interface (GUI) to implement the proposed method. The suggested method provides straightforward and graphical presentation so that practitioners can implement the method and interpret resulting outcomes more easily.

The paper is organized as follows. Section 2 briefly reviews the ALT model and its analytical scheme. Section 3 presents the proposed RR method and an illustrating example is given in Section 4. In Section 5, simulation studies are presented to compare performance of the proposed method with that of the ML method. Concluding remarks are given in Section 6.

2. ANALYSIS OF ACCELERATED LIFE TESTING DATA

Failure-times of testing samples may present variation from item to item, even under the same testing condition. This requires a proper application of statistical methods to analyze the life testing data. The ALT involves physical stresses to hasten product failure within a short time. Typical types of stresses are temperature, voltages, mechanical load, thermal cycling, humidity, and etc. Some well-known relationships are often required to investigate effects of the stresses on lifetimes. For example, the Arrhenius relationship describes the effects of temperature as

$$\tau = A \times \exp(B/T) \quad \cdots \quad (1)$$

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where τ represents nominal life, T is temperature in the absolute Kelvin scale, A and $B(>0)$ are unknown coefficients to be estimated from the data. The coefficients involve product geometry, specimen size and test method, and other factors. The inverse-power relationship between nominal life and voltage stress is

$$\tau = A/V^B \quad \cdots \quad (2)$$

where V is voltage, and A and $B(>0)$ are parameters to be estimated from test data. As multi-stress model, the Eyring relationship is commonly used to describe relationship between lifetimes and temperature along with other accelerating variable S as follows:

$$\tau = (A/T)\times\exp[(B\times S)+(1/T)\times\{C+(D\times S)\}] \quad \cdots \quad (3)$$

where A, B, C, and D are unknown coefficients to be estimated from the ALT data. See Nelson[1] for more details.

Failure-times and/or censoring times at several accelerated stress conditions are then analyzed under a presumed lifetime distribution and a pre-specified stress-life relationship to extrapolate product lifetimes at normal use condition. However, such application of the stress-life relationship to the ALT data implicitly assumes that a scale-accelerated failure-time (SAFT) model[2] holds over test range. The term scale-acceleration means that lifetime at a certain stress level can be obtained by simply multiplying its lifetime at any other stress level by a time-scale factor[2] (or an acceleration factor). Assessment of the model assumption often requires post-mortem analysis that determines whether or not applied stresses generate the same failure mechanisms as normal environment. Additionally, the validity of the SAFT model can be checked indirectly by implementing a statistical hypothesis test on the common shape or scale parameters of the fitted lifetime distribution[1]. Figure 1 shows a general procedure for analyzing the ALT data to make inference on the lifetimes at normal use condition.

