

# GENERATION EXPANSION PLANNING USING BENDERS' DECOMPOSITION AND GENERALIZED NETWORKS

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This paper presents an optimization model and its application to a generation expansion planning problem. The proposed model has a generalized network structure and is exploited effectively by Benders' decomposition algorithm, where a master problem generates trial expansion plans and a set of subproblems compute production cost and system reliability for the trial plan. The applicability of our decomposition algorithm is demonstrated in the case study of Korea's generation expansion planning. The results demonstrate that the model is a practical and flexible tool in solving realistic long-range generation planning problems.

**Keywords:** Benders' decomposition, Generalized network, Generation expansion, Generation planning, Time step approach.

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## 1. INTRODUCTION

Generation expansion planning - deciding what types and sizes of generating plants should be brought into a power system, with the appropriate degree of reliability - is of fundamental importance to electric utilities. For this planning problem, the prices of inputs to supply (including the cost of capital) and the demand are assumed to be exogenous variables and the objective is to minimize the present worth of all the investment and operating costs incurred for power generation over the planning horizon. This problem is referred to as the Generation Planning Problem (GPP) and, because of such factors as variable economic conditions and increase of environmental regulations, the GPP is becoming increasingly critical for the electric power sector. The GPP has received considerable attention for the last three decades, and many mathematical programming models have been proposed (Cote and Laughton, 1979; Louveaux, 1980; Bloom, 1982, 1983a, and 1983b; Park et al., 1985; Evans and Morin, 1986; Youn et al., 1987; Yang and Chen, 1989; Malcolm and Zenios, 1994; Handke et al., 1995; Hobbs, 1995; Zhu and Chow, 1997; Alguacil and Conejo, 2000; Kenfack et al., 2001; McCusker and Hobbs, 2002; Sirikum et al., 2007). By and large, these models tend to be very complex and have required various simplifications and assumptions to render the model solvable, thus yielding simpler models which may poorly approximate the actual problem. For example, unit sizes, spinning reserve, variable heat rate, and treatment of pumped storage hydro plants are seldom considered. In this short paper, therefore, in order to overcome these limitations, we propose a simple yet practical approach, which employs a mixed integer linear program with a generalized network structure. We also propose Benders' decomposition (Benders, 1962; Geoffrion, 1972) based algorithm since its procedure is particularly well suited to take advantage of the special structure of the GPP, i.e., when the capacity expansion is fixed according to a trial plan, the subproblem of minimizing the operating costs of the plants in the trial plan can be solved very simply.

Though most other algorithms must deal with difficult nonlinear optimization programs in their subproblems, the subproblem in our proposed model can be solved without resorting to nonlinear programming by using generalized networks. The rest of this paper is organized as follows. In Section 2, we present mathematical notation used throughout the paper. The generation expansion planning problem is modeled as a mixed integer linear programming model with a generalized network structure in Section 3. Sections 4 and 5 describe the applications of Benders' decomposition and time-step approach, respectively. In Section 6, a case study on Korea's generation expansion planning is presented. Some concluding remarks are presented in Section 7. Finally, Section 8 lists references used in this paper.

## 2. NOTATION

The notation used throughout this paper is stated below:

$T$	number of years in planning horizon
$M$	total number candidate plants for the system expansion
$Q$	set of arcs in network
$L$	set of nodes for plants $i = 1, 2, \dots, M$

$G_i$	set of hourly plant capacity of plant $i \in L$
$\bar{G}_j$	subset of nodes in $\bigcup_{i \in L} \bar{G}_i$ such that node $k \in \bar{G}_j$ supplies demand $j \in R$
$P$	unit cost of energy-not-served
$\bar{P}$	higher unit cost (high penalty cost) of energy-not-served ( $\bar{P} \gg P$ )
$D$	total annual demand plus total spinning reserve
$D_j$	demand # $j$ , $j \in R$
$\bar{D}_j$	spinning reserve # $j$ , $j \in \bar{R}$
$R$	demand nodes, $j \in R$ is a demand
$\bar{R}$	spinning reserve nodes, $j \in \bar{R}$ is a spinning reserve
$f$	fictitious unit for unserved energy
$s$	pumped hydro storage plant
$a$	a super source, that is, total annual capacity
$b$	a super sink, that is, total annual demand
$r(i, j)$	arc $(i, j)$ such that a flow $f_{ij}$ is passed from node $i$ to node $j$
$\alpha_r^t$	coefficients for present-worth calculations from year $t$ to the reference year at a discount rate $r$
$F_i^t$	annual capital and fixed O&M cost of plant type $i$ in year $t$
$N_i^t$	maximum number of plants of type $i$ that can be installed in year $t$
$A_{ij}$	arc parameter between node $i$ and node $j$ ( $0 < A_{ij} \leq 1$ ). Note that if $A_{ij} = 1$ for all $i$ and $j$ , a pure or conventional network formulation exists; if $A_{ij} > 1$ , the flow is augmented (gains); and if $A_{ij} < 1$ , the flow is decreased (losses)
$U_i^t$	capacity of plant # $i$ in year $t$
$C_{ij}^t$	unit cost in arc $(i, j)$ in year $t$
$S_i^t$	annual energy capacity of plant # $i$ in year $t$
$U_{ij}^t$	upper bound on arc flow between node # $i$ and node # $j$ in year $t$
$\varepsilon^t$	upper limit of expected unserved energy in year $t$
$O_t(Y)$	discounted operating cost and unserved energy cost during time period $t$ as a function of technology purchase $Y$
$Y_i^t$	cumulative number of plants of type $i$ available in year $t$
$X_{ij}^t$	flow out of node # $i$ in arc $(i, j)$ in year $t$
$\bar{X}_{af}$	flow into artificial plant $f$ at higher penalty
$u_i$	dual variable corresponding to conservation of flow constraint for node $i$
$v_{ij}$	dual variable corresponding to upper bound on arc $r(i, j)$

### 3. PROBLEM FORMULATION

In its simplest form, the objective of the GPP is to select the plant mix that will minimize the discounted expected sum of all fixed costs, operation costs, and unserved energy costs over the specified planning horizon, subject to certain constraints such as a required reliability of supply. The transmission and distribution network are not represented and parameters such as load levels and prices of fuel and capital are fixed exogenously. The generic formulation of GPP can be stated as:

$$(GPP_0) \text{ Minimize } \sum_{t=1}^T \sum_{i=1}^M \alpha_r^t F_i^t Y_i^t + \sum_{t=1}^T O_t(Y) \quad \dots \quad (1)$$

$$\text{Subject to } 0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t \quad \text{for all } t, i \quad \dots \quad (2)$$

