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DERIVING A BRIDGE TO TRANSFER UNKNOWN (PROBABILISTIC) TIMES TO KNOWN (DETERMINISTIC) TIMES TO ALLOW CONFIDENCE LEVEL ESTIMATES FOR A TIME/COST TRADEOFF: A CRITICAL ANALYSIS OF PERT/CPM PROCEDURES

[3] Keith Krumpe, [1]Joseph Lane, [2]Jimin Lee, [1]Mary Lynn Manns, [1]Claudel McKenzie, [1]Linda Nelms, [1]Mckenna Stockhausen, [1]Lauren Turnburke,
and [1]Robert Yearout

[1]Department of Management & Accountancy, [2]Department of Mathematics, [3]Office of the Dean of Natural Sciences
The University of North Carolina at Asheville, One University Heights, Asheville, NC 28804
Corresponding author's e-mail: {Robert Yearout, yearout@unca.edu}

Network analysis is the examination of activities grouped together by estimated completion times and precedence. Program Evaluation Review Technique (PERT) determines a network's critical path. The ***critical path***, or the longest path through the network, has no "slack" and is the earliest the project can be completed. Included in this stochastic analysis is the critical path's variance. By using the variance and assuming the normal distribution, the analyst can determine the probability for project completion on schedule. Without this confidence level adjustment, there is a 50% probability that the project will be completed on time. Critical Path Method (CPM), which is deterministic, uses the same definition for the critical path and emphasizes time/cost tradeoff. However, the two methods are not completely compatible. Since the 1950s textbook writers and software producers have attempted to combine these methods. The problem occurs when the PERT analysis projects a project out to a specified confidence level and then a project manager places the activities' times into a CPM network. Rather than the cost/time trade-off analysis incorporating calculations that provide a probability of completion at the desired confidence level, the probability of completion is reduced to the original 50%. This may explain why projects are not completed on time. Rather than reconciling the differences, current literature attempts to combine the two and states that there are no differences. This project consisted of the following: conducting a random100 iteration network simulation, developing a heuristic that allocates expected times for each activity, validating the heuristic by testing 60 networks at 90, 95, and 97.5% confidence levels, and conducting a CPM analysis at 95%. Results using the heuristic revealed a successful allocation of projected activity times at 0.00 percent error. This significant research will assist engineers and managers in making more realistic project completion and cost projections. These findings have a potential for initiating changes in operations research/management science textbooks and in project management software.

Significance: This paper examines the differences in PERT and CPM and presents an algorithm that will allow PERT data projected to a given confidence level to be integrated into a CPM cost-trade off analysis. This capability will allow industrial engineers, project managers, and operations managers to better prepare project bids and priority documents.

Keywords: Program Evaluation Review Technique, PERT, Critical Path Method, CPM

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1. INTRODUCTION

Program Evaluation Review Technique (PERT) and Critical Path Method (CPM) were developed independently by Lockheed and DuPont Construction Company respectively in the early 1950's. PERT was first used in the development the Polaris Missile System, while DuPont used CPM to determine time/cost tradeoffs for shortening activity times to compensate for delays in their construction projects. Both techniques were adapted from Taha's work on network analysis (Taha, 2003). Initially PERT displayed activities on the network's arc and CPM placed the activity on the node. Both determined that the definitions of the critical path would be the path though a project where there is no slack. The alternate definition of the critical path is the path through the network which has the shortest time for the completion of all the project's tasks. Since both methods use the same definitions for the critical path, a casual observer would come to the conclusion that PERT and CPM yields the same results. Initially engineers and project managers were very clear in their application of the two methods. Textbooks would be very distinct in keeping the two methods separate (Taha, 2003).

1.1 Pert

PERT is used to determine the probabilities of completing a project, or the converse risk of not completing the project, at the established completion time. By pooling the variances across the critical path and assuming the normal distribution, the analyst can determine the probability for project completion. Without a desired confidence level adjustment, there is 50% probability that the project will be completed on schedule. A proposal to complete a project should have built into the projected time of completion (critical path) an extended time based upon the desired confidence interval and its corresponding risk. For example if a confidence interval of 95% is desired there would be a 5% risk of not completing the project on schedule. Time estimates (t_e) for each activity are calculated using a weighted average as shown in equation 1.

$$\textit{Time Expected}\,(t_e)=\left(\frac{\textit{Optimistic Time}+(4\times\textit{Most Likely Time})+\textit{Pessimistic Time}}{6}\right) \quad \ldots \quad (1)$$

One of the major criticisms of PERT is that estimates have the tendency to be optimistically biased. These optimistic estimates are Beta distributed rather than following the normal distribution pattern (Tversky and Kahnenan, 1974). To correct for this criticism an empirical heuristic was developed to approximate the normal distribution by changing the divisor from 6 to 3.2 and dividing it into the difference between the pessimistic time and the optimistic time (equation 2). After this adjustment, project completion times could be computed for the project by summing the variances across the critical path and applying the central limit theorem.

$$\textit{Variance}\,(\sigma^2)=\left(\frac{(\textit{Pessimistic Time}-\textit{Optimistic Time})}{6}\right)^2 \quad \ldots \quad (2)$$

1.2 CPM

CPM is deterministic in nature and uses known resource estimates to establish the cost of reducing a selected activity time. Like PERT, CPM uses the same definitions for the critical path and slack. The distinguishing feature of CPM is that it identifies trade-offs between time and cost for project activities (Anderson, Sweeney, Williams, Camm, and Martin, 2010). Equations 1 and 2 are not used because of the deterministic nature of CPM.

