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HEURISTIC APPROACH TO WORKFORCE SCHEDULING WITH COMBINED SAFETY AND PRODUCTIVITY OBJECTIVE

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The workforce scheduling with combined safety and productivity objective is introduced. Typically, industrial workers are assigned to perform a set of tasks with certain safety/ergonomics hazard. To reduce their hazard exposures, it is recommended that workers be rotated among the tasks periodically within each day so that the hazard exposures do not exceed a permissible limit. Since workers have unequal skill levels, this job rotation practice can affect total system productivity. Thus, assigning the right workers to the right tasks in each period can help to increase the productivity and, at the same time, achieve the safety objective. In this paper, we present a safety-productivity workforce scheduling model. From a given numerical example, we compare the optimal solutions obtained from the productivity-based, safety-based, and safety-productivity workforce scheduling models. A heuristic approach to the workforce scheduling with combined safety and productivity objective is also developed for solving large-sized problems.

Significance: While job rotation can help to enhance workplace safety, it is likely to reduce the system productivity since workers might be assigned to perform the tasks that they are not competent. This workforce scheduling problem considers both safety and productivity when developing the worker-task-period assignment solution. Thus, both safety and productivity objectives can be simultaneously achieved.

Keywords: Workforce scheduling, optimization, safety, hazard exposure reduction, job rotation

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1. INTRODUCTION

Workers are usually exposed to one or more occupational hazards when they perform their assigned tasks. For their safety, employers are required to eliminate hazard sources, prevent workers from exposure to such hazards, or enforce them to wear effective personal protection equipment. Job rotation is a common practice for reducing the workers' exposure to occupational hazards (Sanders and McCormick, 1993; Asfahl, 2004; Goetsch, 2005). However, its implementation as described in the literature is rather vague, causing most practitioners to believe that job rotation is, literally, a simple task to perform.

Given a set of resources (e.g., workers) and a set of tasks, the problem of allocating resources to tasks so as to achieve the optimal pairing or matching is normally called the assignment problem. The assignment problem can be modeled as a linear programming problem and solved to optimality (Rardin, 1998; Hiller and Lieberman, 2001). When the resources are to be allocated over time, the problem then becomes the scheduling problem. This problem has already been studied extensively, with its emphases on non-human resources in various manufacturing systems such as machines. As for the service sector, workforce scheduling is also an important issue. In the transportation industry, for example, the scheduling of airline crew and bus/train drivers has received considerable attention from researchers (Pinedo and Chao, 1999; Yang and Yu, 2004; Kwan, 2004).

Nanthavanij and Yenradee (1999) proposed a quantitative approach to job rotation by developing a mathematical model for the problem with equal numbers of workers and tasks. Their solution described the work assignments for workers in different periods of the day such that the maximum noise hazard exposure is minimized. They also investigated the effect of work period length on the noise hazard reduction (Nanthavanij and Yenradee, 2000a). Later, they developed a mathematical model to determine the minimum number of workers for job rotation (Nanthavanij and Yenradee, 2000b). For complex safety-based job rotation problem, a genetic algorithm (GA) approach has been applied to obtain the minimax work assignment solution (Nanthavanij and Kullpattaranirun, 2001; Kullpattaranirun and Nanthavanij, 2005).

Yaoyuenyong (2006) showed that when the minimum number of workers for job rotation is to be determined, the workforce scheduling problem is a variant of the bin packing problem, which is an *NP* problem. Yaoyuenyong and Nanthavanij (2006) described a hybrid procedure to determine an optimal workforce without being exposed to excessive noise hazard in the manufacturing environment. Additionally, they developed heuristic job rotation procedures for workers who are exposed to single-limit and multiple-limit occupational hazards (2008).

Jaturanonda and Nanthavanij (2002) studied the multi-objective job rotation problem in which competency and job satisfaction are considered when matching workers to tasks. To achieve the maximum person-job fit, they developed mathematical models to optimize the worker-task assignment to obtain competency-based and preference-based person-job fit (Jaturanonda and Nanthavanij, 2005).

It is seen that the previous research studies in workforce scheduling were performed with only one objective, either to maximize the system productivity or to maximize the worker safety. In this paper, we develop a safety-productivity workforce scheduling model to schedule the minimum number of workers to perform a set of tasks such that the worker's hazard exposure does not exceed the daily permissible limit and the productivity is maximized. We also introduce a heuristic procedure to obtain the safety worker-task-period assignments with the maximum productivity.

2. WORKFORCE SCHEDULING MODELS

Job rotation is an administrative approach to occupational hazard control. Through job rotation, workers are prevented from being consistently exposed to a certain occupational hazard. When safety is of concern, workers are not permitted to be exposed to a given occupational hazard beyond a permissible level within an 8-hour workday. Job rotation allows workers to switch to other tasks (preferably at different work locations) during a workday. When synchronous job rotation is implemented, a workday is divided into discrete work periods and the decision whether to rotate workers or not is made at the end of each work period. While some workers are required to switch to perform other tasks, some are allowed to continue with their current tasks. If task idles are not allowed, at the minimum, the number of workers will equal the number of tasks. However, if the levels of hazard exposure at some tasks are high, it is common to need more workers than the number of tasks for safety job rotation.

