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OPTIMAL CONTROL STRATEGIES FOR ADAPTIVE PRICING IN RIDE-SHARING SERVICES

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Rideshare platforms are an example of economies of sharing where ride requests initiated by riders are fulfilled by car owners through the platform that connects both of them. When demand for a ride is initiated by the customer, the platform checks service providers' (car owners) availability and assigns a fare (ride price) that both the ride requester and provider should agree on to complete the transaction, and the ride service is fulfilled. In this research, optimal pricing strategies for ride-share platforms are considered. The optimal control approach is used to first develop differential equations to model the dynamics of the number of ride requests and for the price rate. Second, we model the total profit as a function of a linear revenue and a nonlinear cost. The optimal rate of change in the ride price is then obtained. Finally, a numerical example and extensive sensitivity analyses not only provide insights into the effect of the system parameter on the model but also lead to managerial implications to help companies determine the best price for each ride.

Keywords: Optimal Pricing; Ride-Share Platforms; Optimal Control; Model Predictive Control; Price Ride Change; Sensitivity Analysis.

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1. INTRODUCTION

A sharing economy is a concept that refers to the collective consumption of goods and services. It refers to the peer-to-peer sharing of resources, from goods and services to labor, and is made possible through digital platforms such as apps and websites, Schor (2016).

There is nothing new about sharing; people have shared their goods since they first started living together in communities, Schor (2014), Belk (2010, 2014a, 2014b). Recently, the sharing economy has become increasingly popular as people are looking for more affordable alternatives to traditional products and services. It has also opened up new opportunities for entrepreneurs to create businesses built on the sharing model. Examples of sharing economy activities include peer-to-peer car sharing, such as with Uber and Lyft; peer-to-peer room rentals, such as Airbnb; and peer-to-peer labor, such as TaskRabbit.

Sharing economy growth will continue without a doubt. Not only are the costs of transportation and other services becoming more affordable but the convenience and flexibility of shared services make it an attractive option for many people. Due to the growing awareness of the benefits of the sharing economy, this trend is expected to grow in the future, Baruffati (2023).

The latest data suggest that private vehicles are used for only 5% of their lifetime, which is a staggering statistic. This, combined with the fact that there are fewer requirements to drive for ride-sharing services, means that there is a greater supply of rides than ever before. This is excellent news for those who are looking to save money on transportation costs.

On short notice, dynamic ride-share systems connect travelers with similar itineraries and schedules. By reducing the number of vehicles used for personal travel and improving the utilization of available seat capacity, these systems may provide significant societal and environmental benefits. A successful dynamic ride-share system requires effective and efficient optimization technology that matches drivers and riders in real time, Agatz *et al*. (2012).

For instance, Uber Technologies Corporation began as a ride-sharing company to disrupt the traditional taxi industry. The company's net revenue has increased 319 times in nine years, from 0.1 billion U.S. dollars in 2013 to 31.9 billion U.S. dollars in 2022. The number of trips Uber drivers completed in 2022 was 7.6 billion, an increase of 20.6% over 2019. In 2022, Uber had 131 million users, up 11% from last year. Uber Eats had 85 million users, Iqbal (2023).

Ride-sharing is a 'matchmaking' service implemented using a digital platform that matches independent drivers with passengers. The process of ride-sharing commences when a passenger uses a ride-sharing mobile app to request a ride by sending a signal to a platform (for example, Uber) containing information about the trip's location and origin. Based on

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demand (number of ride requests) and supply (number of drivers) around the passenger's location, the platform's mobile app calculates an instant fare and sends it to the passenger. The passenger has the option of accepting or rejecting the proposed fare at the next step. After the proposed trip and fare are accepted, the nearest driver is identified and calculated algorithmically and transmitted. A driver can accept or decline the proposal for a trip fare. Upon accepting the offer, he/she will receive instructions on how to pick up the passenger. Once the passenger has arrived at the destination, the transaction ends, Cassey (2017).

Furthermore, a ride-sharing service allows multiple passengers to travel along similar routes (completely or partially overlapped), Psaraftis (1980), which can be static or dynamic. Static ride sharing involves knowing the ride requests, the source and destination location, and the available vehicles in advance, Atahran *et al*. (2014). A non-heuristic algorithm or a heuristic algorithm (approximation) can be used to process the requests after the system has all the necessary information. In contrast, dynamic ride-sharing systems do not receive such information and match passengers with drivers as requests arrive (in real-time), Archetti *et al*. (2016).