The constraint (2) describes the maximum number of plants that can be constructed in each year. This constraint also requires that once plant # $i$  has been installed, then the plant remains available in future years. In this formulation, the decision variables are cumulative capacity additions by plant types. Since the operating cost function  $O_t(Y)$  is linear, this problem is a mixed integer linear programming problem with a special structure. That structure stems from the fact that there exist two different types of decisions in a generation expansion planning. The first type is concerned with the choice of plant-mix, i.e., the quantity of each class of technology to be brought into the system. The second type is concerned with

the operation of these technologies so as to evaluate the cost of running these plants. Therefore, the GPP can be exploited efficiently by Benders' decomposition, where a master problem generates trial expansion plans and a set of subproblems compute production cost and system reliability for the trial plan. Benders' decomposition provides great flexibility by permitting the application of specifically tailored algorithms, which in this case is a generalized network algorithm, to the subproblems. By using a generalized network, the model provides a powerful aid in the solution to a variety of problems concerning the optimal operation and planning of a power system. Generalized networks can successfully model many problems that have no pure network equivalent. The flows that are transmitted across the arcs of a generalized network may be modified by gain/loss factors so that the amount of flow entering an arc is not necessarily equal to the amount of flow leaving the arc. This is made possible by arc multipliers, which can be interpreted in two ways. First, multipliers can be viewed as either a gain (if greater than 1) or a loss (if less than 1) modifying the amount of flow of some particular item. Second, it is possible to interpret the multipliers as transforming one type of item into another (Phillips, 1981). Generalized networks can be used in generation expansion planning problems to model such aspects as fuel to energy conversions, plant deterioration with aging, plant unavailability owing to forced outage and maintenance, deduction of plant capacity taken by station auxiliaries and losses from transmission, and loss of energy of pumped storage hydro plant occurring in pumping and re-generation mode. The model in this paper is primarily concerned with the pumped storage hydro plant and the objective is to supply  $D$  units of flow to the sink node at minimum cost. The mathematical program of the generalized network model can be formulated as follows.

$$O_t(Y) = \text{Minimize} \quad \dots \quad (3)$$

$$\sum_{r(i,j) \in Q} \alpha'_r C_{ij} X_{ij} + \sum_{r(a,r) \in Q} \alpha'_r (PX_{of} + \bar{P}\bar{X}_{of})$$

subject to various constraints as follows:

1) The plant in operation must be sufficient at all times to meet the instantaneous power demand. This insures that the sum of power produced from all available thermal plants, conventional hydro plants, pumped storage hydro plants, and fictitious plant is equal to the demand at that time.

- For node  $j \in R$ :

$$-\sum_{i \in G_j} X_{ij} - X_{sj} - X_{fj} + X_{jb} = 0 \quad \text{for all } j \in R \quad \dots \quad (4)$$

2) The output of each plant cannot exceed its installed capacity. In general, the available capacity, i.e., the installed capacity times the availability factor, is somewhat lower than the installed capacity on account of maintenance and forced outages. Furthermore, the constraints (5) and (8) below consider the spinning reserve, which is the generating capacity that can be called on in a few seconds to supply power in the event of sudden load increases or plant failures.

- For node  $i \in L$ :

$$-X_{ai} + \sum_{j \in G_i} X_{ij} + \sum_{j \in R} X_{ij} = 0 \quad \text{for all } i \in L \quad \dots$$

- Power can be produced by plant # $i$  only if plant # $i$  is built; for arc from super source node  $a$  to node  $i \in L$ :

$$0 \leq X_{ai} \leq S_i Y_i \quad \text{for all } r(a,i), i \in L \quad \dots \quad (6)$$

- Plant cannot be operated at a level above its capacity; for arc from node  $i \in L$  to node  $j \in G_i$ :

$$X_{ij} \leq U_i \quad \text{for all } r(i,j), i \in L, j \in G_i \quad \dots \quad (7)$$

- For spinning reserve nodes ( $\bar{R}$ ):

$$-\sum_{i \in L} X_{ij} + X_{jb} = 0 \quad \text{for all } j \in \bar{R} \quad \dots \quad (8)$$

3) Loss of efficiency of pumped storage hydro plants is represented by using arc multipliers. The arc multiplier ( $A_{is} \leq 1$ ) represents the combined efficiency of the pumping and generating cycle. The constraints also require that no plant be operated above its capacity.

- The pumping mode of pumped storage hydro acts as an additional load; for node  $i$  in set  $G_k$ ,  $k \in L$ , where  $i$  &  $j$  correspond to the same time period;  $s$  is pumped storage hydro plant

$$-X_{ki} + X_{ij} + X_{is} = 0 \quad \text{for all } i \in G_k, j \in R, k \in L \quad \dots \quad (9)$$

- The output (used for pumping energy) of thermal plant cannot exceed its capacity; for arc from node  $i \in G_k$  to node  $s$ , where  $k \in L$ :

$$X_{is} \leq U_i \quad \text{for all } r(i,s), i \in G_k, k \in L \quad \dots \quad (10)$$

- The total amount of energy stored is equal to the total energy generated by pumped storage hydro plant; for node  $s$  (pumped storage hydro plant);  $A_{is}$  is combined efficiency of pumping and generating:

$$-\sum_{k \in L} \sum_{i \in G_k} A_{is} X_{is} + \sum_{j \in R} X_{sj} = 0 \quad \dots \quad (11)$$

- The output (during discharging cycle) of the pumped storage hydro plant is limited by its capacity; for arc from node  $s$  to node  $j \in R$ :

$$X_{sj} \leq U_s \quad \text{for all } r(s, j), j \in R \quad \dots \quad (12)$$

4) In order to guarantee feasibility, a large fictitious generating unit is introduced with unserved energy cost (greater than operating cost of the other units) so that optimally this unit will be used solely as a last resort to prevent load shedding (see Constraint (14)). Constraint (13) represents the reliability standard of the system using the expected unserved energy (EUE) criterion:

- Expected unserved energy (EUE) criterion (upper bound of arc flow  $X_{af}$ ) is defined as  $\varepsilon$ ;  $X_{af}$  = flow into fictitious unit  $f$  at unserved energy cost  $P$ ; for arc from super source node  $a$  to fictitious unit node  $f$ :

$$X_{af} \leq \varepsilon \quad \dots \quad (13)$$

- $\bar{X}_{af}$  = flow into fictitious unit  $f$  at much higher penalty  $\bar{P}$  than unserved energy cost  $P$  so that optimally this flow will be zero; for node  $f$ :

$$-X_{af} - \bar{X}_{af} + \sum_{j \in R} X_{fj} = 0 \quad \dots \quad (14)$$

- The fictitious unit has an infinite capacity (i.e., upper bound of arc flow  $\bar{X}_{af}$  is unlimited):

$$\bar{X}_{af} \leq +\infty \quad \dots \quad (15)$$

- The fictitious unit can supply as much energy as required by the demand node:

$$X_{jf} \leq +\infty \quad \text{for all } j \in R \quad \dots \quad (16)$$

5) Finally, there are a number of other constraints. For example, constraint (17) represents the conservation-of-flow constraint for the sink node. The constraints (18) and (19) denote the capacity restriction constraints for demand and spinning reserve, respectively. Constraint (20) imposes nonnegativity for all arc flows.