1.3 The Problem

Over time it has frequently been accepted that PERT and CPM can be merged into the same method (Taylor, 2007; Larson and Grey, 2011). In their most recent textbook Anderson, Sweeney, Williams, Camm, and Martin (2010) made the following statement:

> *"Today's computerized versions of PERT and CPM combines the best features of both approaches. Thus, distinction between the two techniques is no longer necessary."*

Seventeen different textbooks were examined to determine if PERT and CPM differences were adequately addressed. Of these texts fifteen would state in a single sentence that they were distinctly different but then continue to lump the two together as one technique labeled PERT/CPM. Schroeder, Goldstein, and Rungyusanatham (2010) do an excellent job explaining why the two methods are distinctly different. However, there is no attempt to reconcile how the same project would use both methods and achieve a desired confidence level. Although several software packages make confidence level calculations at the end of a given project, there appears to be no current application that will assist the project engineer or manager before the project begins in determining confidence levels to meet unique delays, unexpected or anticipated, such as inclement weather or a strike. This becomes especially important and complicated when the critical path may vary from the original path at the point of the contingency being examined. The ability to have a reasonable confidence level for completion can be very useful where project duration, early completion bonuses and overrun penalties are significant considerations in competitive bids.

Interviews with several companies that use project management software revealed that their software did not have the ability to translate PERT projected data to a given confidence level and then use those time estimates in a CPM time/cost tradeoff. One large construction company stated that they just used CPM at the 50% confidence level for all their analyses. One of the early graduates of MIT's first operations research graduate program related his experience in working with PERT in the Polaris Missile program. Mr. Lucas stated, '*given the state of the art at that time, none of the attempts to translate PERT activity times were successfully translated into CPM for time cast trade offs.'* A current project manager of a large construction company made the following statement, *"I use over five different software packages and none of them will do a cost trade off analysis with the same probabilities of completion as identified in PERT. The time cost trade off analyses returns the user to the original critical path times which are a 50% probability of on time completion. It would be very useful to be able to transfer data from PERT to CPM without losing desired confidence levels."*

Reconciling Stochastic Data with Deterministic Data

If a project requires a 95% probability of completion (5% acceptable risk), it is not helpful to estimate the range of time of completion based on the individual variances for each task. The sum of the individual adjusted times will be significantly and unrealistically greater than the project completion time. Current practice is to take the unadjusted activity times and apply them to the time/cost tradeoff analysis in CPM and reduce the confidence level back to the original 50% prior to adjusting for 5% risk of not completing the project on time. An approach like this can be problematic when reductions in the project duration are required to meet either anticipated or unexpected contingencies. (Leach, 2003).

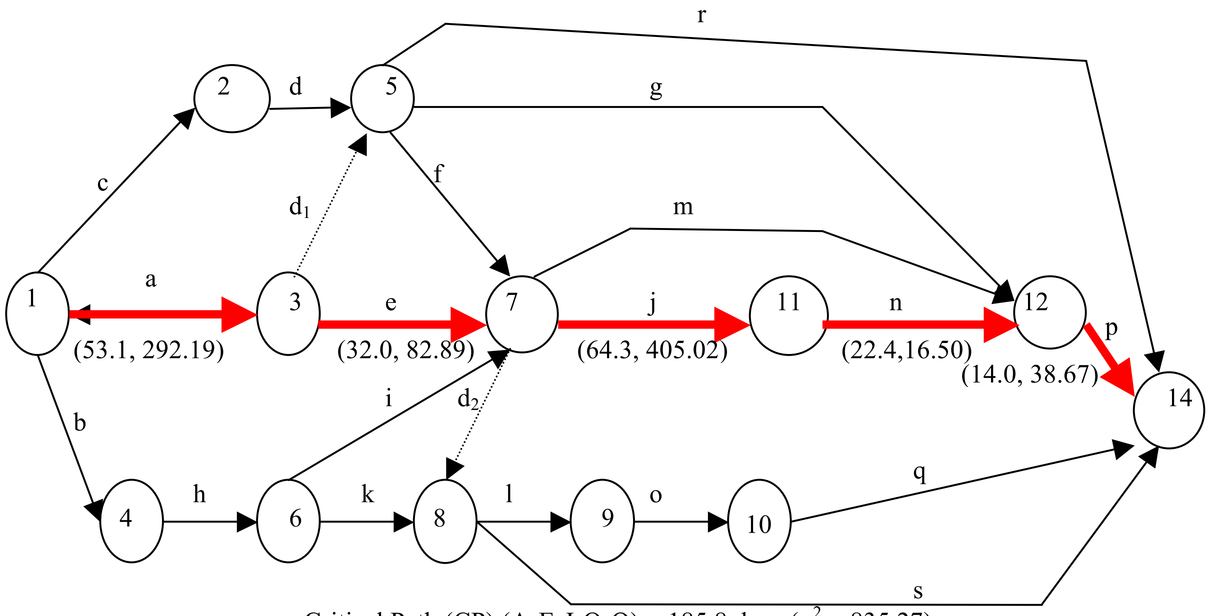
Confidence Levels for Individual Activities

While the variance (σ^2) for a given activity is known, in most applications, the value of a pooled variance ($\sum\sigma^{2s}$) is not known. Since each activity furnishes an estimated σ^2, the best estimate of the variation across the critical path is determined by summing the activities' variation along that path. However, in pooling variances, the assumption is that the estimated σ^2s of the activities are not statistically different one from the other (Snedecor and Cochran, 1980). Since the assumption of equal variances for individual activities is violated (homogeneity), the resulting path will greatly exceed the PERT estimate. Such estimates of extended project times could result in project bids or selection priority no longer being competitive. If a confidence level of 95% is selected to estimate the time required to complete a project (PERT), and then the analyst desires to make a cost/time trade-off analysis (CPM) at that given confidence level, there must be an accurate method to allocate the estimated time for the overall project at that confidence level to each of the different activities that make up the project.. The purpose of this case study was to develop a heuristic that would approximate activity completion times across all tasks to yield the calculated project completion time at a realistic confidence level. The heuristic would distribute the variance associated with the entire project to the activities. The times of the activities could then be transferred as deterministic values into a CPM network.