According to Cassey (2017), the ride-sharing competition involves four aspects: price dynamics, strategic pricing, fixed pricing vs. surge pricing, and information sharing. Pricing strategies can take the form of setting lower prices to take advantage of network effects. It is common to see this strategy in ride-sharing when two or more platforms are in intense competition. Surge pricing involves taking into account current demand and supply conditions at a particular location. Fixed pricing entails a fixed price that does not fluctuate over time as demand and supply conditions change. When information is shared between rival platforms, prices are set based on the collective demand and supply at the location, and the two platforms may still have different market shares based on their different locational distributions.

In this paper, we explore a dynamic pricing system for ride-share platforms, leveraging the optimal control approach to address the inherent complexities of this problem. Optimal control theory, with its extensive track record of successful applications in management science and economics (Sethi, 2019; Al-Matar and Tadj, 2023; Ghorai, 2022), serves as the foundation for our analysis. The primary objective of this research is to develop a robust framework for determining the optimal rate of change in ride prices, aiming to maximize profitability while maintaining system stability and customer satisfaction.

The originality of our research lies in its application of a nonlinear profit function over a specified prediction horizon, where the control variable is dynamically adjusted to reflect real-time changes in the ride-share market. Unlike traditional static pricing models, our approach incorporates both the number of ride requests and the hourly ride price rate as state variables, capturing the fluctuating demand and pricing behavior in a highly competitive environment.

To achieve these objectives, we model the system dynamics using relevant initial value problems, ensuring that our approach can adapt to various market conditions. The model predictive control (MPC) methodology is employed to iteratively calculate the optimal rate of change in the ride price, ensuring responsiveness to market shifts. The results are validated through a series of illustrative examples, demonstrating the effectiveness and practical relevance of our approach.

Furthermore, we conduct sensitivity analyses to provide deeper insights into the factors influencing the system's performance. These analyses offer valuable managerial implications, guiding decision-makers in optimizing pricing strategies under different scenarios. This research not only advances the application of optimal control in dynamic pricing but also contributes to the broader understanding of real-time pricing strategies in the ride-share industry.

2. LITERATURE REVIEW

Different models, data analysis and simulation have been used in several studies to explore ride-sharing's potential. For instance, Tsao *et al*. (2019) developed a model predictive control (MPC) approach to optimize routes for ride-sharing automobility-on-demand (RAMoD) systems. As a result of this approach, self-driving cars can provide coordinated on-demand mobility to improve social welfare. According to a real-world case study, RAMoD can significantly improve social welfare over a single-occupancy Autonomous Mobility-on-Demand (AMoD) system. There are, however, several factors that influence the performance of the MPC algorithm and RAMoD system, including the size and density of the city, the traffic patterns, and user behavior, as well as the amount of capital required to implement such a system.

The work by Yuan and Hentenryck (2021a) introduces a machine-learning model that predicts the optimal solutions for MPCs using an optimization proxy. This study discusses dynamic pricing and vehicle relocation in ride-hailing systems. The system is designed to maximize the number of riders served while minimizing its operational costs. While this model assumes that the demand forecast is accurate, this is not always the case in practice. Thus, it may not be able to generalize well to different cities or regions with different traffic patterns and demand characteristics.

Also, Yuan and Hentenryck (2021b) propose a spatiotemporal pricing framework called AP-RTRS, which controls the waiting time and completion rate of ride-sharing requests. Through the use of Model Predictive Control (MPC), the framework optimizes demand and supply imbalances, pricing, average wait times, and service quality based on geographical location. According to simulation experiments, the pricing optimization model minimizes waiting times while maintaining revenue and geographical equity. While the framework is effective, various factors may affect its effectiveness, including the accuracy of demand forecasting, rider willingness to pay higher prices during peak hours, and the availability of drivers. Moreover, the framework may not apply to all ride-hailing/ride-sharing systems in general, as different systems may have different characteristics and constraints.

As well, Yuan and Hentenryck (2021c) propose a hybrid approach that combines a machine-learning model with a high-fidelity optimization to improve ride-sharing system performance. The proposed approach achieves a 27% reduction in average waiting time compared to the original relocation model due to its higher fidelity. The results suggest that this hybrid approach may provide an appealing avenue for certain. In addition, the predicted solutions may not always be feasible due to physical constraints that must be met due to the high-dimensional decision space and sparse decision space and the difficulty of capturing patterns with large amounts of data.