- For super sink node  $b$ :

$$\sum_{j \in (R \cup \bar{R})} X_{jb} = D \quad \dots \quad (17)$$

- For arc from node  $j \in R$  to  $b$ :

$$0 \leq X_{jb} \leq D_j \quad \text{for all } r(j, b), j \in R \quad \dots \quad (18)$$

- For arc from node  $j \in \bar{R}$  to  $b$ :

$$0 \leq X_{jb} \leq \bar{D}_j \quad \text{for all } r(j, b), j \in \bar{R} \quad \dots \quad (19)$$

- For all arcs  $r(i, j)$ :

$$X_{ij} \geq 0 \quad \text{for all } r(i, j) \quad \dots \quad (20)$$

The several important assumptions inherent in this particular formulation are as follows. First, all arc multipliers are real positive numbers. Second, the lower bounds on all arc flows are zero. Third, generation capacities and demands have been combined to form a single source node  $a$  and a single sink node  $b$ . This can be done by creating a super source and a super sink. Finally, unlike pure network formulations, the total output flow from the super source (gross demand) need not equal the total input flow to the super sink (net demand) due to flow adjustments caused by the arc multipliers (Phillips, 1981). The gross demand is unknown until the final solution is obtained. In our model, the losses of flow ( $A_{ij} < 1$ ) through a given arc result from the loss of energy of pumped storage hydro plants. In order to impose the reliability constraint, we introduce an artificial generating unit with infinite capacity in the existing generating system with a high running cost (unserved energy cost  $P$ ) so that the artificial plant will be used only when the demand cannot be satisfied. This approach allows quite a lot of flexibility in view of the fact that the model presented here is capable of being converted to a loss-of-load probability (LOLP) constraint easily, merely by summing the number of unsatisfied demands in the network problem. While the expected unserved energy (EDE) index represents the expected amount of unserved energy during the year, the LOLP index indicates the probability that some portion of load will not be satisfied by the available generating capacity. More specifically, LOLP is defined as the proportion of days or hours per year (e.g., 0.5 days per year, or 12.0 hours per year) when insufficient generating capacity is available to serve all the daily or hourly loads. The loss-of-load probability can be derived as follows:

$$\text{LOLP} = \sum_{j=1}^R P_j \quad \dots \quad (21)$$

where  $P_j$  is  $\theta_j /$  total number of demand, and  $\theta_j$  is 1 if plant capacity is not sufficient for demand  $j \in R$  ( $X_{jT} > 0$ ), otherwise 0. Hence, this model is able to convert easily to a LOLP index for system reliability by using alternative constraint (22) instead of constraint (13):

$$\sum_{j=1}^R P_j \leq \delta \quad \dots \quad (22)$$

where  $\delta$  represents upper limit of LOLP. This convertibility suggests that this model is a versatile tool for the capacity expansion planning. Now embedding the single period problem of computing  $O_t(Y)$  into the generic model ( $GPP_0$ ), we obtain the complete GPP as follows:

$$(GPP_1) \text{ Minimize} \quad \dots \quad (23)$$

$$\sum_{t=1}^T \sum_{i=1}^M \alpha_r^t F_i^t Y_i^t + \sum_{t=1}^T \left[ \sum_{r(i,j) \in Q} \alpha_r^t C_{ij}^t X_{ij}^t + \sum_{r(a,f) \in Q} \alpha_r^t (P^t X_{af}^t + \bar{P}^t \bar{X}_{af}^t) \right]$$

subject to the constraints from (4) through (20) and (2). Note that parameters and variables in the program ( $GPP_1$ ) are indexed by year  $t$ , and all the constraint sets can be written in more compact form:

$$\sum_{j \in R} X_{ij}^t - \sum_j A_{ji}^t X_{ji}^t = 0 \quad \text{for all } i \neq a, b; t \quad \dots \quad (24)$$

$$\sum_j X_{jb}^t = D^t \quad \text{for all } j \in (R \cup \bar{R}), t \quad \dots \quad (25)$$

$$0 \leq X_{ai}^t \leq S_i^t Y_i^t \quad \text{for all } r(a, i), i \in L, t \quad \dots \quad (26)$$

$$0 \leq X_{ij}^t \leq U_{ij}^t \quad \text{for all } r(i, j), t, i \neq a \quad \dots \quad (27)$$

$$0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t \quad \text{for all } t, i \quad \dots \quad (28)$$

The constraints (24) and (25) represent conservation-of-flow constraints for the transshipment nodes and sink node, respectively. The constraint (27) denotes arc capacity restriction. The constraints in (26) are relatively small in number, but significantly complicate the mathematical program. There are  $T \cdot M$  sets of these constraints in the overall program, and it is the presence of these constraints coupling the  $X$  and  $Y$  variables, which make the mathematical program large scale.

#### 4. BENDERS' DECOMPOSITION

If the vector of investment decision variables  $Y_i^t$  were fixed, then the problem of selecting production decision variables  $X_{ij}$  would reduce to disjoint generalized network problems, one for each year. We can define an optimal value function for period  $t$ , including fixed costs, operating costs, and energy-not-served costs, given a trial investment plan  $Y = (Y_1, Y_2, \dots, Y_M)$ :

$$(GPP_2) \quad V_t(Y) = \sum_{i=1}^M F_i Y_i + \text{Minimize} \quad \dots \quad (29)$$

$$\left[ \sum_{r(i,j) \in Q} C_{ij} X_{ij} + \sum_{r(a,f) \in Q} (P X_{af} + \bar{P} \bar{X}_{af}) \right]$$

subject to the constraints from (24) through (28). The index  $t$  and the present-worth coefficients  $\alpha_r^t$  have been dropped from the objective function for clarity of notation; however, the fixed cost  $F_i$ , the unit operating cost  $C_{ij}$ , the utilization levels  $X_{ij}$ , and the demand  $D$  as well as the unserved energy cost  $P$  all depend on the year  $t$ . Trial values of  $Y_i$  will be determined by solving an integer linear program called the master problem. The production cost and reliability of this trial plan are determined in a set of subproblems, one for each year in the planning horizon, using the generalized network algorithm. The total annual fixed cost of the plants, plus the optimal generalized network costs, is the value of the function