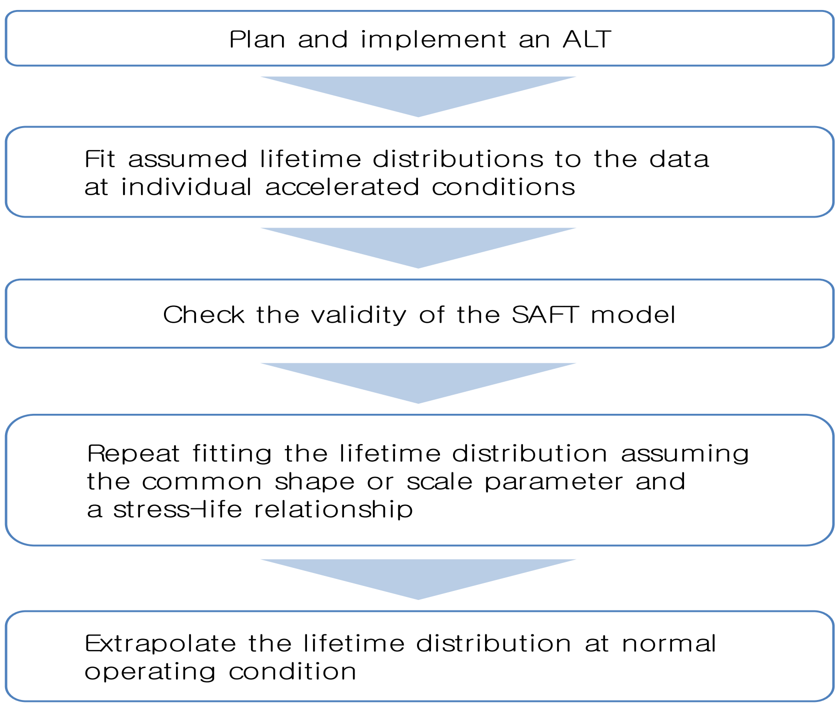


Figure 1. Flowchart for extrapolating use-condition lifetimes from the ALT data

3. WEIGHTED RANK REGRESSION WITH DUMMY VARIABLES

The rank regression has been commonly used to analyze life testing data in engineering practice and commercial software. The rank regression determines a best-fit straight line for failure-time data plotted on a probability plot using the least squares method. For this purpose, it takes advantage of a linear relationship between the cumulative distribution function (CDF) and the failure-times. For example, the CDF of a Weibull distribution can be linearized as

$$\ln[-\ln(1-F(t))] = -m\ln\eta + m\ln t \quad \cdots \quad (4)$$

for the CDF of Weibull failure-times $F(t)$ at time t, where m and η are shape and scale parameters of the Weibull distribution, respectively. For a lognormal distribution, the linearly transformed CDF is

$$\Phi^{-1}(F(t)) = -(\mu/\sigma) + (\ln t/\sigma) \quad \cdots \quad (5)$$

where $\Phi(\cdot)$ is the standard normal CDF, μ and σ are respective location and scale parameters of the lognormal distribution.

Suppose that an accelerated test is conducted at l higher stress levels $S_i (i = 1,2,\cdots,l)$. The stress S_i can contain multiple stresses, that is, $S_i = (S_{i1}, S_{i2}, \cdots, S_{is})$ for *s*-dimensional multiple-stress vector. At each S_i, n_i units are put on test, then the total number of testing items $N = \sum_{i=1}^{l} n_i$. Further, the ordered samples $t_{(i1)} < t_{(i2)} < \cdots < t_{(in_i)}$ are obtained at *i*th stress level S_i. The ordered sample $t_{(ij)} (j = 1,2,\cdots,n_i)$ is either a failure-time or a censored time. Several methods have been proposed to estimate the CDF at $t_{(ij)}$. The commonly used methods are Bernard's median rank estimator[8] as

$$\hat{F}(t_{(ij)}) = \frac{j-0.3}{n_i+0.4} \quad \cdots \quad (6)$$

and the Herd-Johnson estimator[9-10]

$$\hat{R}(t_{(ij)}) = \frac{n_i - j + 1}{n_i - j + 2}\hat{R}(t_{(i,j-1)}) \quad \cdots \quad (7)$$

where $\hat{F}(t_{(ij)}) = 1 - \hat{R}(t_{(ij)})$ and $\hat{R}(t_{(i0)} = 0) = 1$. Then, the unknown distribution parameters can be obtained by regressing t on $F(t)$ with a simple linear model as (see table 1 for the details)

$$y_{ij} = \beta_{i0} + \beta_{i1}x_{ij} \quad \cdots \quad (8)$$

Corresponding regression analysis should use different weights for each observation $t_{(ij)}$ since variance of the response variable in the model (8) depends on its observations. In this study, we employ the following weight factor[11]

$$w_{ij} = 3.3 \times \hat{F}(t_{(ij)}) - 27.5 \times \left[1 - \left(1 - \hat{F}(t_{(ij)})\right)^{0.025}\right] \quad \cdots \quad (9)$$

As mentioned in Section 2, one should check if the SAFT model is valid over entire range of testing conditions when the ALT scheme is employed. This can be done by checking that the fitted lifetime distributions at each testing conditions have the same shape parameter (m) for the Weibull distribution and scale parameter (σ) for the lognormal distribution. This means the slopes in (8) should be identical but the intercepts differ among each stress levels.

Table 1. Variables and parameters in the simple regression model (8)

	Variables	Parameters
Weibull	$y = \ln[-\ln(1-F(t))],\ x = \ln t$	$\beta_0 = -m\ln\eta,\ \beta_1 = m$
lognormal	$y = \Phi^{-1}(F(t)),\ x = \ln t$	$\beta_0 = -\mu/\sigma,\ \beta_1 = 1/\sigma$