In Chen and Cassandras (2020), authors propose a dynamic vehicle assignment strategy that reduces passengers' waiting times and travel times in a real-time ride-sharing system. In order to overcome the "curse of dimensionality" in RSS optimization formulations, authors adopted an event-driven Receding Horizon Control (RHC) approach that reacts to real-time random events. Both waiting times and travel times are optimized by modeling the RSS as a discrete event system. Despite the paper's focus on reducing the complexity of the vehicle assignment problem, it does not address how profit can be optimized by changing the ride price rate at an optimal rate.

An interactive bathtub model is proposed by Sadeghi and Smith (2019) for explaining the traffic dynamics of ride-sourcing vehicles in undifferentiated streets that are exclusively serviced by ride-sourcing services. Using this model, only basic inputs are required. To model ride-sourcing vehicle movements accurately, it considers three states: idling, picking up and collecting, and delivering. It also takes into account travel time, waiting time, and service time. The interactive bathtub model provides a framework for developing effective traffic management strategies for cities served by ride-sourcing vehicles, but it has not been used to minimize waiting and travel times for travelers.

To minimize the wait time of customers, two indirect control methods are introduced by Fan (2020) to optimize the location of drivers waiting. However, both the sharing information control method and the pay-to-control method are only found to have near-optimal controls with the algorithm proposed. In order to further optimize customer waiting time, dynamic pricing and routing incentives have not yet been investigated.

Two indirect control methods are also proposed by Yengejeh and Smith (2021); information on the location of other waiting drivers is shared with a subgroup of drivers, and drivers are paid to relocate. Using approximation and LP-rounding algorithms, the model is further modified to optimize waiting time; these algorithms manipulate drivers' decisions to relocate to a desired waiting location, thereby minimizing maximum customer and driver wait times. As the proposed model is combinatorial in nature, it is still unable to handle scalability issues, including the number of customers and drivers in the system. A real-life scenario where the number of available drivers and the number of ride requests changes over time is also overlooked in the model, which could further optimize a driver's profit (by applying dynamic pricing).

In Luo and Saigal (2017), authors propose a dynamic pricing approach to on-demand ride-sharing that addresses dimensionality and improves efficiency in pricing decisions by using continuous-time, continuous-space methods. This approach is shown to provide efficient pricing decisions, which can lead to increased revenue for ride-sharing platforms. In large-scale systems, however, such a method may be difficult to implement due to the computation effort required.

Beojone *et al*. (2024) introduce a multi-layer control strategy for efficiently repositioning empty ride-hailing vehicles, aiming to bridge the gap between proactive repositioning strategies and micro-management. They demonstrate their framework's effectiveness and efficiency in reducing average passenger waiting times and abandonment rates through experimental validation using an agent-based simulator on a real network.

As ride-sharing becomes an increasingly popular mobility option, Shen *et al*. (2023) focus on the modeling and control of large-scale multimodal systems in an automated and shared environment. The authors develop a macroscopic fundamental diagram–based traffic flow model to capture the demand-supply relationship and traffic dynamics at an aggregated network level, considering private cars, taxis, and both single- and multioccupancy ride-sharing vehicles. The model incorporates multimodal meeting functions for passenger-vehicle matching, enabling the formulation of optimization strategies for region-level dispatching and vehicle relocation. Experimental results demonstrate that the proposed model effectively reproduces traffic dynamics and multimodal interactions under various conditions, while the control-based dispatching strategies enhance the efficiency of all modes, reduce travel costs for riders, and improve service levels for passengers.

Most literature addresses the optimization problem of matching passengers with drivers in static settings. Mejjaouli and Tadj (2023) introduce a dynamic approach, using differential equations to model the system's evolution with the aim of maximizing profit over the planning horizon. They apply optimal control theory to determine the ideal rate of change in ride pricing.

This brief literature review shows that even though pricing in ride-sharing has been considered from different perspectives, many questions remain unanswered, and there is still room for research to fill gaps, such as the lack of models covering the ride-sharing problem, including state and control variables and the scarcity of continuous-time analysis. Also, while considerable attention has been given to traffic dynamics and minimizing customer waiting and service times, there is a notable lack of focus on profit optimization for ride-sharing platform operators. This critical aspect remains underexplored in the literature, presenting an opportunity for future research to address the financial sustainability and efficiency of ride-sharing platforms.