2. METHODOLOGY AND RESULTS

2.1 Develop a network

A project with an estimated duration of six months was selected as the test case. A sample network (Figure 1) (Yearout et. al., 2009) was developed with 19 activities to include two dummy activities. Times expected are given in days. The optimistic (a), most likely (m), and pessimistic (b) times were obtained by a restricted random number generation. The random number generation was created by averaging one hundred iterations of network activities from the random simulation. From these averages the activity times expected (equation 1) and variances (equation 2 adjusted for optimistic bias) were calculated (Appendix A). The critical path and project variance (pooled over the critical path) were obtained by using the project management module in STORM (Edmonds, et. al. 2001). Data for the activities on the critical path (marked in red) are listed under the appropriate activity (time expected (t_e), variance (σ^2)). Complete listing of all activities' times expected and variances are listed in the Appendix A Table.



Critical Path (CP) (A-E-J-O-Q) = 185.8 days (σ^2 = 835.27)

Figure 1. Selected Network for Analysis at 0.50 Probability of Completion

2.2 Assign a Confidence Level

To obtain a 95% confidence level that ensures an acceptable risk of 5% ($\alpha = 0.05$) of not completing the project on schedule, equation 3 was applied.

$$Z_{0.95} = \left(\frac{(X - CP)}{\sqrt{\sigma^2}} \right) \quad \ldots \quad (3)$$

Where X is equal to the desired completion time, CP is the Critical Path at 50% probability, and Z is the number of standard deviations away from the mean.

This gave an adjusted critical path of 233.2 days ($\sigma^2 = 835.27$)(Figure 2) to insure a confidence level of 95% (or only accepting 5% risk) of not completing the project on time.

2.3 Apply the Confidence Level to each Activity

In an initial approach to determine the critical path's projected completion time to meet the 95% confidence level, the authors applied 1.645 standard deviations away from the mean statistic to each activity. The sum of these individual projected activities resulted in a critical path duration of 295.4 days which was an extended time of 62.1 days greater than the 95% confidence interval's critical path. This extension of the project's duration at a 95% confidence level (26.6% error) is not acceptable for estimation purposes (Figure 2).

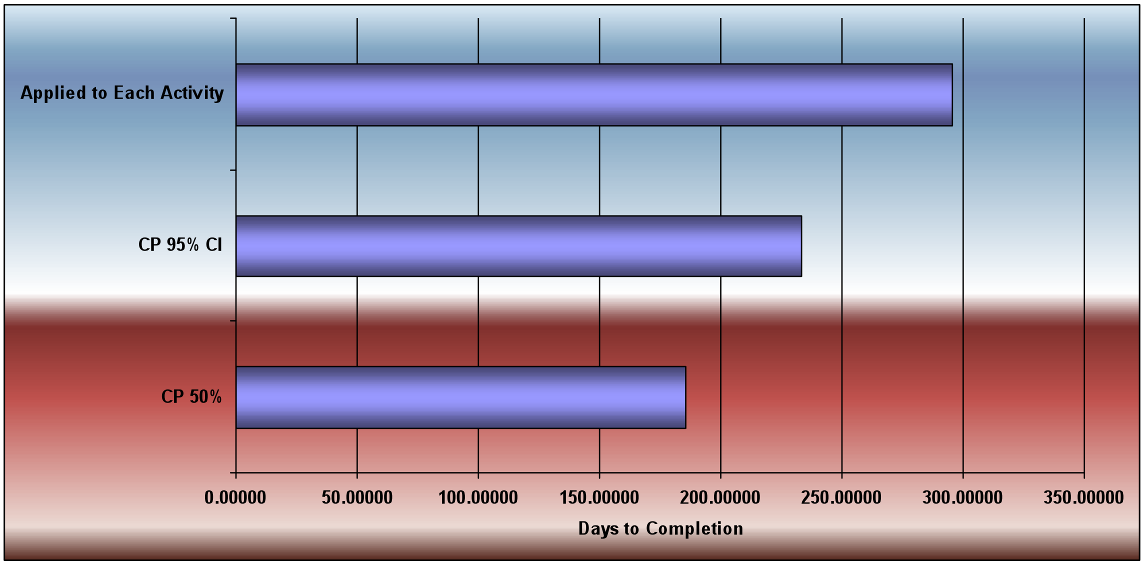


Figure 2. Comparison for Estimated Project Durations at a 50% Confidence Level, at a 95% Confidence Level and at a 95% Confidence Level Applied to each Individual Activity on the Critical Path without the heuristic

2.4 Develop a Correction Factor by Simulation

To use the initial variance, but distribute it to the separate activities a simulation spreadsheet was developed using the random number generator for times expected (paragraph 2.1). Then a crude algorithm was developed from the results of the simulation to create a correction factor to properly allocate the extended critical path durations to the project's corresponding critical path activities. By applying the Golden Section Search technique (Kiefer, 1953) to the 100 simulations, a correction factor was derived. The correction factor's best solution heuristic yielded an average 4.5% error ($\sigma = 0.06$ %).

2.5. Develop a Heuristic

Since it is not practical to apply a simulation analysis to every network and its desired confidence level, it is apparent that a universal heuristic is required that would be appropriate to any network and confidence level. Below is the derivation of the heuristic (equation 4). Note that the following are holding variables; that are defined as follows;

Let $P(X_i < a_i) = d$ for $i = 1, \cdots, n$. That is, we are $d \cdot 100\%$ confident that the length of time to complete the ith path is less than a_i for $i = 1, \cdots, n$.

Then each a_i can be written as following.

$$a_i = \mu_i + \Phi^{-1}(d) \cdot \sigma_i \quad \text{for} \quad i = 1, \cdots, n.$$

Let $P(\sum_{i=1}^{n} X_i < K) = c$. That is, we are $c \cdot 100\%$ confident that the total length of time to complete is less than K. Since $X_i \sim N(\mu_i, \sigma_i)$ for $i = 1, \cdots, n$ and $X_1, \cdots, X_n$ are independent, $\sum_{i=1}^{n} X_i \sim N(\sum_{i=1}^{n} \mu_i, \sqrt{\sum_{i=1}^{n} \sigma_i^2})$.