$V_t$  at the point  $Y$ . Our original problem is therefore equivalent to minimize the program ( $GPP_2$ ). By linear programming duality theory, the optimal value of the generalized network problem is equal to that of its dual linear program ( $GPP_3$ ):

$$(GPP_3) \quad V_t(Y) = \sum_{i=1}^M F_i Y_i + \text{Maximize} \quad \dots \quad (30)$$

$$\left[ Du_b - \sum_{i=1}^M S_i Y_i v_{ai} - \sum_{r(i,j) \in Q, i \neq a} U_{ij} v_{ij} \right]$$

subject to

$$u_i - A_{ij} u_j - v_{ij} \leq C_{ij} \quad \text{for all } r(i, j), i \neq a, j \neq b \quad \dots \quad (31)$$

$$-A_{ai} u_i - v_{ai} \leq C_{ai} \quad \text{for all } r(a, i), i \in (L \cup f) \quad \dots \quad (32)$$

$$u_j + u_b - v_{jb} \leq C_{jb} \quad \text{for all } r(j, b), j \in (R \cup \bar{R}) \quad \dots \quad (33)$$

$$u \text{ unrestricted, } v \geq 0 \quad \dots \quad (34)$$

The discussion to follow will be eased by defining the set of feasible dual solutions of the constraints from (31) through (34) in the production subproblem for year  $t$ . Since this set is a convex polyhedron, then it can be represented in terms of a generally large but finite number  $K$  of basic dual feasible solutions. Now if  $W_t^k \equiv \{(u^1, v^1), (u^2, v^2), \dots, (u^k, v^k)\}$  denotes the set of the first  $k$  basic solutions (omitting an index  $t$  in  $(u, v)$  for clarity), where  $(u^k, v^k)$  represents a feasible vector of dual variables for the  $k$ -th instance of the production subprogram for year  $t$ , and if  $W_t \equiv \{(u^1, v^1), (u^2, v^2), \dots, (u^k, v^k), \dots, (u^K, v^K)\}$  denotes the entire set of all basic feasible solutions of dual problem for period  $t$ , the program ( $GPP_3$ ) becomes ( $GPP_4$ ):

$$(GPP_4) \quad V_t(Y) = \sum_{i=1}^M F_i Y_i + \quad \dots \quad (35)$$

$$\text{Maximize}_{(u, v) \in W_t} \left[ Du_b - \sum_{i=1}^M S_i Y_i v_{ai} - \sum_{r(i,j) \in Q, i \neq a} U_{ij} v_{ij} \right]$$

subject to the constraints from (31) through (34). The program ( $GPP_4$ ) cannot be directly solved since the sets  $W_t$  are not explicitly available. However, each time the set of disjoint production subprograms are solved under a trial investment plan, it is possible to generate one of the vectors  $(u^k, v^k)$  of dual variables belonging to the set  $W_t$ . Hence, it is possible to construct a subset of basic feasible dual solutions,  $W_t^k$  denoting the set of the first  $k$  basic feasible solutions, of  $W_t$  and to solve a relaxed equivalent program by carrying out the maximization in the program ( $GPP_4$ ) over  $W_t^k$  rather than all vectors in  $W_t$ . The equivalent program may be then restated as:

$$(GPP_5) \quad V_t^k(Y) = \sum_{i=1}^M F_i Y_i + \quad \dots \quad (36)$$

$$\begin{aligned} \text{Maximize}_{(u, v) \in W_t^k} & \left[ Du_b - \sum_{i=1}^M S_i Y_i v_{ai} - \sum_{r(i,j) \in Q, i \neq a} U_{ij} v_{ij} \right] \\ & = \text{Maximize}_{n=1, \dots, k} \left[ \sum_{i=1}^M \alpha_i^n Y_i + \beta^n \right] \end{aligned}$$

where, for each dual basic feasible solution  $(u, v) \in W_t^k$ , we define  $\alpha_i^k = F_i - \sum_{i=1}^M S_i v_{ai}$  and  $\beta^k = Du_b - \sum_{r(i,j) \in Q, i \neq a} U_{ij} v_{ij}$

In the evaluation of a trial solution  $Y$ , the production decision variables  $X_{ij}$  are suppressed and production cost is expressed as a function of the investment decision variables  $Y_i$  by bringing explicitly into play the dual variables. Thus  $V_t(Y)$  is the maximum of a large number of linear functions of  $Y$ , that is, a convex piecewise-linear function. This generation expansion planning algorithm based on Benders' decomposition therefore will involve an iterative procedure where, at each iteration, a two stage process is employed. At the first stage of each Benders' iteration, a trial expansion plan  $Y$  is to be computed by solving:

$$(GPP_6) \text{ Minimize } \sum_{i=1}^T V_i^k(Y^t) \quad \dots \quad (37)$$

subject to the constraints (2) and integer restriction on  $Y$ . Because  $V_i^k(Y^t)$  is represented by only a small subset of the linear supporting functions, i.e.,  $\sum_{i=1}^M \alpha_{ii}^k Y_i^t + \beta_i^k$ , we instead are solving a relaxation of this problem. The equivalent integer linear program is:

$$(GPP_7) \text{ Minimize } \sum_{i=1}^T Z^t \quad \dots \quad (38)$$

subject to

$$Z^t \geq \sum_{i=1}^M \alpha_{ii}^n Y_i^t + \beta_i^n \quad \text{for all } n=1,2,\dots,k;t \quad \dots \quad (39)$$

$$0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t \quad \text{for all } t,i \quad \dots \quad (40)$$

$$Y \text{ integer} \quad \dots \quad (41)$$

where a new continuous variable  $z^t$  has been introduced, and  $k$  linear supports are used to approximate each  $V_i$ . This relaxed program is referred to as the master problem, which in this case is a mixed integer linear program with  $T \cdot M$  integer variables and is used to generate trial solutions for the optimal generation expansion plan. The optimal solution to the master problem delivers both a feasible investment plan and a lower bound to the minimal cost for the equivalent program. In the second stage of each iteration, the subproblems are solved to determine the minimum cost of operation and the reliability under the trial solution just obtained in master problem. The solution of subproblem also yields optimal dual multipliers, which estimate the changes in production cost resulting from marginal changes in the trial plant capacities. These dual multipliers are used to form new constraints that are added to the master problem, which is then re-solved to determine a new trial expansion plan. The process continues, alternately solving the master problem and subproblems, until the algorithm has found an optimal expansion plan or one that is known to be within an acceptable tolerance of optimality. In this way, solving the complex original program for generation expansion planning is reduced to the iterative solution of an integer linear program and a set of generalized network problems. The objective function value of the master problem always provides a lower bound on the cost of the optimal solution, since the master problem is a relaxation of the original problem. Furthermore, the cost of any trial solution that is feasible in the original problem provides an upper bound on the cost of the optimal solution. In the master problem, the feasible region of the problem is represented by an outer polyhedral approximation. Therefore, in general, trial solutions generated by solving the Benders' master problem may be infeasible in the original problem. To avoid infeasibility, our model has introduced a large artificial generating unit in the existing production system with a high running cost (i.e., unserved energy cost). By this means, trial solutions are guaranteed to be feasible.