This study proposes an inferential procedure using dummy variables for testing the hypothesis that l regression models have a common slope. The regression model with dummy variables has been introduced in many applications to compare different regression equations (e.g., see Neter *et al.*[12]). First, define $(l-1)$ dummy variables corresponding to the l groups, each taking on the values 0 and 1 such as

$$\mathrm{d}_i = (\underbrace{0,0,\ldots,0}_{\sum_{k=1}^{i} n_k}, \underbrace{1,1,\ldots,1}_{\sum_{k=i+1}^{l} n_k})^T, \quad i = 1,2,\ldots,l-1,$$

and fit the following model to the pooled data over all of stress levels:

$$y_{ij} = \beta_{D0} + \beta_{D1}x_{ij} + \beta_{D2}d_{1,m_i+j} + \beta_{D3}d_{2,m_i+j} + \cdots + \beta_{D,l}d_{l-1,m_i+j}, i = 1,\cdots,l, j = 1,\cdots,n_i, \quad \cdots \quad (10)$$

where $m_i = \sum_{k=0}^{i-1} n_k$ and $n_0 = 0$. In the integrated model (10), each observation has different combination of 0 and 1 on the dummy variables according to testing condition, i.e., the observations at S_i is assigned $(\underbrace{1,1,\cdots,1}_{(i-1)},\underbrace{0,0,\cdots,0}_{(l-i)})$ for their dummy variables. This structure for dummy variables permits a common slope but different intercept terms (i.e., $\beta_{i0} = \beta_{D0} + \sum_{k=2}^{i} \beta_{Dk}$) all over the stress levels. Satisfactory fit with (10) implies that the SAFT model is valid, along with the same slope.

Let SSE_D be the residual sum of squares from (10). Fit l separate regression equations of (8) to each testing condition and find SSE_T by adding the residual sums of squares from each separate equation. Next, compute the test statistic F_0 and conclude that at $(1-\alpha)\times 100\%$ confidence level, the SAFT model is adequate if

$$F_0 = \frac{(SSE_D - SSE_T)/(df_D - df_T)}{SSE_T / df_T} < F_{\alpha,(df_D - df_T),df_T} \quad \cdots \quad (11)$$

where $df_D = N-(l+1)$, $df_T = N - 2\times l$, and $F_{\alpha,(df_D-df_T),df_T}$ is the αth percentile of the F-distribution with $(df_D - df_T)$ and df_T degrees of freedom. After checking validity of the SAFT model, the coefficients in the stress-life relationship can be estimated by fitting the regression model with the common slope $\beta_{D1}(=\beta_{11} = \cdots = \beta_{l1})$ and the different intercept β_{i0}'s, such as

$$y_{ij} = \beta_{i0} + \beta_{D1}x_i, \; i = 1,2,\cdots,l, \; j = 1,2,\cdots,n_i, \quad \cdots \quad (12)$$

to the pooled data. In the regression model (12), the intercept term β_{i0} accounts for the effect that the stress variables have on the failure-time distribution.

4. ILLUSTRATING EXAMPLE

The proposed method in previous section will be illustrated by analyzing a set of real ALT data: transformers in Nelson[1]. Ten samples were tested at each of three accelerated voltage levels; 35.4kV, 42.4kV, and 46.7kV. Table 2 shows lifetime data including some censored observations. The purpose of this experiment is to estimate the lifetime distribution at design voltage (15.8kV).

Table 2. Transformer test data (Nelson[1])

Voltage	Hours
35.4kV	40.1 59.4 71.2 166.5 204.7 229.7 308.3 537.9 1002.3^+ 1002.3^+
42.4kV	0.6 13.4 15.2 19.9 25.0 30.2 32.8 44.4 50.2^+ 56.2
46.7kV	3.1 8.3 8.9 9.0 13.6 14.9 16.1 16.9 21.3 48.1^+

+ censored observation

The Weibull-inverse-power model was assumed for the lifetime-stress relationship, and median rank method is used to estimate the CDF. We developed a special program with its own GUI, written in Visual Basic[13] language, to implement all the computation so that practitioners can easily exploit the proposed method (its “Startup window” is shown in Figure 2). The succeeding figures in this section were all constructed by this program.

The estimated common shape from the integrated model (10) is 1.0986 and the resulting SSE_D is 1.9061. Individual regression at each stress levels produces