Then K can be written as following.

$$K = \sum_{i=1}^{n} \mu_i + \Phi^{-1}(c) \sqrt{\sum_{i=1}^{n} \sigma_i^2}.$$

$\sum_{i=1}^{n} a_i = K$ if we let $P(\sum_{i=1}^{n} X_i < \sum_{i=1}^{n} a_i) = c$.

Since $a_i = \mu_i + \Phi^{-1}(d) \cdot \sigma_i$ for $i = 1, \cdots, n$ and $K = \sum_{i=1}^{n} \mu_i + \Phi^{-1}(c) \sqrt{\sum_{i=1}^{n} \sigma_i^2}$, we have the following equation.

$$\begin{aligned} \sum_{i=1}^{n} \mu_i + \Phi^{-1}(d) \cdot \sum_{i=1}^{n} \sigma_i &= \sum_{i=1}^{n} \mu_i + \Phi^{-1}(c) \sqrt{\sum_{i=1}^{n} \sigma_i^2} \qquad \cdots \\ \Phi^{-1}(d) \cdot \sum_{i=1}^{n} \sigma_i &= \Phi^{-1}(c) \sqrt{\sum_{i=1}^{n} \sigma_i^2} \\ \Phi^{-1}(d) &= \Phi^{-1}(c) \cdot \frac{\sqrt{\sum_{i=1}^{n} \sigma_i^2}}{\sum_{i=1}^{n} \sigma_i} \end{aligned} \tag{4}$$

Figure 3 shows a comparison between those critical path completion times illustrated in Figure 2 (Figure 1 network) with the heuristic applied to all activities on the critical path Note that the CP with the 95% confidence level and the CP with the heuristic applied to the activities on the critical path are identical.

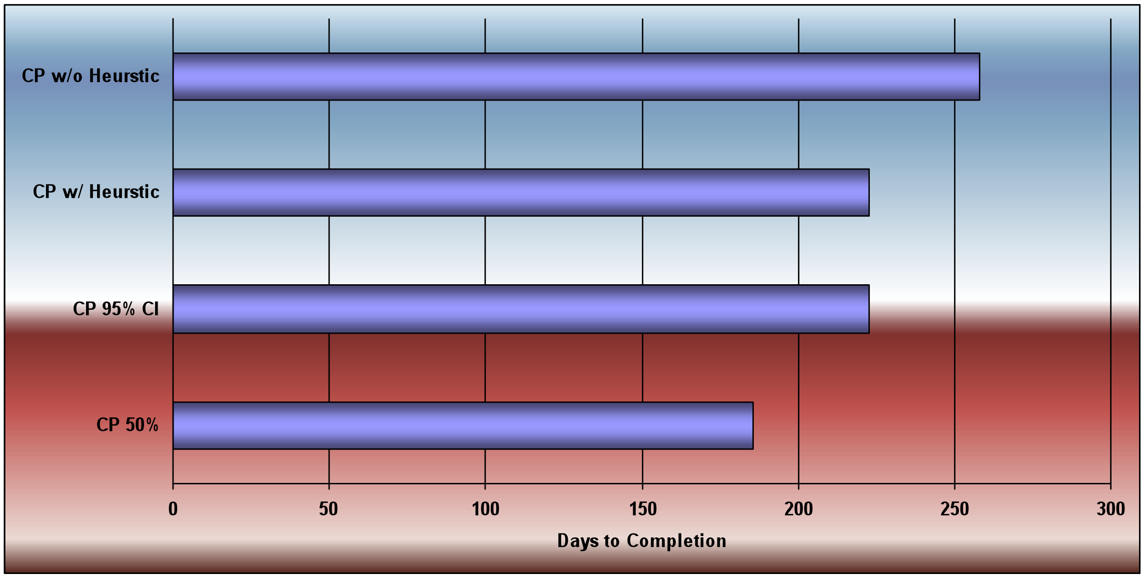


Figure 3. Comparison for Estimated Project Durations (figure 2) with the Heuristic Added to Each Activity on the Critical Path

2.6 Test the Heuristic

Sixty different networks, including the network illustrated by figure 2, taken from 17 current textbooks were analyzed at 90%, 95%, and 97.5% confidence levels. The heuristic was applied to each network. The margin of error between the projected completion time for the critical path using PERT and the sum of each activity with the applied heuristic along that path was $0.1X10^{-8}$ or statistically insignificant. The mean margin of error for applying a given confidence level to each activity versus applying the heuristic to the confidence level was at confidence levels 0.1425 ($\sigma = 0.029903$) at 90%, 0.1738 ($\sigma = 0.034876$) at 95%, and 0.1963 ($\sigma = 0.040795$) at 97.5% confidence levels respectively. Satterthwaite's Approximation with Bonferroni's Adjustment (Millikin and Johnson, 1984) was then used to determine significant differences at the 5% significance level (Figure 4). At the 90%, 95%, and the 97.5% there was a statistically significant difference between the margin of error for each confidence level (Figure 4). Note that, as the confidence level increases, the mean margin of error and variation between using and not using the heuristic also increases.