Conventionally, workers are assigned to the tasks that they are highly competent to perform so as to maximize the work system productivity. Competency is an ability to perform the task well or effectively. In general, a standard measure is used to assess the level of competency among workers. It is typical for a worker to be able to perform several tasks but with different levels of competency. As a result, it is preferable to assign each worker to the task that he/she can perform best, and not rotating him/her to other tasks.

Although job rotation can help to enhance workplace safety by reducing the worker's exposure to occupational hazard, it can affect the work system productivity since the competency in performing the task is not considered. In fact, assigning a given task to the worker who can perform it effectively will definitely help to increase the system productivity. If this consideration is included in job rotation, it is anticipated that the enhanced workplace safety and increased system productivity can be simultaneously achieved.

In this paper, we assume that the competency level is classified into five competency scores, namely, 1 to 5, where score 1 represents the lowest competency level and score 5 the highest competency level. When a worker performs several tasks in one day, a total competency score is determined by summing his/her corresponding task competency scores. As for the worker-task-period assignment solution, a grand total competency score is the sum of the workers' total competency scores.

Next, we discuss two conventional approaches, namely, productivity-based assignment and safety-based workforce scheduling. Later, we present the safety-productivity workforce scheduling model.

2.1 Productivity-based Assignment Problem

The productivity-based assignment problem is a problem dealing with allocating a group of workers to perform a set of tasks so as to maximize the pairing or matching between them. When the work system productivity is of concern, the problem objective then is to maximize the grand total competency score.

Assumptions

The productivity-based assignment problem has the following assumptions.

1. Each worker can perform all tasks, but with different skill levels.
2. Each worker can be assigned to perform only one task.
3. Each task requires only one worker to perform.
4. The number of workers is equal to or greater than the number of tasks. That is, worker idles are allowed.
5. No task idles are allowed.

Mathematical Model

The following notation is used.

- M number of available workers in the worker team
- n number of tasks
- s_{ij} competency score of worker i when performing task j
- x_{ij} 1 when worker i is assigned to task j
  0 otherwise

Then,

$$\text{Maximize } \sum_{i=1}^{M}\sum_{j=1}^{n} s_{ij}x_{ij} \quad \dots \quad (1)$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le 1 \quad \text{for } i = 1,.., M \quad \dots \quad (2)$$

$$\sum_{i=1}^{M} x_{ij} = 1 \quad \text{for } j = 1,.., n \quad \dots \quad (3)$$

$$x_{ij} = (0, 1) \quad \text{for } \forall\ i, j \quad \dots \quad (4)$$

Safety-based Workforce Scheduling Problem

The safety-based workforce scheduling problem is intended to determine the optimal worker-task-period assignment solution such that the required number of workers is minimized and each worker's hazard exposure level does not exceed a daily permissible limit. Here it is assumed that this permissible limit is the same for every worker. Although the problem does not consider the productivity issue at first, the workers' competency scores are used in computing the grand total competency score of the resulting worker-task-period assignment solution.

Assumptions

The safety-based workforce scheduling problem has the following assumptions.

1. Each worker can be assigned to perform only one task in each work period.
2. Each task requires only one worker to perform in each work period.
3. The number of workers is equal to or greater than the number of tasks. That is, worker idles are allowed.
4. No task idles are allowed.
5. The worker's hazard exposure in an 8-hour workday must not exceed the daily permissible limit.

Mathematical Model

The additional variables are defined as follows.

- p number of work periods per workday
- L permissible daily hazard exposure level
- h_j hazard exposure amount per work period of task j
- x_{ijk} 1 when worker i is assigned to task j in period k
  0 otherwise
- y_i 1 when worker i is selected from the worker team
  0 otherwise

Then,

$$\text{Minimize } \sum_{i=1}^{M} y_i \quad \dots \quad (5)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{p} h_j x_{ijk} \le L \quad \text{for } i = 1,.., M \quad \dots \quad (6)$$

$$\sum_{j=1}^{n} x_{ijk} \leq 1 \quad \text{for } i = 1,.., M;\ k = 1,.., p \qquad \text{...} \qquad (7)$$

$$\sum_{i=1}^{M} x_{ijk} = 1 \quad \text{for } j = 1,.., n;\ k = 1,.., p \qquad \text{...} \qquad (8)$$

$$x_{ijk} \leq y_i \quad \text{for } \forall\ i, j, k \qquad \text{...} \qquad (9)$$

$$x_{ijk}, y_i = (0, 1) \quad \text{for } \forall\ i, j, k \qquad \text{...} \qquad (10)$$

2.3 Workforce Scheduling Problem with Combined Safety and Productivity Objective

The workforce scheduling problem with combined safety and productivity objective is intended to maximize the work system productivity while assuring the safety of all workers. In terms of the occupational safety, the result must yield a worker-task-period assignment solution that none of the workers receives the hazard exposure beyond the daily permissible limit. To maximize the work system productivity, the tasks must be selectively assigned to workers so as to yield the maximum grand total competency score. Note that the number of workers used in this problem can be determined by solving the model in Section 2.2.2 or obtained from the M-LPT swap heuristic described in Yaoyuenyong and Nanthavanij (2006).