$$SSE_T = SSE_{35.4} + SSE_{42.4} + SSE_{46.7} = 0.1624 + 0.7584 + 0.2555 = 1.1763,$$

and the test statistics (11) is calculated as

$$F_0 = (1.1906 - 1.1763) \times 2/(1{,}1763 \times 20) = 6.2049.$$

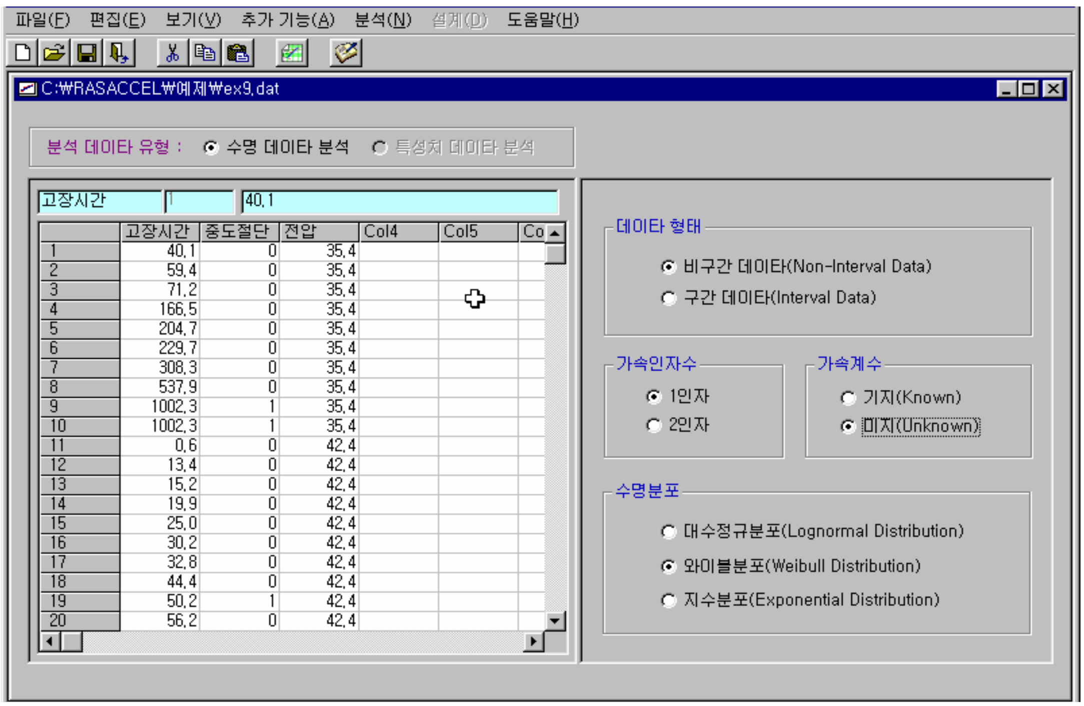


Figure 2. Start-up window for implementing the proposed method

For $F_{0.05,2,20} = 3.4928$, we conclude that the slopes in the three regression lines are not identical. The estimated slope of the line at the highest level is quite different from those of the other two stress levels (see Figure 3).

Deleting the highest level from the whole data gives $F_0 = 1.1969 < F_{0.05,1,13} = 4.6672$, concluding that slopes of the other two lines are the same. The common shape parameter at lower two stress levels is estimated as 0.9763. After incorporating the inverse power model into the Weibull equation (4), with some manipulation, the equation (12) becomes

$$y_{ij} = \ln[-\ln(1 - F(t_{(ij)}))] = -m\ln\eta_i + m\ln t_{(ij)} \quad \cdots \quad (13)$$

$$= -m(\ln A - B\ln V_i) + m\ln t_{(ij)} = \beta_0 + \beta_1 x_{1,ij} + \beta_2 x_{2,ij}$$

where $(x_{1,ij}, x_{2,ij}) = (\ln V_i, \ln t_{(ij)})$, $\beta_0 = -m\ln A$, $\beta_1 = mB$, and $\beta_2 = m$.