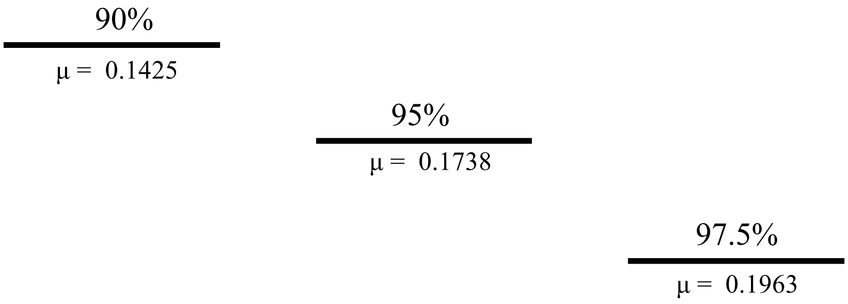


Figure 4. Pair-wise Comparison Analysis at a 0.05 Statistical Significance Level for Confidence Levels of 90%, 95%, and 97.5%

3. APPLICATION AND DISCUSSION OF RESULTS

To illustrate why it is important to use appropriate expected times to establish the desired PERT confidence level of accepting the risk of not completing the project on schedule, the sample network (Figure 1) was crashed using CPM. CPM is an iterative method that reduces an activity on the critical path to the absolute minimum amount of time (crash time) that can be obtained by applying additional resources. The additional resources above the normal cost to complete an activity are labeled crash costs. The crash rate is the incremental cost associated with reducing the activity time by one unit of time. Total cost without compressing the project is $127,500. The standard methodology for approaching the problem is to reduce the activity with the cheapest crash rate on the critical path to its minimal point. The importance of applying the heuristic to the critical path activities (Yearout et. al., 2009) is illustrated by CPM analyses comparing the situations with the heuristic not applied, the heuristic applied to only those activities on the critical path, and the heuristic applied to all activities in the network. It was determined that failure to apply the heuristic to all activities and all potential paths (Yearout et. al., 2009) could result in costly error (McCoy, Yearout, and Patch, 2004). For this reason the heuristic was applied to all activities on the potential critical paths. Since the heuristic is based upon the total variance of the critical path, if the critical path is changed and the variance calculated at the desired confidence level changes, the means for the individual activities would be slightly different. After all paths were analyzed, the mean of the means was calculated and that value was used for the activity time for heuristic application. For this illustration the confidence level chosen was 95% or only accepting a risk of 5% of not completing the project as scheduled. Appendixes A and B give the data for both not applying the heuristic (50% Confidence Level) and applying the heuristic at a 95% Confidence Level. Figure 5 illustrates the initial CPM solution with the heuristic applied. Since activity 'a' is the cheapest activity on the critical path, it will be the first activity in the network to be crashed. Table 1: illustrates the differences between applying the heuristic to only the critical path versus the heuristic applied to all activities in the network. Figure 6 depicts the results of the three CPM analyses discussed above. It becomes quite obvious that the 50% confidence level analysis will give the analyst a false sense of project duration.

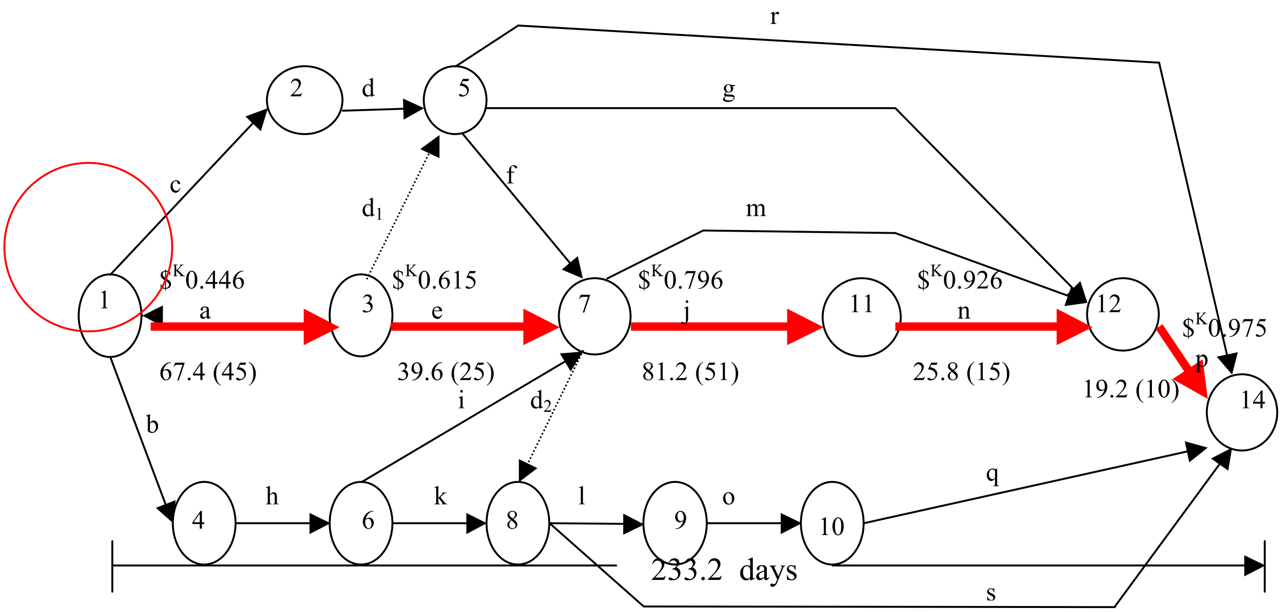


Figure 5. CPM Analysis 1: Network With Projected Times to the 95% Confidence Level

Note: Activity 'a' the Cheapest on the Critical Path is Crashed in the 1st Iteration for all three CPM analysis.

Table 1. Results of CPM Analysis 2 and 3 Cost Trade Off

	Heuristic Applied to All Activities			**Heuristic Only Applied to Activities on CP**		
Iteration	**CP**	**Duration**	**Cost ('$000)**	**CP**	**Duration**	**Cost ('$000)**
0	a-e-j-n-p	233.2	127.500	a-e-j-n-p	233.2	127.500
1	c-d-f-j-n-p	219.3	137.500	a-e-j-n-p	210.8	137.500
2	a-e-j-n-p	189.1	161.500	c-d-f-j-n-p	197.6	146.500
3	a-e-j-n-p	180.6	177.500	c-d-f-j-n-p	167.4	170.500
4	c-d-f-j-n-p	169.8	186.500	c-d-f-j-n-p	156.6	180.500
5	c-d-f-j-n-p	159.0	196.500	c-d-f-j-n-p	147.4	189.500
6	c-d-f-j-n-p	149.8	205.500	a-c-d-e-f-j-n-p	146.0	205.500
7	a-e-j-n-p	146.9	213.000			
8	a-c-d-e-f-j-n-p	146.0	216.676			

The reason that the calculations where the heuristic was applied to all activities took two additional iterations and the estimated cost was $11,176.00 more than the original estimate was that the critical path change came in the 1st iteration rather than the 2nd iteration. This 5% error is a result of not applying the heuristic to all activities and potential critical paths.