Assumptions

The workforce scheduling problem with optimal safety and productivity has the following assumptions.

1. Each worker can perform all tasks, but with different skill levels.
2. Each worker can be assigned to perform only one task in each work period.
3. Each task requires only one worker to perform in each work period.
4. The number of workers must not exceed the minimum number of workers obtain earlier.
5. No task idles are allowed.
6. The worker's hazard exposure in an 8-hour workday must not exceed the daily permissible limit.

Mathematical Model

An additional variable is defined as follows.

m^* number of workers required for job rotation

Then,

$$\text{Maximize} \quad \sum_{i=1}^{M}\sum_{j=1}^{n}\sum_{k=1}^{p} s_{ij} x_{ijk} \qquad \text{...} \qquad (11)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{p} h_j x_{ijk} \leq L \quad \text{for } i = 1,.., M \qquad \text{...} \qquad (12)$$

$$\sum_{j=1}^{n} x_{ijk} \leq 1 \quad \text{for } i = 1,.., M;\ k = 1,.., p \qquad \text{...} \qquad (13)$$

$$\sum_{i=1}^{M} x_{ijk} = 1 \quad \text{for } j = 1,.., n;\ k = 1,.., p \qquad \text{...} \qquad (14)$$

$$x_{ijk} \leq y_i \quad \text{for } \forall\ i, j, k \qquad \text{...} \qquad (15)$$

$$\sum_{i=1}^{M} y_i = m^* \qquad \text{...} \qquad (16)$$

$x_{ijk}, y_i \;=\; (0, 1) \qquad \text{for } \forall\, i, j, k \qquad \ldots \qquad (17)$

2.4 Workforce Scheduling Index

To compare the assignment/scheduling solutions from the three mathematical models in Sections 2.1-2.3, two indices are proposed: (1) productivity index I_p, and (2) safety index I_s. The productivity index I_p is intended to measure an effectiveness of the solution based on the productivity (as reflected by the grand total competency score). The total productivity is determined from all resulting worker-task pairs and normalized by the product of the number of tasks and that of work period. The productivity index I_p can be computed from

$$I_p \;=\; \frac{1}{n \cdot p}\left[\sum_{i=1}^{M}\sum_{j=1}^{n}\sum_{k=1}^{p} s_{ij}x_{ijk}\right] \qquad \ldots \qquad (18)$$

Since job rotation is not applied in the productivity-based assignment model, the number of work period per day p equals 1. Note that I_p cannot exceed the maximum competency score.

The safety index I_s is a comparative measure of the safety of those workers assigned to perform the tasks. Since hazard exposure presents a risk of illness or accident, the lower the exposure is, the safer the worker. When considering the worker team as a collective not as individuals, it is important that every team member is equally safe. In other words, the workers' hazard exposures should be ideally equal and within the daily permissible limit. If any worker's hazard exposure exceeds such limit, the safety requirement is not met. When all hazard exposures are within the limit, a variance of the hazard exposures could then be used to measure the variability of the workers' safety levels. Letting H_i be the daily hazard exposure amount of worker i, $\bar{H}$ be an average hazard exposure amount determined from all workers, m' be the number of assigned workers, and **M** be a set of workers assigned to the tasks, the safety index I_s is defined as follows.

$$I_s \;=\; \frac{1}{m'-1}\left[\sum_{i\in \mathrm{M}}\left(H_i - \bar{H}\right)^2\right] \qquad \ldots \qquad (19)$$

where $H_i \leq L$ for $\forall\, i \in \mathbf{M}$. Also, note that the lower the safety index I_s is, the better the workforce scheduling solution.

3. NUMERICAL EXAMPLE

Consider a workplace wherein there are eight tasks (T1, T2, …, T8) to be performed. These tasks are performed at different locations in the workplace. A team of twelve workers (W1, W2, …, W12) can be assigned to perform the given eight tasks. Table 1 shows the competency score matrix based on the given twelve workers and eight tasks.

Table 1. Competency score matrix

Worker	Task							
	T1	T2	T3	T4	T5	T6	T7	T8
W1	2	3	5	3	4	5	3	2
W2	5	5	5	2	3	2	4	4
W3	5	4	2	5	4	3	3	5
W4	4	4	3	2	5	3	5	4
W5	5	2	5	3	2	5	5	5
W6	5	2	4	3	3	3	4	4
W7	4	3	2	2	2	5	2	3
W8	2	5	5	3	4	4	5	3
W9	2	5	4	5	4	3	4	5
W10	3	2	4	5	5	5	4	5
W11	5	5	3	3	4	4	3	4
W12	3	2	5	4	3	4	5	5

In this paper, workplace noise is considered as the occupational hazard. The daily permissible noise exposure that any worker is allowed to receive is 90 dB*A* (8-hour time weighted average sound level), which is equivalent to a daily noise dose of 1.0000. We also assume that an 8-hour workday is divided into four 2-hour work periods. When performing task j and being exposed to noise level L_j (in dB*A*), the noise exposure per work period (or noise weight h_j) is

$$h_j = \frac{1}{p} \cdot 2^{\frac{L_j - 90}{5}} \qquad \ldots \qquad (20)$$

The assumed noise levels and their computed noise weights at the eight task locations are shown in Table 2.