3. MODEL PREDICTIVE CONTROL APPROACH

In this work, the problem of obtaining the optimal ride price over a time horizon for ride-share platforms is addressed. Ride-share platforms generate income through connecting drivers with ride requesters that pay a ride fare. The ride fare is then divided between the platform and the ride provider according to pre-agreed terms. In the developed model, a decision-making horizon where the number of ride providers and the number of ride requests change over time. A growth rate μ of the number of ride requests is considered. This reflects the increase in customer base and adopters of the platform over time. Also, the utility factor for passengers, which is the difference between the reference price and the shown price, is considered when building the model. Actually, the reference price is the price that the customer finds acceptable for a certain product or service within a specific market. In terms of cost, there is a cost associated with serving each customer, denoted as ϵ in our model. This cost covers the cost of developing and maintaining the app, marketing, hiring drivers and training them, … etc. Another cost to be considered pertains to changing the ride price. This cost is captured by β in the developed model and will be considered as quadratic, as the marginal impact of changing the ride price usually increases as the amount of change price increases (Kumar and Sethi, 2009).

Model Predictive Control is a powerful tool for managing dynamic systems where conditions are constantly changing. By predicting future outcomes and optimizing control actions accordingly, MPC can significantly improve the efficiency and effectiveness of systems like dynamic pricing in ride-share platforms. MPC uses a mathematical model of the system to predict its future behavior over a finite prediction horizon. This model describes how the state of the system evolves over time in response to changes in control inputs (like ride pricing in a ride-share platform). At each control step, MPC solves an optimization problem. The goal is to determine the sequence of future control actions (e.g., rate changes in ride prices) that optimize a given objective function, such as maximizing profit or minimizing cost, over the prediction horizon. The objective function typically balances different factors like revenue, customer demand, and operational constraints.

To describe the model, Table 1 summarizes the parameters and variables of the model.

Table 1. Parameters and Variables

H:	Length of planning horizon
T:	Length of prediction horizon
$x(t)$:	Number of ride requests at time t after checking the price (state variable)
$p(t)$:	Ride price rate (per hour) at time t (state variable)
$u(t)$:	Rate of change in the ride price rate (control variable)
ϕ:	Sensitivity of riders to the ride price rate
μ:	Market growth rate for ride sequesters
π:	Utility factor for passengers for difference between the reference price and shown price
ε:	Cost of serving riders
β:	Cost of changing the price rate

Let $[0, H]$ represent the planning horizon, and for any $t_0 > 0$, $T > 0$ with T much smaller than H, let $[t_0, t_0 + T]$ represent the prediction horizon. The state of the system at time t is denoted by $x(t)$, the number of ride requests after checking the price, and $p(t)$, the hourly ride price rate. The control variable is $u(t)$, the rate of change in the ride price rate. To describe the evolution of the system, let μ be the market growth rate for ride sequesters, ϕ be the sensitivity of riders to the ride price rate, π be the utility factor for passengers for the difference between the reference price and shown price, and R be the reference price. Since the number of ride requests subscribers decreases (resp., increases) with an increase (resp., decrease) in the ride price rate, the dynamics of the system are described by the differential equations:

$$\frac{dx(t)}{dt} = \mu - \phi u(t) + \pi\,[R - p(t)], \qquad x(0) = x_0, \tag{3.1}$$

$$\frac{dp(t)}{dt} = u(t), \qquad p(0) = p_0, \tag{3.2}$$

where x_0 and p_0 are known constants.

To determine the optimal control and state variables, let ε be the cost of serving riders and let β be the cost of changing the price rate. The revenue $p(t)x(t)$ and the nonlinear cost $\varepsilon x(t) - \beta u(t)^2$ are used to determine the total profit to maximize during the prediction interval:

$$\max J(t_0, u) = \int_{t_0}^{t_0+T} [p(t)x(t) - \varepsilon x(t) - \beta u(t)^2] dt. \tag{3.3}$$