## 5. TIME STEP APPROACH

The dynamics of the electric power sector and the interaction of the investment decisions over time are among the factors responsible for complicating the generation expansion planning model, as they require that the model be solved simultaneously across all time periods in the planning horizon. In our Benders' decomposition algorithm, at each iteration we must solve the (current) master problem, obtaining a trial expansion plan  $Y = (Y^1, Y^2, \dots, Y^T)$  where each  $Y^t = (Y_1^t, Y_2^t, \dots, Y_M^t)$ . Then, for each year  $t$ , a subproblem is solved and a  $(k+1)^{\text{st}}$  linear support is generated. The disadvantage of Benders' decomposition algorithm is the size of the master problem, with its  $T \cdot M$  variables. For example, if there are 6 possible types of candidate plants in each year and a 20 year planning horizon, the master problem has 120 integer variables. If each integer variable is represented by several binary variables (e.g.,  $Y = y_1 + 2y_2 + 4y_3 + 8y_4$  if  $Y \leq 15$ , where  $y_1$  through  $y_4$  are binary variables) to represent the number of plants to be added for each type, this further increases the size of the problem that must be solved. As an alternative approach, therefore, the problem can be solved as a series of related 1-year problems, with the expansion program of a given year constituting the input to the optimization problem of the following year. This model is referred to as the time-step model (Levin et al., 1983), or the myopic decision rule (Louveaux, 1980), to distinguish it from the dynamic models that view the GPP simultaneously over time.

Clearly, the time-step approach offers significant computational savings over the dynamic approach, since it deals with each period separately. The main drawback of this approach is that it cannot account for future developments in

determining present investment decisions, thus yielding solutions that might not be optimal in the dynamic sense. Yet, there might be models under which this shortcoming can be overcome, so that the time-step model provides solutions that are nearly identical to the dynamic-model solutions. The purpose of this section is to investigate and formulate a model under which the solutions of the two models coincide for the generation expansion planning problem.

There are two types of time-step approaches to solve the capacity expansion decision problem. In the "forward procedure," the calculations proceed year by year, starting from the first year of the planning horizon, adding purchased technologies each year, and ending when the expansion plan for the final year of the planning horizon has been determined. At each year, the optimal (minimum cost) way to achieve each level of cumulative purchased capacity is calculated. In the forward version of the time-step approach, then, rather than solving the program ( $GPP_6$ ), we instead, using the Benders' decomposition algorithm, solve Minimize  $V_1(Y^1)$  subject to the constraints (2) and integer restriction on  $Y$ . Then, fixing  $Y^1$  at the optimal value of this planning problem, we solve Minimize  $V_2(Y^2)$  subject to the constraint (2) and integer restriction on  $Y$ . Then we fix  $Y^2$ , and recursively solve Minimize  $V_3(Y^3)$ , etc. This algorithm is, of course, a myopic or "short-sighted" decision model. Each minimization is performed to minimize only the cost for year  $t$ , disregarding the cost of future years. Hence we may obtain nonoptimal solutions in which the added generation capacity employs a technology which might have a lower initial capital cost but higher operating costs in later years, and thus a higher total cost during the planning horizon, than does the technology employed in the optimal expansion plan. Thus, it is then difficult to design an algorithm which recursively constructs the optimal solution through the forward method.

On the other hand, the "backward procedure" starts at the last year and then proceeds backward to the first year of the planning horizon. The main difference between the forward and backward methods is that, while the forward method adds purchased capacity from the beginning of the planning horizon, the backward method subtracts the purchased capacity from the end of the planning horizon. Lessening the shortcoming of the myopic condition would require that the cost function for each year  $t$  include an estimate of the value in future years of any generation capacity added in year  $t$ . This is accomplished to a certain extent in the backward version of the time-step approach. Consider the problem ( $GPP_8$ ):

$$(GPP_8) \text{ Minimize } \sum_{i=1}^T V_i(Y^i) \quad \dots \quad (42)$$

subject to

$$0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t \quad \text{for all } t, i \quad \dots \quad (43)$$

$$Y \text{ integer} \quad \dots \quad (44)$$

This problem is separable except for the constraints (43), which link two successive years; if these constraints were relaxed, then the problem becomes separable, i.e., each year could be optimized independently of the others. Lagrangian techniques can be useful, in the relaxation of the linking constraints. We associate Lagrangian multiplier  $(u_i^t)' \geq 0$  with the constraint  $0 \leq Y_i^t - Y_i^{t-1}$ , which requires that capacity, once added, remains available for later years, and  $(u_i^t)'' \geq 0$  with the constraint  $Y_i^t - Y_i^{t-1} \leq N_i^t$ , which specifies the maximum number of plants that can be constructed in each year. Then for any multiplier vector  $u = (u', u'')$ , the problem becomes;

$$(GPP_9) \phi(u) = \text{Minimize} \quad \dots \quad (45)$$

$$\sum_{i=1}^T V_i(Y^i) + \sum_{i=1}^T \sum_{i=1}^M \{(u_i^t)'\} (Y_i^{t-1} - Y_i^t) + \sum_{i=1}^T \sum_{i=1}^M \{(u_i^t)''\} (Y_i^t - Y_i^{t-1} - N_i^t)$$

$$\text{subject to integer restriction on } Y \quad \dots \quad (46)$$

(Note that if the constraint is violated, i.e., if  $(Y_i^{t-1} - Y_i^t) > 0$  or  $(Y_i^t - Y_i^{t-1}) > N_i^t$ , the added terms act to penalize the cost function since  $u_i^t = ((u_i^t)', (u_i^t)'') \geq 0$ . If one of these two constraints is violated, the other must be slack and so the Lagrangian multiplier corresponding to the slack constraint must be zero, according to complementary slackness conditions, giving a net positive penalty added to the objective function.) This problem then separates;

$$(GPP_{10}) \phi(u) = \quad \dots \quad (47)$$

$$\sum_{i=1}^T \left[ \text{Minimize } V_i(Y^i) + \sum_{i=1}^M \{(u_i^{t+1})' - (u_i^t)'\} Y_i^t + \sum_{i=1}^M \{(u_i^t)'' - (u_i^{t+1})''\} Y_i^t + \sum_{i=1}^M \{(u_i^t)''\} N_i^t \right]$$

$$\text{subject to integer restriction on } Y^t \quad \dots \quad (48)$$



where  $u^0 = 0$  and  $u^{T+1} = 0$ .