Table 2. Noise levels (L_j, dBA) and noise weights per work period (h_j)

Task	L_j	h_j	Task	L_j	h_j
T1	88	0.1895	T5	97	0.6598
T2	92	0.3299	T6	94	0.4353
T3	83	0.0947	T7	89	0.2176
T4	85	0.1250	T8	84	0.1088

3.1 Productivity-based Assignment Solution

To obtain the optimal worker-task assignment solution, the assignment model in Section 2.1.2 is solved to optimality using an optimization software program called ILOG CPLEX 11.1.1. The solution shows that from the given worker team, eight workers are selected to perform these eight tasks. The worker-task assignment solution is summarized in Table 3.

Table 3. Optimal worker-task assignment solution

Worker	Task	Noise Level (dBA)	Competency Score	Worker	Task	Noise Level (dBA)	Competency Score
W1	T3	83	5	W7	T6	94	5
W2	T1	88	5	W8	-	-	-
W3	T4	85	5	W9	T2	92	5
W4	T5	97	5	W10	T8	84	5
W5	-	-	-	W11	-	-	-
W6	-	-	-	W12	T7	89	5

Using Eq. 18, the productivity index I_p of the above worker-task assignment solution is equal to 5.00 which is also the highest I_p. A very high I_p is expected since the model has been formulated to obtain the solution with the highest productivity. In terms of safety, the above worker-task assignment solution fails to provide adequate safety since workers W4, W7, and W9 receive the daily noise exposure that exceeds 90 dBA. Since not all hazard exposures are within the permissible limit, I_s cannot be determined.

3.2 Safety-based Workforce Scheduling Solution

Using job rotation, a sufficient number of workers are to be rotated among the eight tasks such that their daily noise exposure amount (i.e., daily noise dose) does not exceed 1.0000. Depending on the noise levels at the task locations, the number of workers could be as small as eight persons, or larger than that if the noise levels are high. The mathematical model described in Section 2.2.2 is solved and the optimal solution shows that nine workers are required for job rotation. Table 4 shows the worker-task-period assignments for the nine workers and their total competency scores. Workers W1, W4, and W11 are not included in those workers for job rotation. Among the selected nine workers, there are four workers who receive a three-period work assignment. Also, it can be seen that none of the nine workers receives the daily noise dose beyond 1.0000.

From Eqs. 18 and 19, the workforce scheduling indices are computed: I_p = 3.94 and I_s = 0.0337. This low productivity index is due to the fact that the scheduling of tasks to workers is not based on productivity, but on safety only.

Table 4. Safety-based worker-task-period assignments

Worker	Work Period				Noise Dose	Sum of Competency Score
	1	2	3	4		
W1	-	-	-	-	-	-
W2	T3	T3	T5	T4	0.9742	15
W3	T4	T6	T1	T7	0.9674	16
W4	-	-	-	-	-	-
W5	T7	T2	T7	T1	0.9546	17
W6	-	T1	T4	T5	0.9743	11
W7	T1	-	T6	T2	0.9547	12
W8	T8	T5	T8	-	0.8774	10
W9	T2	T8	T3	T6	0.9687	17
W10	T5	T7	-	T3	0.9721	13
W11	-	-	-	-	-	-
W12	T6	T4	T2	T8	0.9990	15

3.3 Safety-Productivity Workforce Scheduling Solution

Readers can see that the assignment and workforce scheduling solutions shown in Sections 3.1 and 3.2 are based on two *pure* considerations. The former is only concerned with productivity whereas the latter is based only on safety. The mathematical model described in Section 2.3.2 utilizes a so-called *hybrid* consideration since it considers both safety and productivity issues when finding the workforce scheduling solution. It should be noted that between the two issues, safety must come first since it is required by the safety law. That is, the workforce scheduling solution must firstly comply with the safety requirement even though the productivity may not be as high as that of the productivity-based solution.

The optimal safety-productivity workforce scheduling solution also requires nine workers for job rotation. However, a different set of workers are selected from the worker team. As expected, none of the selected nine workers receives the daily noise dose beyond 1.0000. Table 5 shows the optimal worker-task-period assignment solution. Based on the safety-productivity workforce scheduling model, the workforce scheduling indices are: I_p = 4.84 and I_s = 0.0350.

Table 5. Safety-productivity worker-task-period assignments

Worker	Work Period				Noise Dose	Sum of Competency Score
	1	2	3	4		
W1	T3	-	T6	T6	0.9653	15
W2	T2	T1	T2	T3	0.9440	20
W3	-	T5	T4	T4	0.9098	14
W4	T5	T8	-	T7	0.9862	14
W5	T6	T6	T8	-	0.9794	15
W6	-	-	-	-	-	-
W7	-	-	-	-	-	-
W8	T7	T7	T7	T2	0.9827	20
W9	T4	T3	T5	T8	0.9883	18
W10	T8	T4	T3	T5	0.9883	19
W11	T1	T2	T1	T1	0.8984	20
W12	-	-	-	-	-	-

4. HEURISTIC PROCEDURE

In this section, we introduce a heuristic procedure to solve the safety-productivity workforce scheduling problem so that all workers are not exposed to any given occupational hazard beyond the daily permissible limit and the work system productivity is maximized. The heuristic procedure consists of three phases as follows.