Thus, the problem is to determine the optimal rate of change in the price that maximizes the objective function (3.3) subject to the state equations (3.1)-(3.2). The integral in (3.3) is calculated by dividing the prediction interval $[t_0, t_0 + T]$ into m intervals of equal length $h = T/m$ and employing the trapezoid formula. Using matrix notation, the above objective function becomes:

$$J(t_0, u) = M(t_0) + G(t_0)^\top U(t_0) - U(t_0)^\top Q U(t_0),$$

where

$$M(t) = \frac{h}{2} \begin{Bmatrix} - h\pi(2\alpha + m)p(t)^2 + h(2\alpha + m)(\mu + \pi R + \varepsilon\pi)p(t) + \\ 2mp(t)x(t) - 2m\varepsilon x(t) - h\varepsilon(2\alpha + m)(\mu + \pi R) \end{Bmatrix}$$

is independent of the control variable,

$$U(t) = \begin{bmatrix} u(t) \\ u(t+h) \\ \vdots \\ u(t + (m-1)h \end{bmatrix}, \quad G(t) = \begin{bmatrix} g(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad Q = \begin{bmatrix} q & 0 & \cdots & 0 \\ 0 & h\beta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & h\beta \end{bmatrix},$$

with

$$\begin{aligned} q &= \frac{h}{2}(\beta + 2h^2\gamma\phi + m^2h^2\phi), \\ g(t) &= \xi\_0 + \xi_1 x(t) + \xi_2 p(t), \end{aligned}$$

and

$$\begin{aligned} \xi_0 &= h^3\gamma\mu + h^3\gamma\pi R + h^2\varepsilon\phi\,\alpha - \frac{m^2h^3\mu}{2} - \frac{m^2h^3\pi R}{2} - \frac{mh^2\varepsilon\phi}{2}, \\ \xi_1 &= h^2\alpha - \frac{mh^2}{2}, \\ \xi_2 &= -h^3\gamma\pi - h^2\phi\alpha - \frac{mh^2\phi}{2} - mh\pi. \end{aligned}$$

Note that the matrix Q is positive definite. Therefore, the global minimum of J is reached at $U(t_0)$ given by:

$$U(t) = \frac{1}{2} Q^{-1} G(t).$$

In receding horizon, we obtain $u(t)$ as $u(t) = [1, 0, 0, \cdots, 0]U(t)$, which yields:

$$u(t) = \frac{\xi_0 + \xi_1 x(t) + \xi_2 p(t)}{2q}. \tag{3.4}$$

Since the control variable $u(t)$ is found in terms of the state variables $x(t)$ and $p(t)$, we substitute its expression in the state equations, which yields:

$$\begin{aligned}\frac{dx(t)}{dt} &= \left(\mu+\pi R-\frac{\phi\xi_0}{2q}\right)-\frac{\phi\xi_1}{2q}x(t)-\left(\frac{\phi\xi_2}{2q}+\pi\right)p(t),\\ \frac{dp(t)}{dt} &= \frac{\xi_0}{2q}+\frac{\xi_1}{2q}x(t)+\frac{\xi_2}{2q}p(t).\end{aligned}$$

Using matrix notation, let $y(t)=[x(t)\quad p(t)]^\top$, $y_0=[x_0\quad p_0]^\top$,

$$A=\begin{bmatrix}a_{00} & a_{01}\\ a_{10} & a_{11}\end{bmatrix}=\begin{bmatrix}-\frac{\phi\xi_1}{2q} & -\left(\frac{\phi\xi_2}{2q}+\pi\right)\\ \frac{\xi_1}{2q} & \frac{\xi_2}{2q}\end{bmatrix},\qquad b=\begin{bmatrix}b_1\\ b_2\end{bmatrix}=\begin{bmatrix}\mu+\pi R-\frac{\phi\xi_0}{2q}\\ \frac{\xi_0}{2q}\end{bmatrix}.$$

Then, the above differential system can be written as:

$$\frac{d}{dt}y(t)=Ay(t)+b,\qquad y(0)=y_0.$$

The matrix A has two eigenvalues given by:

$$\begin{aligned}m_1 &= \frac{a_{00}+a_{11}-\sqrt{a_{00}^2-2a_{00}a_{11}+a_{11}^2+4a_{01}a_{10}}}{2},\\ m_2 &= \frac{a_{00}+a_{11}+\sqrt{a_{00}^2-2a_{00}a_{11}+a_{11}^2+4a_{01}a_{10}}}{2}.\end{aligned}$$

The corresponding eigenvectors are:

$$V_1=\begin{bmatrix}v_1\\ 1\end{bmatrix},\qquad V_2=\begin{bmatrix}v_2\\ 1\end{bmatrix},$$

where

$$\begin{aligned}v_1 &= \frac{a_{00}-a_{11}-\sqrt{a_{00}^2-2a_{00}d+a_{11}^2+4a_{01}a_{10}}}{2},\\ v_2 &= \frac{a_{00}-a_{11}+\sqrt{a_{00}^2-2a_{00}d+a_{11}^2+4a_{01}a_{10}}}{2}.\end{aligned}$$