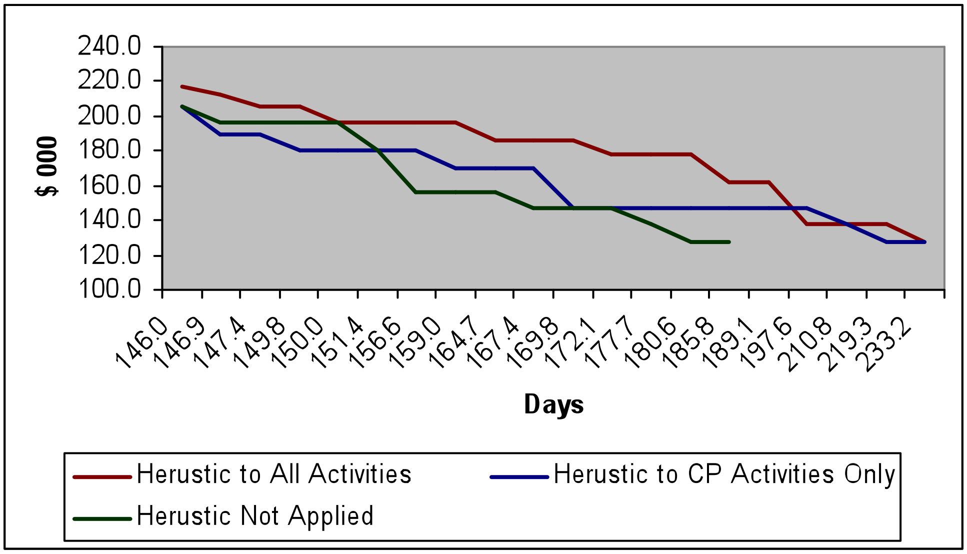


Figure 6. Graphical Illustration of the three CPM Analysis

However, to illustrate why applying the heuristic to all activities is important, note the choke point at event number 7 (Figure 7). When the 95% confidence level is applied to all activities in the network, activity 'c' becomes the critical activity that changes the critical path after the second iteration. If the heuristic is not applied to activity 'c', then that activity does not become critical until 147.4 days. This is an error in project planning of 22.5%. What that means to the project planner is a very high potential that any delay in activity 'c' such as a strike, inclement weather, equipment failure, vendor failure to meet scheduled delivery, or material shortages will delay the entire project to an extent that they might be unaware could happen if they have not applied the heuristic . By applying the heuristic, project managers and engineers are given better opportunity to adjust their activities or plan for contingencies that might arise. The insight gives them a greater capacity to meet the acceptable risk levels that are specified by the project proposal.

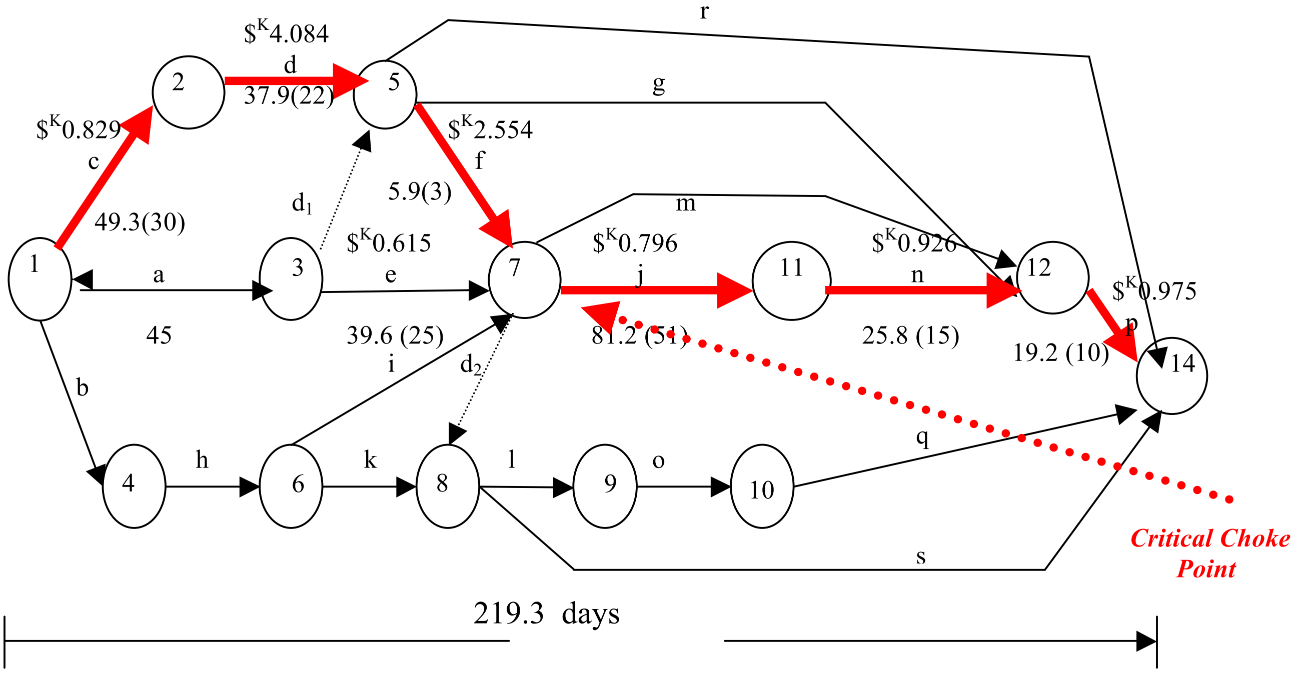


Figure 7. CPM First Iteration with New Critical Path

Note: Activity 'j' will be crashed at the second iteration and activity 'c' becomes critical.