Phase I: Determining the minimum number of workers
Phase II: Selecting workers for job rotation
Phase III: Scheduling the selected workers

4.1 Determining the Minimum Number of Workers

The first phase of the heuristic procedure is intended to determine the minimum number of workers for job rotation. The procedure below was developed by Yaoyuenyong (2006) to determine the lower bounds when dealing with single-limit occupational hazards. It was modified from the lower bounds for the bin packing problem developed by Martello and Toth (1990a, 1990b).

The lower bound *LB* of the number of workers for job rotation is determined from

$$LB = \max\{LB_1, LB_2, n\} \quad \ldots \quad (21)$$

where n = number of tasks.

Finding LB_1

Letting t_j be subtask of task j and **I** be a set of subtasks of all tasks to be assigned in individual work periods ($|\mathbf{I}| = n \cdot p$), LB_1 can be obtained from

$$LB_1 = \left\lceil \frac{\sum_{t_j \in \mathrm{I}} h_j}{L} \right\rceil \quad \ldots \quad (22)$$

Finding LB_2

Firstly, the subtasks t_j for $\forall j$ are separated into three sets according to their w_j's and α where $0 \le \alpha \le 0.5$.

Given

$$J1 = \{t_j \in \mathrm{I} : w_j > 1 - \alpha\}, \quad \ldots \quad (23)$$

$$J2 = \{t_j \in \mathrm{I} : 0.5 < w_j \le 1 - \alpha\}, \quad \ldots \quad (24)$$

$$J3 = \{t_j \in \mathrm{I} : \alpha \le w_j \le 0.5\}, \quad \ldots \quad (25)$$

then

$$L(\alpha) = \quad \ldots \quad (26)$$

$$|J_1| + |J_2| + \max\left\{0, \left\lceil \sum_{t_j \in J_3} w_j - \left(|J_2| - \sum_{t_j \in J_2} w_j\right)\right\rceil\right\}$$

Finally, LB_2 is obtained from

$$LB_2 = \max\{L(\alpha) : 0 \le \alpha \le 0.5\} \quad \ldots \quad (27)$$

4.2 Selecting Workers for Job Rotation

As far as safety is concerned, all workers are identical when being exposed to single-limit hazards. That is, it does not matter who in the worker team will be selected for job rotation. Thus, the selection of workers will be based on the workers' competency. The selection procedure is summarized as follows

Step 1: Let $s_{\max}$ be the highest competency score. For each worker in the worker team, determine the number of $s_{\max}$'s that the worker has. If none of the workers has the competency score of $s_{\max}$, then determine the number of ($s_{\max}$-1)'s. If none still exists, then consider ($s_{\max}$-2). And, so forth.

Step 2: Arrange workers in the worker team according to non-increasing order of the number of $s_{\max}$'s from Step 1. In case there are ties, list the worker whose number of the highest competency score is larger first. Then, consider ($s_{\max}$ -1). And, so forth.

Step 3: Draw one worker at a time from the rearranged list of those in the worker team until the number of selected workers equals *LB*.

4.3 Scheduling the Selected Workers

The heuristic procedure tries to generate a set of safety worker-task-period assignments for *LB* workers, *n* tasks, and *p* periods. In these assignments, if there is no worker whose sum of hazard exposures from all periods exceeds the permissible limit L, these assignments are feasible. If at least one worker has the sum of hazard exposures which exceeds L, the procedure then tries to generate the assignments for LB+1 workers. Again, if the feasible assignments for LB+1 workers are not found, the procedure continues to generate the assignments for LB+2 workers. And, so forth. The procedure stops when it is able to generate the feasible assignments for the increased number of workers.

The procedure can be divided into three steps.

Step 1: *Generating worker-task-period assignments*

Suppose that there are LB workers being considered. Construct an assignment table with LB rows (for LB workers) and p columns (for p periods). Re-list all n tasks in non-increasing order of h_j, and consider the n tasks accordingly. Let l_i be the sum of h_j's from all tasks currently assigned to worker i in all work periods. When considering task j, the p copies of task j (representing task j for period 1, task j for period 2, and so on) must be assigned. To assign each copy of task j, do the following:

(1) Find any worker i whose l_i is currently the smallest. In case of ties, select worker i whose s_{ij} is the largest.

(2) Find any period k that (a) worker i has not yet been assigned and (b) no other copy of task j has been assigned to any worker in period k. If there are ties, break the tie arbitrarily. If task j cannot be assigned to any of the available periods of worker i, try to resolve the conflict by re-shuffling copies of task j among previously assigned workers. If the conflict cannot be resolved, skip to the next worker.

Then, assign this copy of task j to row i and column k of the assignment table. Keep on assigning the copies of each task until all tasks have been assigned. If the resulting work assignments for LB workers are feasible (i.e, all l_i's do not exceed L), then stop. If the resulting assignments for LB workers are not feasible, go to the next step.