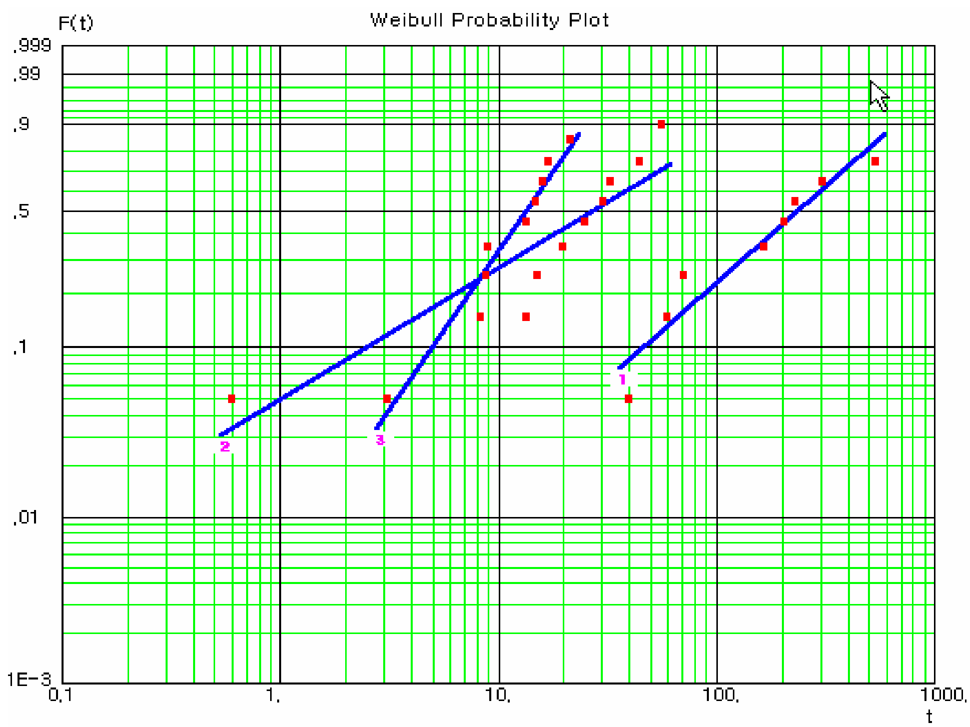


Figure 3. Estimated Weibull distributions (1: 35.4kV, 2: 42.4kV, 3: 46.7kV)

The next step to evaluate the voltage effect on the transformer lifetimes is to fit (13) to the pooled data from the two lower stress levels, giving $\hat{A} = 8.29 \times 10^{21}$ and $\hat{B} = 12.521$. Figure 4 shows the estimated stress model under inverse-power-law relationship. Estimated lifetime distributions at the two stress levels are also plotted in Figure 5, along with lifetime distribution at the design stress (left straight line).