4. CONCLUSIONS

Although PERT and CPM are widely discussed as being the same technique, investigation reveals that the commonality is that they both use the same definition for determining the critical path. Initially PERT was used to establish the probable completion time of a project within a specified confidence level to minimize project overruns. As mentioned earlier the initial project duration without applying a statistical projection is a 50% probability of completion on schedule, but the technique is easily adapted to statistical calculations that can establish a higher confidence level. CPM was developed to analyze the cost/time tradeoff of activities and did not attempt to apply statistical projection. Since the pooled variance for the critical path is for the total project duration, there has not been a technique developed that will adjust the estimates of individual activity times to meet the desired confidence level. Common practice is to use the times expected for each activity and place them into a CPM analysis. The results are project durations that do not correspond to the to the desired confidence level. Thus tradeoff projections at 50% confidence level will not match the probability of completion at a given confidence level projected by a PERT analysis.

By applying the heuristic presented above both the times expected and the cost/time tradeoffs can be adjusted to the specified confidence level. For this study a 95% confidence level or a 5% risk of not completing the project on time was tested. The heuristic for a 95% confidence level was used to illustrate that the analyst can reconcile the differences in project duration between a PERT and a CPM network. Its utility is immediately apparent for writing project bids. By ensuring that all activities reflect the critical path time adjusted to a given confidence level (PERT) prior to placing those times into a CPM network, the analyst will acquire the capability to more accurately meet unforeseen contingencies once the project is underway. For a project whose daily costs can be in the thousands, this is extremely important. Planning the original project with a reasonable level of confidence and then being able to adjust the project schedule once it is in motion could prevent the loss of time and money. The savings in both time and money could be substantial. Much of the project management software and many discussions in text books do not show that this issue has been addressed. It would most helpful if a software package was developed using this heuristic to reconcile the differences between the two methods and provide the benefits described above.

5. ACKNOWLEDGEMENT

The authors' express their gratitude for Fielding Lukas for his insights and experiences during the Polaris Missile project. It became apparent to Mr. Lukas that PERT and CPM are indeed significantly different and that without a bridge between the two techniques time cost/trade off could not be accomplished at a given confidence level other than 50%. Mr. Lukas graduated from the United States Naval Academy in 1945 and was one of the first graduates from MIT's MS in Operations Research in 1948.

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Appendix A: Times Expected (t_e) and Variances (σ^2) (Times are Days) at 50% and 95% Probability of Completion. Critical Path is in Red

Activity	Times Expected (t_e) At 0.50 Probability of Completion	Times Expected (t_e) At 0.95 Probability of Completion	Variances (σ^2)
A	**53.1**	**67.4**	**292.19**
B	10.7	12.3	3.29
C	37.9	49.3	162.56
D	28.8	37.9	104.85
E	**32.0**	**39.6**	**82.89**
F	4.7	5.9	1.81
G	44.8	55.2	144.75
H	9.6	12.4	9.64
I	14.0	19.3	35.50
J	**64.3**	**81.2**	**405.02**
K	10.2	14.5	28.89
L	13.5	18.8	36.25
M	42.7	53.8	156.51
N	**22.4**	**25.8**	**16.50**
O	4.6	5.6	1.27
P	**14.0**	**19.3**	**38.67**
Q	9.8	14.0	21.97
R	29.8	36.6	59.26
S	26.6	30.5	18.96
D_1 & d_2	0.0	0.0	0.00
CP Total	185.8	233.2	

Appendix B: Critical Path Method (CPM) Times and Crash Rates for Cost Trade-Off (Heuristic applied to all Activates and Potential Paths). Critical Path is in Red.

Activity	Times Expected 95% Probability of Completion	Crash Times (t_{ec}) Expected	Normal Cost ($000) ($N_c$)	Crash Cost ($000) ($C_c$)	Crash Rate $\frac{(C_c - N_c)}{(t_e - t_{ec})}$
A	**67.4**	**45.0**	**5.0**	**15.0**	**0.446**
B	12.3	7.0	3.0	15.0	2.276
C	49.3	30.0	5.0	21.0	0.829
D	37.9	22.0	2.0	67.0	4.084
E	**39.6**	**25.0**	**6.0**	**15.0**	**0.615**
F	5.9	3.0	2.5	10.0	2.553
G	55.2	39.0	2.5	5.0	0.151
H	12.4	6.0	3.5	9.0	0.861
I	19.3	12.0	2.0	6.0	0.548
J	**81.2**	**51.0**	**24.0**	**48.0**	**0.796**
K	14.5	5.0	8.0	10.0	0.200
L	18.8	8.0	4.0	8.0	0.370
M	53.8	34.0	24.0	36.0	0.605
N	**25.8**	**15.0**	**10.0**	**20.0**	**0.926**
O	5.6	4.0	8.0	9.0	0.628
P	**19.3**	**10.0**	**9.0**	**18.0**	**0.975**
Q	14.0	10.0	4.0	7.5	0.870
R	36.6	30.0	3.5	6.0	0.377
S	30.5	26.0	1.5	6.0	0.994
D_1 & d_2	0.0	0.0	0.0	0.0	0.000
Total	233.2		127.5	331.5	

BIOGRAPHICAL SKETCH

Keith Krumpe, Dean of Natural Sciences and Professor of Chemistry at the University of North Carolina at Asheville (UNCA), BS in chemistry from Allegheny College and Ph.D. in organic chemistry at Emory University. He completed a post-doctoral research fellowship at the University of Pittsburgh before taking a position at UNCA in 1992. He is actively engaged in undergraduate research that focuses on the synthesis of biologically active molecules and the development of new synthetic methodologies. He has also served on the editorial staff of the NCUR Proceedings for over 10 years and currently serves as the Onsite Editor at each conference. Keith is also actively engaged with SENCER as a Summer Institute faculty member, as a senior associate, and as a co-director of the newly formed SENCER Center of Innovation – South.