Step 2: *Exchanging tasks to make the assignments feasible*

The procedure tries to make any currently infeasible assignments feasible by exchanging tasks among workers in any work period. It starts with any worker, say, worker a, whose $l_a > L$. Then, it tries to exchange jobs currently assigned to worker a and another worker's jobs, say, worker b, so that $l_a \leq L$ and $l_b \leq L$. The procedure searches for any exchange in all possible pairs among workers. If the procedure can make the assignments feasible, then stop. If not, increase the number of workers by 1 and return to Step 1.

Step 3: *Improving the solution by transforming to the assignment problem*

The procedure tries to improve the current solution by transforming the solution to a sub-problem close to the classical assignment problem. Again, M is the number of available workers in the worker team. Usually, the solution generated in Step 2 requires a lesser number of workers, say, m_2. Letting P_u be a task-period schedule currently assigned to worker u from the solution generated in Step 2, call it pattern u. Let c_{iu} be the sum of competency scores of worker i providing that worker i is assigned to follow the task-period schedule P_u, or pattern u. Thus,

$$c_{iu} = \sum_{j \in P_u} s_{ij} \qquad \ldots \qquad (28)$$

Then, solve the following sub-problem:

$$\text{Maximize } Z = \sum_{i=1}^{M} \sum_{u=1}^{m_2} c_{iu} y_{iu} \qquad \ldots \qquad (29)$$

subject to

$$\sum_{u=1}^{m_2} y_{iu} \leq 1 \qquad \text{for } i = 1,.., M \qquad \ldots \qquad (30)$$

$$\sum_{i=1}^{M} y_{iu} = 1 \qquad \text{for } u = 1,.., m_2 \qquad \ldots \qquad (31)$$

$$y_{iu} \geq 0 \qquad \text{for } i = 1,.., M;\ u = 1,.., m_2 \qquad \ldots \qquad (32)$$

where y_{iu} denotes that worker i is assigned to follow the task-period schedule pattern u.

Because this sub-problem has the structure close to the classical assignment problem; therefore, an optimal integer solution for this sub-problem can be obtained in polynomial time and y_{iu} will be either 0 or 1. Hence, y_{iu} equals 1 if worker i is chosen to follow the task-period schedule pattern u, 0 otherwise.

4.4 Illustration

Consider the same example as that used in Section 3. The heuristic procedure described in Sections 4.1 – 4.3 is employed to find the worker-task-period assignments.

In the first phase, the lower bound of the number of workers for job rotation is determined using the procedure described in Section 4.1. From Eq. (22), we obtain $LB_1 = 9$. Next, from the given h_j's, we consider the following α's: 0.4353; 0.3299; 0.2176; 0.1895; 0.1250; 0.1088; 0.0947. Using Eqs. (23) – (27), we obtain $L(0.4353) = 6$; $L(0.3299) = 6$; $L(0.2176) = 7$; $L(0.1895) = 8$; $L(0.1250) = 8$; $L(0.1088) = 9$; $L(0.0947) = 9$. Then, we have $LB_2 = \max\{6,6,7,8,8,9,9\} = 9$. Finally, the lower bound LB is obtained from $LB = \max\{9,9,8\} = 9$.

In the second phase, we use the selection procedure described in Section 4.2 to select nine workers from the worker team. A list of workers according to non-increasing order of the number of competency scores of 5 is W5, W10, W9, W3, W8, W12, W2, W11, W4, W1, W6, and W7, among which the first nine workers are selected for job rotation.

In the third phase, we use Steps 1 and 2 of the procedure described in Section 4.3 to determine the worker-task-period assignments for workers W2, W3, W4, W5, W8, W9, W10, W11, and W12. The *initial* solution presented in Table 6 shows that the productivity index I_p is 4.44 and the safety index is $I_s = 0.0286$. So far, the procedure is also able to assign those eight tasks to nine workers without exposing them to the noise hazard beyond the daily permissible limit.

Following Step 3 of the procedure, the solution yields nine task-period schedule patterns as follows: $P_1 = \{2, 2, 1, 8\}$; $P_2 = \{8, 5, 1\}$; $P_3 = \{5, 1, 8\}$; $P_4 = \{6, 7, 7, 3\}$; $P_5 = \{4, 8, 3, 5\}$; $P_6 = \{3, 4, 2, 6\}$; $P_7 = \{1, 5, 4\}$; $P_8 = \{6, 4, 2\}$; $P_9 = \{7, 3, 6, 7\}$. Also, $M = 12$ and $m_2 = 9$.

Next, generate the matrix c_{iu} for all i's and u's. For example, if $i = 6$ and $u = 2$, we have

$$c_{62} = \sum_{j \in P_2} s_{6j} = s_{68} + s_{65} + s_{61} = 4 + 3 + 5 = 12$$

Table 7 shows an assignment table for the twelve workers (for i = 1 to 12) and nine task-period schedule patterns (for u = 1 to 9) with the value shown in cell (i, u) being c_{iu}. By formulating this assignment table as the sub-problem in Eqs. (29) – (32) and solve it to optimality, the optimal worker-pattern assignment solution can be obtained (see Table 8). The productivity index I_p and safety index I_s are 4.59 and 0.0286, respectively. Readers can see that in the *improved* solution, I_p is increased by 3.38% whereas I_s remains unchanged. Table 9 summarizes the comparison among the results from the three mathematical models and the heuristic procedure.