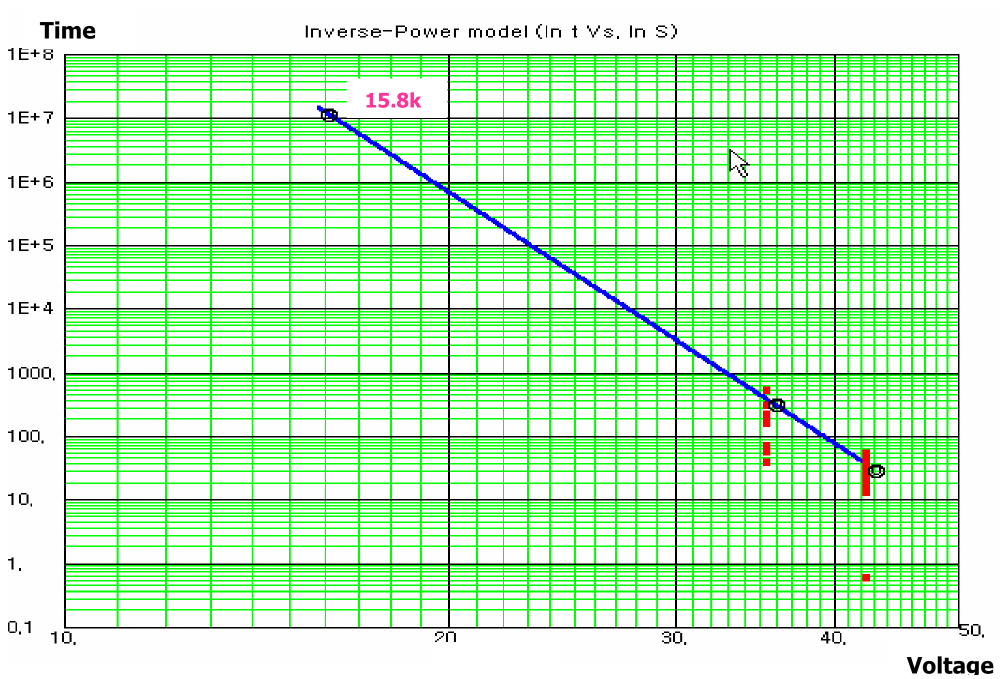


Figure 4. Estimated stress-model of the inverse power law

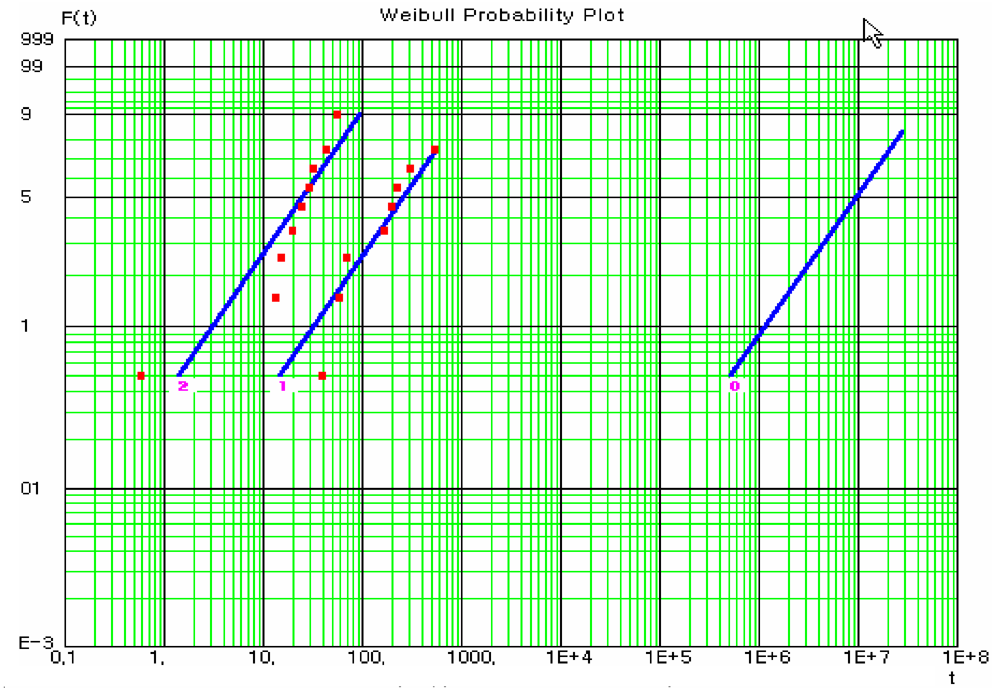


Figure 5. Estimated Weibull distributions assuming a common shape parameter (0: 15.8kV, 1: 35.4kV, 2: 42.4kV)

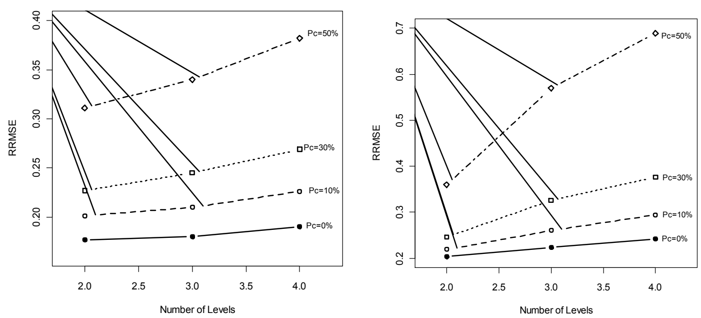
5. SIMULATION RESULTS

The rank regression method is intuitively appealing because it requires simple calculation for parameter estimation. Meanwhile, it may introduce bias in predicting the lifetimes. The bias is caused by its two-step analysis including the nonparametric CDF computation and succeeding estimation of model parameters (Ross[4], Skinner *et al.*[5], and Somboonsavatdee *et al.*[6]). In this section, we use simulation studies to evaluate the performance of the proposed method and compare it with that of the ML method. From this, we suggest a general guideline as to what situation the propose method can be a potential alternative over the ML method.

Random samples from the Weibull and the lognormal distribution with specified parameter values (m =1.0, 2.0, and 5.0 for Weibull; σ =0.25, 0.5, and 3.0 for lognormal) were generated. The acceleration model was assumed as $\ln\eta = 0 - 2.5x$ for the Weibull, and $\mu = 1 - 2.5x$ for the lognormal, where x is a standardized stress level such that the use level x_0 becomes 0 and the highest level x_h becomes 1. The number of stress levels l was varied from 2 to 4, and different combinations of stress level were assumed such as $(0.5, 1.0)$ for $l = 2$, $(0.3, 0.7, 1.0)$ for $l = 3$, and $(0.25, 0.5, 0.75, 1.0)$ for $l = 4$. We considered the equal number of samples ($n_i = 10, 15, 25$) assigned to each stress level. The samples generated at each stress level were classified as either failure-times or censoring observations such that desired levels of censoring $P_C (= 0\%, 10\%, 30\%, 50\%)$ were satisfied.