**Lemma 1.** The expression  $\sum_{t=1}^T \sum_{i=1}^M \{(u_i^{t+1})' - (u_i^t)'\} Y_i^t$  is same as  $\sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)'\} (Y_i^{t-1} - Y_i^t)$ .

$$\begin{aligned} \text{Proof. } & \sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)'\} (Y_i^{t-1} - Y_i^t) = -\sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)'\} Y_i^t + \sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)'\} Y_i^{t-1} = -\sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)'\} Y_i^t + \sum_{t=0}^{T-1} \sum_{i=1}^M \{(u_i^{t+1})'\} Y_i^t \\ & = \sum_{t=1}^T \sum_{i=1}^M \{(u_i^{t+1})' - (u_i^t)'\} Y_i^t + \sum_{i=1}^M \{(u_i^1)'\} Y_i^0 - \sum_{i=1}^M \{(u_i^T)'\} Y_i^T = \sum_{t=1}^T \sum_{i=1}^M \{(u_i^{t+1})' - (u_i^t)'\} Y_i^t \quad (\text{Since } Y_i^0 = 0 \text{ and } (u_i^{T+1})' = 0) \end{aligned}$$

**Lemma 2.** The expression  $\sum_{t=1}^T \sum_{i=1}^M \{(u_i^{t+1})'' - (u_i^t)''\} Y_i^t - \sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)''\} N_i^t$  is same as  $\sum_{t=1}^T \sum_{i=1}^M \{(u_i^t)''\} (Y_i^t - Y_i^{t-1} - N_i^t)$ .

*Proof.* It is straightforward and omitted since it is very similar to Lemma 1.

$\phi(u)$  represents the optimal cost with the added Lagrangian terms. For any  $u=(u', u'') \geq 0$ , it is easily shown that  $\phi(u)$

provides a lower bound on the optimal cost. The Lagrangian dual problem is that of assigning values to the Lagrangian multipliers  $u$  so as to maximize this lower bound, i.e.,

$$\text{Maximize } \phi(u) \quad \dots \quad (49)$$

$$\text{subject to the nonnegativity restriction on } u \quad \dots \quad (50)$$

To do so would, however, require a large amount of computation. The problem would be solved for the initially assigned values of  $u$ , which would involve applying Benders' decomposition algorithm for each year  $t = 1, 2, \dots, T$  to obtain an expansion plan  $Y = (Y^1, Y^2, \dots, Y^T)$ . This plan typically would violate the relaxed constraints, i.e., for one or more values of  $t$  and  $i$ ,  $Y_i^t < Y_i^{t-1}$  or  $Y_i^t - Y_i^{t-1} > N_i^t$ . The Lagrangian multiplier for each of these violated constraints would be increased, thereby increasing the penalty for these violations. At the same time, the Lagrangian multipliers for slack constraints, i.e.,  $i$  &  $t$  such that  $0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t$  might be decreased. Then the problem would be solved again to evaluate  $\phi(u)$  for this new set of multipliers  $u$ , again requiring the application of Benders' decomposition algorithm to minimize

$$V_t(Y^t) + \sum_{i=1}^M \{(u_i^{t+1})' - (u_i^t)'\} Y_i^t + \sum_{i=1}^M \{(u_i^t)'' - (u_i^{t+1})''\} Y_i^t + \sum_{i=1}^M \{(u_i^t)''\} N_i^t \quad \text{for all } t = 1, 2, \dots, T$$

Obtaining the maximum value of  $\phi$  might require  $\phi(u)$  to be computed for each of a sequence of values of  $u$ , a large computational effort. Instead, in our backward version of the time-step approach, we will not find the optimal values of  $(u', u'')$  but will attempt to estimate the differences  $\{(u_i^{t+1})' - (u_i^t)'\}$  of the optimal values without subsequently adjusting them.

However, we will let  $u'' = 0$ , making no attempt to estimate the optimal differences  $\{(u_i^t)'' - (u_i^{t+1})''\}$  and  $\{(u_i^t)'' N_i^t\}$ . That is, the relaxed problem will be solved for a single vector  $u$ , as follows.

$$\text{For } t = T: \text{ Minimize } \quad \dots \quad (51)$$

$$V_T(Y^T) + \sum_{i=1}^M 0 \cdot Y_i^T \quad (\text{i.e., let } u = 0)$$

$$\text{subject to integer restriction on } Y^T \quad \dots \quad (52)$$

This then selects the final generation capacity & mix of technologies at the end of the planning period. Now let the coefficient of  $Y_i^{T-1}$  in the Lagrangian function be estimated by the optimal dual LP variable  $v_{ai} \leq 0$  for the constraint  $0 \leq X_{ai}^T \leq S_i^T Y_i^T$  in the generalized network problem solved at stage  $t = T$ . (The units will be \$/MW, and will provide an estimate of the marginal value per MW in the trial plant capacities.) Let  $Y_i^{T*}$  be the optimum at the stage  $T$ , and  $-S_i v_{ai}$  be the estimated reduction in cost in year  $T$  if  $Y_i^{T-1}$  plants of type  $i$  have been built.

$$\text{For } t = T-1: \text{ Minimize } V_{T-1}(Y^{T-1}) + \sum_{i=1}^M S_i v_{ai} Y_i^{T-1} \quad \dots \quad (53)$$

$$\begin{aligned} & \text{subject to} \\ & 0 \leq (Y_i^{T*} - Y_i^{T-1}) \leq N_i^{T-1} \quad \dots \quad (54) \end{aligned}$$

$$Y \text{ integer} \quad \dots \quad (55)$$

Thus, this problem in essence reduces the cost in year  $T-1$  by the "value" of the generation capacity in the year  $T$ . At each stage  $t$  thereafter, let the coefficient  $v_{ai}$  in the Lagrangian objective function be the dual LP variable from stage  $t+1$ . Note that in this modified Lagrangian relaxation procedure, the linking constraints  $0 \leq (Y_i^t - Y_i^{t-1}) \leq N_i^t$  are in fact imposed, so that the capacity expansion plan which is computed will be feasible  $T$  (although it is not necessarily optimal in the original problem Minimize  $\sum_{t=1}^T V_t(Y^t)$ ). By a recursive manner, this method yields solutions almost identical to the solutions obtained by an equivalent dynamic model that views the generation expansion problem simultaneously over time.

## 6. CASE STUDY

In order to illustrate the implementation of Benders' decomposition algorithm and generalized networks to the GPP, the optimization algorithm has been applied to the generation expansion planning in the Southern part of Korea. The features of this case study are described and compared with the results from the Wien Automatic System Planning Package (WASP) model, a deterministic dynamic programming model which is being used in Korea Electric Power Corporation (KEPCO). Korea is poorly endowed with energy resources, and depends heavily on imports such as oil and bituminous coal. Starting from the oil crises of the 1970s, diversification of energy sources was actively sought to mitigate the impacts of oil crises in the future. This diversification strategy was most pronounced in the power sector, resulting in the active introduction of nuclear power plants and bituminous coal fired units. This trend will continue through the 2010s and is likely to continue for the next decade. At the end of 1995, the area's total installed generation capacity is 32,184 MW. Breakdown by plant type is shown in Table 1. The historical rate of electricity demand growth has been approximately 9% annually between 1980 and 1990. The recorded peak demand in 1995 was 29,878 MW. By 2006, the annual growth rate was 6.6%, while the peak demand was 58,120 MW. In the face of such a high rate of growth in demand, the area's power sector needs to introduce such generating units as pumped storage hydro plants, LNG-fired combined cycle plants, bituminous coal fired units, and nuclear power plants. The characteristic data for these units are summarized in Table 2.