Table 6. *Initial* worker-task-period assignments (from Steps 1 and 2)

Worker	Work Period				Noise Dose	Sum of Competency Score
	1	2	3	4		
W1	-	-	-	-	-	-
W2	T2	T2	T1	T8	0.9581	19
W3	T8	-	T5	T1	0.9581	14
W4	T5	T1	T8	-	0.9581	13
W5	T6	T7	T7	T3	0.9652	20
W6	-	-	-	-	-	-
W7	-	-	-	-	-	-
W8	T4	T8	T3	T5	0.9883	15
W9	T3	T4	T2	T6	0.9849	17
W10	T1	T5	-	T4	0.9743	13
W11	-	T6	T4	T2	0.8902	12
W12	T7	T3	T6	T7	0.9652	19

Table 7. Data for the worker-pattern assignment sub-problem

Worker	Schedule Pattern								
	1	2	3	4	5	6	7	8	9
1	10	8	8	16	14	16	9	11	16
2	19	12	12	15	14	14	10	9	15
3	18	14	14	11	16	14	14	12	11
4	16	13	13	16	14	12	11	9	16
5	14	12	12	20	15	15	10	10	20
6	13	12	12	15	14	12	11	8	15
7	13	9	9	11	9	12	8	10	11
8	15	9	9	19	15	17	9	12	19
9	17	11	11	15	18	17	11	13	15
10	12	13	13	17	19	16	13	12	17
11	19	13	13	13	14	15	12	12	13
12	12	11	11	19	17	15	10	10	19

Table 8. *Improved* worker-task-period assignment solution (from Step 3)

Worker	Work Period				Noise Dose	Sum of Competency Score
	1	2	3	4		
W1	-	-	-	-	-	-
W2	T2	T2	T1	T8	0.9581	19
W3	T1	T5	T4	-	0.9743	14
W4	T5	T1	T8	-	0.9581	13
W5	T7	T3	T6	T7	0.9652	20
W6	-	-	-	-	-	-
W7	-	-	-	-	-	-
W8	T3	T4	T2	T6	0.9849	17
W9	-	T6	T4	T2	0.8902	13
W10	T4	T8	T3	T5	0.9883	19
W11	T8	-	T5	T1	0.9581	13
W12	T6	T7	T7	T3	0.9652	19

Table 9. Summary of results from the example

	Workforce Scheduling Model			Heuristic Procedure
	P[1]	S[2]	S-P[3]	
Productivity index I_p	5	3.94	4.84	4.59
Safety index I_p	-	0.0337	0.0350	0.0286
Minimum number of workers	8	9	9	9

[1]Productivity-based assignment model; [2]Safety-based workforce scheduling model; [3]Safety-productivity workforce scheduling model

5. COMPUTATION EXPERIMENT

Fifteen hypothetical workforce scheduling test problems (P1 – P15) were generated. The number of workers range between 6 and 24 persons and the number of tasks range between 4 and 16 tasks. For each test problem, the numbers of workers and tasks were randomly assigned. Also, the worker-task competency scores were randomly generated, with the scores ranging between 1 and 5. Table 10 shows the numbers of workers and tasks assigned to each test problem.

Table 10. Fifteen test problems for the computation experiment

	Test Problem														
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15
Number of workers	6	8	11	9	9	12	12	14	14	15	21	23	23	24	24
Number of tasks	4	5	5	6	6	8	8	9	9	10	14	15	15	16	16

Firstly, the test problems were formulated as the safety-productivity workforce scheduling problem. They were solved by ILOG CPLEX 11.1.1. Among them, twelve problems (P1-P7, P9-P12, and P14) could be solved to optimality. For problems P8 and P13, the solutions that could not be guaranteed to be optimal were obtained and they were worse than the best solutions (when all integer variables were relaxed) by 3.5% and 1.05%, respectively. For problem P15, the best solution could not be found even after 9,000 seconds. The program was terminated due to an out-of-memory error status. The results (I_p, I_s, minimum number of workers, and computation time) are shown in Table 10.

Then, the heuristic procedure was applied to solve the fifteen test problems. The *improved* worker-task-period assignment solutions could be obtained relatively quickly irrespective of the problem size. In our computation experiment, all fifteen test problems were solved successively from problems P1 to P15. The total computation time (for all 15 test problems) was less than one second. Assuming that all problems required equal computation time, the average computation time was approximately 0.07 second. Their results are also shown in Table 11. Readers can see that the heuristic procedure was able to solve large-sized workforce scheduling problems with safety and productivity consideration and obtained good results (when compared to those from ILOG CPLEX).