Based on the simulated data, we estimated scale parameter η_0 in the Weibull distribution (or location parameter μ_0 in the lognormal distribution) at use condition ($x_0 = 0$) using both the proposed method and the ML method. We repeated the simulation for 1,000 times. To evaluate relative performance of the proposed method, we calculated relative efficiencies (REs), which defined as the ratio of the mean squared error (MSE) of the ML method to that of the proposed method. Resulting REs are summarized in Table 3 for the Weibull distribution and Table 4 for the lognormal distribution.

According to the tables, the proposed approach seems fairly comparable to the ML in estimating the use-condition lifetimes (η_0 and μ_0) even with small samples as long as censoring is not heavy. In fact, no significant difference exists between the two methods for the case of lognormal distribution. Note that the proposed method is more efficient than the ML method in estimating the common shape parameter (m) of Weibull distribution when $l = 2$. Similar results have been reported in the works of Ross[4], Skinner *et al.*[5], and Somboonsavatdee *et al.*[6] for the case of $l = 1$ with small sample sizes. In other cases, as expected, the ML method performs significantly better than the proposed method for the estimation of Weibull shape parameter. Also, the MSE of the ML method is much smaller in all cases of estimating the scale parameter (σ) of lognormal distribution. Such disadvantage that the rank regression has over the ML method becomes more clear when more than 2 stress levels are employed. We observed that the rank regression is likely to produce the larger MSE as the number of stress levels (and experimental size) increases (see Figure 6). In conclusion, under light censoring, the proposed method may be an alterative over the ML method in extrapolating the lifetimes at the use condition from small-scaled accelerated testing data with limited number of stress levels.



(a) Weibull ($m=2$) (b) lognormal ($\sigma=0.5$)

Figure 6. Plots of relative root mean square errors (RRMSE, $\sqrt{\mathrm{MSE}(\hat{m})}/\hat{m}$ or $\sqrt{\mathrm{MSE}(\hat{\sigma})}/\hat{\sigma}$) vs. number of stress levels ($n_i = 15$)

Table 3. REs of the proposed shape estimator ($\hat{m}$) and use-condition scale estimator ($\hat{\eta}_0$) for Weibull distribution

			$l=2$		$l=3$		$l=4$	
n_i	m	P_C	RE_m	RE_{η_0}	RE_m	RE_{η_0}	RE_m	RE_{η_0}
10	1	0.0	1.201	0.924	0.785	0.897	0.622	0.896
		0.1	1.263	0.866	0.822	0.833	0.633	0.810
		0.3	1.439	0.711	0.838	0.566	0.650	0.519
		0.5	1.660	0.389	0.978	0.235	0.644	0.185
	2	0.0	1.143	0.952	0.817	0.925	0.604	0.932
		0.1	1.209	0.903	0.814	0.879	0.610	0.882
		0.3	1.350	0.768	0.885	0.692	0.647	0.660
		0.5	1.659	0.520	0.946	0.370	0.694	0.319
	5	0.0	1.136	0.922	0.887	0.944	0.598	0.949
		0.1	1.226	0.896	0.900	0.934	0.599	0.914
		0.3	1.398	0.838	0.939	0.804	0.612	0.701
		0.5	1.701	0.719	1.049	0.531	0.677	0.415
15	1	0.0	1.017	0.878	0.782	0.919	0.593	0.913
		0.1	1.077	0.852	0.774	0.846	0.605	0.822
		0.3	1.127	0.706	0.773	0.723	0.592	0.658
		0.5	1.429	0.454	0.822	0.280	0.608	0.244
	2	0.0	1.028	0.944	0.790	0.930	0.581	0.929
		0.1	1.079	0.912	0.797	0.899	0.561	0.878
		0.3	1.105	0.839	0.799	0.800	0.566	0.759
		0.5	1.333	0.636	0.864	0.431	0.594	0.361
	5	0.0	1.031	0.911	0.704	0.947	0.601	0.939
		0.1	1.102	0.898	0.742	0.912	0.601	0.906
		0.3	1.130	0.887	0.743	0.839	0.586	0.807
		0.5	1.429	0.734	0.831	0.554	0.602	0.436
25	1	0.0	0.996	0.929	0.727	0.907	0.643	0.933
		0.1	1.004	0.890	0.743	0.889	0.627	0.914
		0.3	1.063	0.750	0.721	0.702	0.602	0.710
		0.5	1.141	0.585	0.704	0.504	0.574	0.485
	2	0.0	0.980	0.919	0.725	0.933	0.591	0.943
		0.1	0.994	0.899	0.734	0.916	0.585	0.941
		0.3	1.042	0.835	0.708	0.800	0.567	0.791
		0.5	1.116	0.756	0.707	0.615	0.561	0.590
	5	0.0	0.941	0.921	0.780	0.919	0.631	0.960
		0.1	0.982	0.914	0.784	0.919	0.636	0.945
		0.3	1.069	0.872	0.774	0.830	0.575	0.826
		0.5	1.136	0.829	0.776	0.706	0.556	0.646
Average			1.187	0.810	0.805	0.759	0.606	0.737