**Table 1. Installed Capacity by Plant Type (Source: Power Development Plan of KEPCO, 1995. 12)**

Plant Type	Capacity (MW)
Conventional Hydro	1,493
Pumped Storage Hydro	1,600
Coal-Fired	7,820
Oil-Fired	6,009
Nuclear (PWR & PHWR)	8,616
LNG-Fired	6,646

- PWR: Pressurized Water Reactor
- PHWR: Pressurized Heavy Water Reactor

**Table 2. Data for Candidate Plants (Source: Power Development Plan of KEPCO, 1995. 12)**

Plant Type	Capacity (MW)	FOR (%)	Maintenance (Day/Year)	Construction Cost (\$/KW)	Fuel Cost (¢/10 <sup>6</sup> Kcal)
Combined cycle	450	6.0	45	639	1,986.8
Coal 500 MW	500	7.0	45	1,287	682.8
Coal 800 MW	800	9.0	52	1,165	682.8
PWR	1,000	6.5	60	1,920	177.6
PHWR	700	5.5	39	2,049	96.3
PSTR	200	0	0	781	-

- Combined cycle plants burn LNG (Liquefied Natural Gas)
- PSTR: Pump Storage Hydro Plant
- FOR: Forced Outage Rate

For the case study, a planning horizon is chosen from the year 1995 to 2010 and the base year for present-worth is the year 1995. To ensure appropriate system reliability, the expected unserved energy (EUE) is constrained not to exceed 0.1369% of annual demand based on loss-of-load probability equal to 0.5 days/year. The cost of capital (discount rate) is assumed to be 8.5%. Bloom (1984) suggests that the unserved energy cost be the cost of operating the most fuel-expensive

candidate unit to produce the required extra energy, plus the cost of installing the most capital-expensive candidate unit to the required extra capacity. In this case study, the unserved energy cost is selected to be the operation cost of LNG-fired combined cycle plant plus the construction cost of nuclear power plant (PHWR). The forecasted demand data are from the official power development plan of KEPCO and are shown in Table 3. Further information about this case study is available at the webpage: [http://ie.nmsu.edu/sohn\\_gpp/casestudy\\_gpp.htm](http://ie.nmsu.edu/sohn_gpp/casestudy_gpp.htm).

**Table 3. Forecasted Load (Source: Power Development Plan of KEPCO, 1995. 12)**

Year	Peak Load (MW)	Energy Demand (GWH)	Year	Peak Load (MW)	Energy Demand (GWH)
1995	29,878	181,529	2003	51,332	311,208
1996	32,603	199,402	2004	53,710	325,634
1997	35,482	217,394	2005	56,001	339,648
1998	38,388	234,463	2006	58,120	353,119
1999	41,032	250,250	2007	60,281	366,109
2000	43,694	265,975	2008	62,404	379,366
2001	46,277	281,218	2009	64,473	392,443
2002	48,862	296,462	2010	66,478	405,509

Before proceeding to the results of the case study, some special conditions under which this program evaluation is conducted should be noted. First, this model sets up system input data describing the characteristics of all plants in the system at the start of the study period as well as those already committed to be added during the study as fixed planned plants, i.e. whose investment decision is not to be included in the decision variables. These fixed-planned plants include existing units and committed units which are already under construction, as well as units scheduled to be retired during the study period. It should be pointed out that although the investment costs of the fixed-planned plants are not included in the objective function, their operating and maintenance costs and fuel costs are included. Thus, the selection of fixed-planned plant additions as well as the characteristics of the existing system has an influence on the operating conditions of plants in the generating system expansion plan being evaluated.

Secondly, in most systems to be studied, the existing and committed plants on the interconnected system will consist of many plant types with a number of generating units in each plant type. In order to reduce the computational time required to simulate the operation of all of these individual plants, it is advisable to group plants which have approximately the same capacity, heat rates, forced outage rate and maintenance requirements, and the same type of fuel and fuel costs. Thirdly, this model is executed in a framework in which the time-step approach is employed. A significant portion of the reserve capacity being installed never incurs an operating cost. Under this condition, the optimization program will always fill this portion of required capacity with the plants which are cheapest in terms of their investment cost, that is, peak-loaded plants, such as gas turbine and combined cycle plants. In most cases, peak-loaded plants are not utilized at their full capacity, which implies that their purchase in the previous year may be less desirable from the viewpoint of the time-step approach. Thus when one employs the time-step approach without considering that a significant portion of peak-loaded plants are seldom utilized at their full capacity, then a decrease in combined cycle plants would likely occur along with an increase in intermediate fossil capacity. Consequently, we assume that the "utilization levels" of the generation capacity of combined cycle plants are same as those of base-loaded plants.

Finally, the maximum number of plants that can be added for each plant type and for each year under the planning horizon is determined by taking into account the available plant sizes in that year. In this case study, the maximum number of additions for each plant type is 5 units per year. During the first several years in the planning horizon, the annual maximum number of additions for each type should be set at zero because of the construction lead time. Construction lead times are assumed to be three years for combined cycle plants, four years for coal-fired plants, and six years for nuclear plants. In this case study, three generation expansion plans, denoted as Plans A, B, and C, were obtained. Each plan refers, respectively, to the investment plans obtained from the forward version of time-step approach, the backward version of time-step approach, and the WASP model (see Figures 1 through 3).

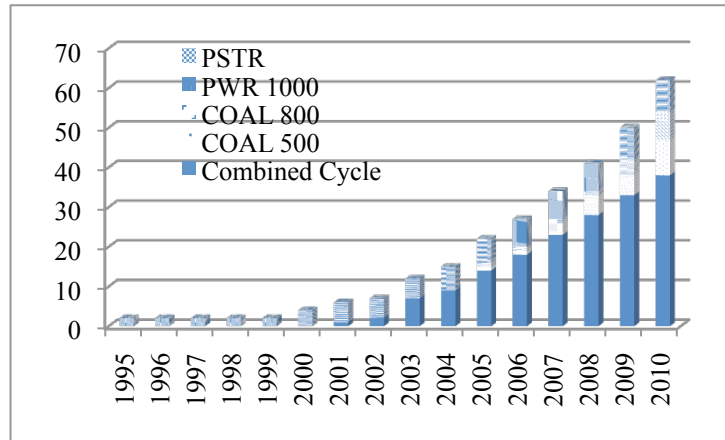


Figure 1. Cumulative Capacity by Plant Type (Forward Procedure)