Table 11. The worker-task-period assignment solutions from ILOG CPLEX and heuristic procedure

Test Problem	Productivity Index I_p		Safety Index I_s		Min. number of workers		Computation Time (sec)	
	CPLEX	Heuristic	CPLEX	Heuristic	CPLEX	Heuristic	CPLEX	Heuristic
P1	3.63	3.63	0	0	4	4	0.17	0.07
P2	4.65	4.25	0.1590	0.0256	5	5	0.11	0.07
P3	4.75	4.45	0.1207	0.0434	10	10	167.00	0.07
P4	4.54	3.96	0.1413	0.0043	6	6	0.11	0.07
P5	4.71	4.50	0.1081	0.0829	8	8	0.08	0.07
P6	4.84	4.44	0.1080	0.0173	8	8	0.11	0.07
P7	4.59	4.44	0.0415	0.0257	10	11	86.32	0.07
P8	4.64[a]	4.61	0.0321[a]	0.0185	9	9	356.87[b]	0.07
P9	4.56	4.19	0.1240	0.0223	13	13	0.70	0.07
P10	4.85	4.40	0.0742	0.0146	10	10	20.55	0.07
P11	4.89	4.59	0.0312	0.0105	14	14	69.61	0.07
P12	4.98	4.67	0.0996	0.0090	15	15	1.54	0.07
P13	4.82[a]	4.55	0.0364[a]	0.0305	18	19	470.69[b]	0.07
P14	5.00	4.64	0.1214	0.0083	16	16	0.08	0.07
P15	-	4.66	-	0.0211	-	24	-	0.07

[a]Not guaranteed to be an optimal solution. [b]The computation time until CPLEX reached the given percentage of the best solution.

6. CONCLUSION

In this paper, we present the safety-productivity workforce scheduling problem wherein both safety and productivity issues are considered concurrently. When the safety-productivity workforce scheduling problem is concerned with only the productivity issue, it is basically the classical assignment problem which will yield the solution with the highest total productivity. However, it is likely that workers might be exposed to certain occupational hazard beyond the daily permissible limit. On the other hand, when the safety-productivity workforce scheduling problem considers only the safety issue, the problem is then reduced to the workforce scheduling problem in job rotation. While the worker-task-period assignment solution yields the safety assignments, the total productivity is usually low since the evaluation of person-job fit is not included in the solution procedure. When both issues are concurrently considered, it is necessary to strictly comply with the safety regulation before attempting to maximize the total productivity.

Three mathematical models are presented for the productivity-based assignment problem, safety-based workforce scheduling problem, and safety-productivity workforce scheduling problem, respectively. Two workforce scheduling indices, namely, productivity index I_p and safety index I_s, are proposed. A relatively small-sized problem (with eight tasks and twelve workers) is solved using the three models separately. When the productivity-based assignment model is employed, the worker-task assignment solution typically results in the highest total productivity (I_p = 5.00) while the safety requirement is not satisfied. On the other hand, when the safety-based workforce scheduling model is employed, the resulting worker-task-period assignments meet the safety requirement with the safety index I_s of 0.0337. However, the total productivity is lowest (I_p = 3.94). The safety-productivity workforce scheduling is able to generate the worker-task-period assignment solution that both meets the safety requirement and yields high productivity (I_s = 0.0350 and I_p = 4.84).

A heuristic procedure is introduced to solve large-sized safety-productivity workforce scheduling problems. The procedure consists of three phases: (1) determining the lower bound of the number of workers for job rotation, (2) selecting workers from the given worker team, and (3) scheduling the selected workers. In the third phase, an initial solution is firstly generated. Then, it is re-formulated as the assignment problem to find a new solution with the improved productivity index. From the computation experiment, it is seen that the average productivity index of the solutions from the heuristic procedure is 6.72% lower than that of the solutions from CPLEX (with a standard deviation of 3.22%); whereas, in terms of safety, the average safety index of the solutions from the heuristic procedure is 66.92% lower (better) than that of the solutions from CPLEX (with a standard deviation of 30.71%).

Readers can see that the safety-productivity workforce scheduling model yields the solution that shares the advantages of both so-called *pure* models. The solution of the heuristic procedure also shares such advantages. The heuristic procedure is simple and is practical for solving large-sized workforce scheduling problems. Owing to its systematic computation nature, the procedure can be coded as a computer program to obtain the solution quickly and conveniently.

7. INDUSTRY APPLICATION

For industrial systems wherein workers are allowed to perform a variety of tasks, job rotation is a practical approach to reducing occupational hazard exposure. On a daily basis, workers might be assigned to perform different tasks (also at different locations) to alleviate the hazard exposure. Generally, there still are two issues that can hinder the implementation of job rotation: (1) workers' flexibility, and (2) appropriate work schedules for the workers. In terms of workers' flexibility, skill training can be provided to increase the workers' flexibility and improve their competency so that they can be assigned to perform various tasks satisfactorily. Regarding appropriate work schedules, the method described in this paper can be applied to determine appropriate worker-task-period assignments for all involved workers so that not only the safety objective is achieved but the total system productivity is maximized.

Although job rotation might have been implemented in many work systems, it is suspected that those who are responsible tend to search for the work schedules for involved workers without using the optimization approach. Although the safety objective must be achieved, the number of workers assigned to the tasks could be too many; thus, increasing the production cost. Furthermore, it is doubtful if the rotation of workers ever considers the person-job fit. The workforce scheduling with safety and productivity consideration can help to find the minimum number of workers for job rotation, the work schedules with relatively equal hazard exposures among the worker team without exceeding the daily permissible limit, and those that maximize the system productivity.

The heuristic procedure is efficient since it requires less than one second in finding the worker-task-period assignment solution even for the problem with 16 tasks and 24 workers. It is also effective in providing good solutions when comparing them to those from a well-known optimization tool, CPLEX.

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