Table 4. REs of the proposed scale estimator ($\hat{\sigma}$) and use-condition location estimator ($\hat{\mu}_0$) for lognormal distribution

			l = 2		l = 3		l = 4	
n_i	σ	P_C	RE_σ	RE_{μ_0}	RE_σ	RE_{μ_0}	RE_σ	RE_{μ_0}
10	0.25	0.0	0.685	0.976	0.423	0.955	0.343	0.941
		0.1	0.720	0.982	0.400	0.974	0.318	0.964
		0.3	0.696	0.978	0.344	0.970	0.257	0.973
		0.5	0.678	0.955	0.278	0.817	0.197	0.730
	0.50	0.0	0.620	0.954	0.442	0.938	0.355	0.926
		0.1	0.651	0.971	0.411	0.961	0.322	0.955
		0.3	0.646	0.988	0.337	0.972	0.260	0.976
		0.5	0.607	0.966	0.273	0.825	0.201	0.718
	3.00	0.0	0.657	0.949	0.426	0.975	0.343	0.932
		0.1	0.673	0.962	0.402	0.982	0.319	0.961
		0.3	0.679	0.982	0.320	0.960	0.247	0.974
		0.5	0.667	0.990	0.261	0.805	0.192	0.714
15	0.25	0.0	0.660	0.958	0.505	0.954	0.383	0.933
		0.1	0.707	0.971	0.475	0.969	0.356	0.962
		0.3	0.712	0.975	0.427	0.970	0.316	0.968
		0.5	0.667	0.958	0.326	0.823	0.227	0.660
	0.50	0.0	0.671	0.967	0.495	0.951	0.381	0.956
		0.1	0.701	0.976	0.476	0.968	0.360	0.979
		0.3	0.711	0.978	0.428	0.971	0.320	0.986
		0.5	0.676	0.976	0.315	0.809	0.229	0.722
	3.00	0.0	0.669	0.959	0.480	0.964	0.387	0.943
		0.1	0.705	0.967	0.458	0.974	0.359	0.981
		0.3	0.699	0.969	0.412	0.987	0.311	0.985
		0.5	0.671	0.971	0.315	0.854	0.227	0.738
25	0.25	0.0	0.680	0.953	0.543	0.954	0.455	0.965
		0.1	0.718	0.961	0.542	0.961	0.454	0.975
		0.3	0.710	0.973	0.458	0.968	0.357	0.960
		0.5	0.713	0.961	0.400	0.906	0.300	0.833
	0.50	0.0	0.699	0.955	0.552	0.938	0.457	0.940
		0.1	0.735	0.961	0.562	0.951	0.447	0.951
		0.3	0.744	0.966	0.460	0.972	0.365	0.963
		0.5	0.710	0.963	0.390	0.907	0.311	0.857
	3.00	0.0	0.745	0.959	0.515	0.959	0.441	0.949
		0.1	0.772	0.961	0.532	0.968	0.434	0.954
		0.3	0.744	0.969	0.476	0.973	0.354	0.967
		0.5	0.733	0.967	0.418	0.909	0.300	0.856
Average			0.693	0.967	0.424	0.936	0.330	0.910

6. CONCLUSION

In this work, we propose a new rank regression model to extrapolate the product lifetimes at normal operation environment from accelerated testing data. The main goal in this article is not to recommend new estimators over the MLE, but to explore and extend conventional rank regression techniques to accelerated life testing data when the ML method is not available. We particularly emphasize its accessibility and ease to implement by general practitioners. Based on the simulation results, we confirm that employing the proposed method, rather than the ML method, does not bring substantial depreciation in performance of extrapolating the use-condition lifetimes even with small samples as long as censoring is not heavy. Further works may include theoretical investigation along with extensive simulation studies in order to understand statistical convergence properties of the proposed method.

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