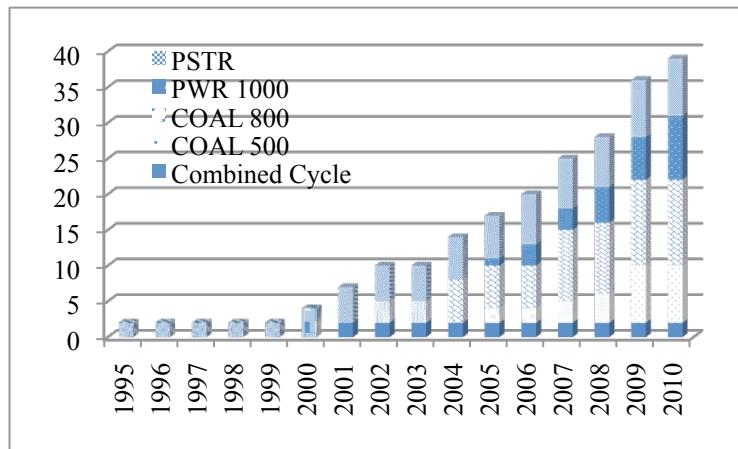


Figure 2. Cumulative Capacity by Plant Type (Backward Procedure)

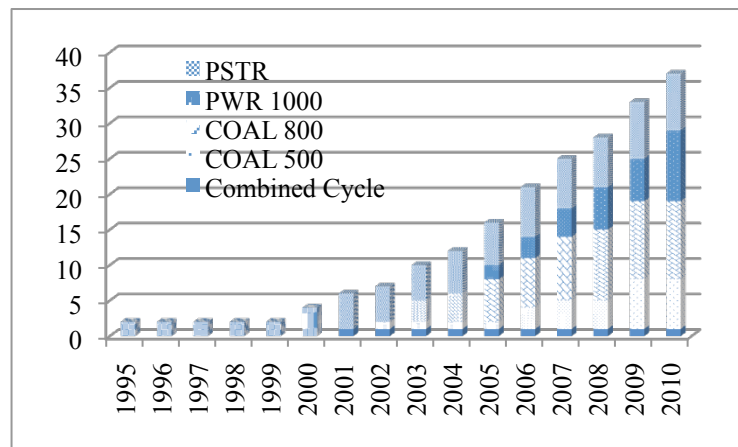


Figure 3. Cumulative Capacity by Plant Type (WASP Model)

After analyzing the case study results, five observations can be made. First, we can compare the two types of time-step approaches for solving the capacity expansion decision problem. In the myopic or "forward procedure," the optimization is performed to minimize only the cost for each year, disregarding the cost of future years. Hence we obtain nonoptimal solutions in which the added generation capacity includes many more of the combined cycle plants which have a lower initial capital cost but higher operating costs in later years, and thus a higher total cost during the planning horizon, than does the technology employed in the solution found by the "backward procedure." The plan generated by the forward procedure calls for a continuous growth in the size of combined cycle plants until the maximum available plant size (in this case, 5 units per year) is reached. At the same time, we find that base-loaded plants such as nuclear power plants are not introduced at all. On the other hand, in the backward version of the time-step approach, dual multipliers, which estimate the future "values" or utility of the generation capacity, are used to yields solutions almost identical to the solutions obtained by a dynamic model, namely the WASP model, that views the generation expansion problem simultaneously over time.

Secondly, the capacity expansion plan from the time-step approach (backward procedure) has a lower risk over time than does the WASP model's plan. The explanation for this particular difference is not clear; it may be either because of some discrepancy in defining the actual reliability requirement (i.e., Expected Unserved Energy for our optimization model and Loss of Load Probability for the WASP model) and unserved energy cost considerations (i.e., unserved energy cost is not considered in the WASP model) or because of some discrepancy in constraints such as the maximum number of additions for each plant type. However, from some partial sensitivity analysis on the reliability constraints, it is our belief that, when the discrepancy is uncovered and corrected, one will not find any significant differences in the plant sizing decisions or in the mix of installed capacity.

Thirdly, because of the required construction lead time and sufficient capacity of fixed-planned plants (committed plants), there is no additional purchase of capacity during the first several years. This explains the fact that, in all three plans, the EUE (Expected Unserved Energy) exceeds the limit during the first three years. The fourth observation deals with plant mix decisions. Neither of the expansion plans obtained from our optimization model nor the plan obtained from the WASP model call for PHWR (Pressurized Heavy Water Reactor) since the economy of PHWR technology is inferior to those of the other technologies. Finally, the mix in 2010 of the capacity installed (including existing and fixed planned capacity) since 1995 was computed for Plans B and C, i.e. for the time-step (backward procedure) plan and for the WASP plan. The results are given below in Table 4. In conclusion, the results obtained from the case study are certainly encouraging, in particular, the result that the plan generated by the backward version of the time-step approach may prove to be almost the same as that of the dynamic WASP model.

**Table 4. Plant Mix Comparison**

Plant Type	Backward Time-Step Program		WASP Model	
Nuclear Power	26,129 MW	32.93 %	27,129 MW	34.52 %
Bituminous Coal	30,000 MW	37.81 %	28,700 MW	36.51 %
Anthracite Coal	800 MW	1.01 %	800 MW	1.02 %
LNG	14,219 MW	17.92 %	13,769 MW	17.52 %
Oil-fired	2,220 MW	2.79 %	2,220 MW	2.82 %
Hydro	1,682 MW	2.12 %	1,682 MW	2.14 %
PSTR	4,300 MW	5.42 %	4,300 MW	5.47 %
Total Capacity	79,350 MW	100 %	78,600 MW	100 %

## 7. CONCLUSIONS

This paper has presented a new approach for generation expansion planning of an electric utility. We have employed the Benders' decomposition principle, a mixed integer linear program, and a generalized network program. This approach is well suited to examining utility planning issues such as plant deterioration with aging, plant unavailability owing to forced outage and maintenance, investment decision on pumped hydro storage plant, and environmental regulations. This model can deal with the short term unit commitment problem as well as long-term capacity expansion planning. In the subproblem algorithm (generalized networks), production allocation decisions are carried out on an individual plant basis so as to provide the information of plant operations that system planners need in their decision making. Furthermore, we present a time-step approach, which offers significant computational savings over a dynamic approach that views the generation expansion problem simultaneously over the entire planning horizon. Through the case study, we have found that this time-step approach yields solutions almost identical to the solutions obtained by an equivalent dynamic model. This model significantly reduces the effort presently expended by system planners in solving the generation expansion planning

problem. More specifically, the model of our study eliminates the need for performing the tedious and computationally costly trial and error searches - vis a vis the dynamic WASP model - in order to obtain optimal investment plans. In the dynamic WASP model, the annual maximum of additions (tunnels) for each candidate plant over a planning horizon is limited by the curse of dimensionality, thus trial and error searches are required. Consequently, these features of the optimization model of our study will allow the system planners to carry out more flexible and effective decision making than have been previously possible.

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