Joseph Lane, Bachelor's degree in Industrial and Engineering Management from University of North Carolina at Asheville (UNCA), emphasis in statistics and project management.

Since graduation he has worked briefly as Facility Manager using basic management skills locally. Currently continuing five year self employment in construction as a General Contractor. Management skills used include scheduling, human resources, accounting, project management, web design, advertising/marketing, leadership roles, and project manager.

Jimin Lee Assistant Professor of Statistics, University of North Carolina at Asheville (UNCA), B.S., Mathematics, Kyungpook National University in Korea, M.S., University of North Carolina at Charlotte(UNCC), Ph.D., UNCC. Dr. Lee won the College of Arts & Sciences and Graduate School Award for Excellence in Teaching by a Graduate Teaching Assistant in a Mathematics or Sciences Discipline at UNCC. Dr. Lee has published several papers in national and international peer reviewed journals such as Statistical Sinica and Journal of Applied Statistics. Her research interests are in the areas of Biostatistics, Biology, Epidemiology, and Industrial Engineering.

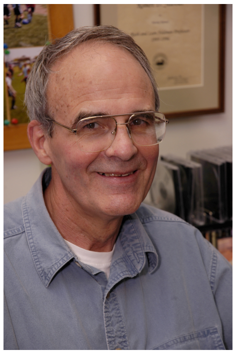
Mary Lynn Manns, PhD, Associate Professor Management University of North Carolina at Asheville where she teaches courses in Management Information Systems, Operations Research/Management Science, and Change Leadership. She is the co-author of the book *Fearless Change: Patterns for Introducing New Ideas*, which documents the successful strategies used by leaders of change in many different types and sizes of organizations throughout the world. She does numerous presentations and consultations on the topic of leading organizational change in and outside the United States.

Claudel B. McKenzie Chair, Department of Management and Accountancy and Professor of Accounting, B.S. Accounting, Mars Hill College; M.B.A., Western Carolina University. CPA and CMA. Distinguished Teaching Award Recipient 1991 and Board of Governors Award for Excellence in Teaching 1998. In additional in publishing in many accountancy journals, she has published several professional papers and case studies that reflect course objectives. In recent years she has become more interested in researching industrial and manufacturing issues and has published in the *International Journal of Industrial Ergonomics* and in the *Proceedings for the International Conference on Industrial Engineering Theory, Applications and Practice*. Mrs. McKenzie also won the Ruth and Leon Feldman Professorship Distinction for Outstanding Scholarship and Service for the 2005-2006 year.

Linda Nelms, Professor of Management, University of North Carolina at Asheville (UNCA), B.A., Literature, UNCA; M.B.A., University of North Carolina at Chapel Hill. North Carolina, C.P.A., C.M.A., C.P.I.M. Mrs. Nelms awards include the Ruth and Leon Feldman Professorship for Outstanding University Service and the University Service Council Award. She has published several articles in top tier peer reviewed international journals such as *The Management Accountant* and The *Journal of Financial Planning* conference proceedings include the Annual *International Journal For Industrial Engineering Theory, Practice, and Application* Conference. Mrs. Nelms has served as the Director of UNCA's Undergraduate Research Program. Her research interests are in the areas of managerial and accounting ethics, economic analysis, inventory, and other related topics that span the gap between economics, accounting and engineering.

Mckenna Stockhausen will graduate from the University of North Carolina at Asheville in May 2010 with a BS in Business Administration and a minor in Economics. Ms. Stockhausen was a recipient of the **Samuel J. Millar Soccer Scholarship** and the **Mortimer Kahn Scholarship** in 2008-2008 and also received the **Helen W. and Frederick R. Eckley Jr. Management & Leadership Scholarship** for the 2009-2010 year. She is a scholarship athlete and senior captain of the UNC-Asheville's Women's Soccer Team (NCAA Division I) program and represented UNC-Asheville at the Big South Leadership Conference in 2008 and 2009. After graduation, Mckenna plans to gain work experience for 2-5 years before attending graduate school.

Lauren Turnburke will graduate Cum Laude from the University of North Carolina at Asheville in May 2010 with a BS in Business Administration and a minor in Economics. A member of Beta Gamma Sigma (BGS), she received the university's BGS scholarship in 2009-2010. Ms. Turnburke represented UNC-Asheville at the BGS National Leadership Forum in February 2010. She received the UNC-Asheville **Management Endowment Scholarship** and the **Eckley Management Leadership Scholarship** in 2008-2009. She is a scholarship athlete on UNC-Asheville's Women's Soccer Team (NCAA Division I) program. After graduation, Lauren plans to gain work experience for 2-5 years before attending graduate school.



Robert Yearout Professor of Industrial Engineering Management, University of North Carolina at Asheville (UNCA), B.S.C.E., Virginia Military Institute, M.S.S.M., University of Southern California, M.S., Ph.D., Kansas State University. LTC (US Army Special Forces, Retired). Dr. Yearout's awards include the Ruth and Leon Feldman Professorship for Outstanding Research, Distinguished Teaching Award, UNCA Distinguished Teacher of the Year, University Service Council Award, Board of Governors of the University of North Carolina Award for Excellence in Teaching, and Annual University Research Council Award for Scholarly and Creative Achievement. Dr. Yearout has published a significant number of articles in national and international peer reviewed journals such as *IEEE Transactions on RELIABILITY*, *International Journal of Industrial Ergonomics*, and the *International Journal For Industrial Engineering Theory, Application, and Practice* in subject areas directly related to his teaching expertise. He is the editor for *National Conferences on Undergraduate Research (NCUR) Proceedings* and a member of the Editorial Board for the *International Journal For Industrial Engineering Theory, Practice, and Application*.