Using standard methods, we readily get the optimal solutions of the differential system, which are the optimal state variables:

$$\begin{aligned}x(t) &= \frac{v_1}{v_1-v_2}\left[\left(x_0-p_0v_2+\frac{b_1-b_2v_2}{m_1}\right)e^{-m_1(t_0-t)}-\frac{b_1-b_2v_2}{m_1}\right]\\ &\quad+\frac{v_2}{v_1-v_2}\left[\left(-x_0+p_0v_1-\frac{b_1-b_2v_1}{m_2}\right)e^{-m_2(t_0-t)}-\frac{b_1-b_2v_1}{m_2}\right]\end{aligned}\tag{3.5}$$

$$\begin{aligned}p(t) &= \frac{1}{v_1-v_2}\left[\left(x_0-p_0v_2+\frac{b_1-b_2v_2}{m_1}\right)e^{-m_1(t_0-t)}-\frac{b_1-b_2v_2}{m_1}\right]\\ &\quad+\frac{1}{v_1-v_2}\left[\left(-x_0+p_0v_1-\frac{b_1-b_2v_1}{m_2}\right)e^{-m_2(t_0-t)}-\frac{b_1-b_2v_1}{m_2}\right]\end{aligned}\tag{3.6}$$

Substituting (3.5)-(3.6) into (3.4) yields the optimal control variable:

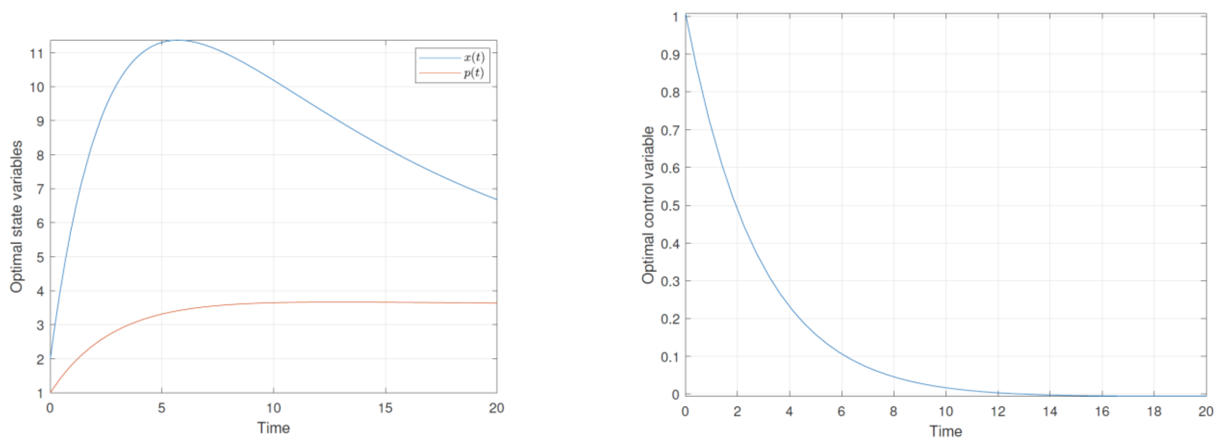
$$
\begin{aligned}
u(t) \quad = \quad & \frac{\xi_0}{2q} + \frac{\xi_1}{2q}\left\{\frac{v_1}{v_1 - v_2}\left[\left(x_0 - p_0 v_2 + \frac{b_1 - b_2 v_2}{m_1}\right) e^{-m_1(t_0 - t)} - \frac{b_1 - b_2 v_2}{m_1}\right]\right. \\
& \left. + \frac{v_2}{v_1 - v_2}\left[\left(-x_0 + p_0 v_1 - \frac{b_1 - b_2 v_1}{m_2}\right) e^{-m_2(t_0 - t)} - \frac{b_1 - b_2 v_1}{m_2}\right]\right\} \\
& + \frac{\xi_2}{2q}\left\{\frac{1}{v_1 - v_2}\left[\left(x_0 - p_0 v_2 + \frac{b_1 - b_2 v_2}{m_1}\right) e^{-m_1(t_0 - t)} - \frac{b_1 - b_2 v_2}{m_1}\right]\right. \\
& \left. + \frac{1}{v_1 - v_2}\left[\left(-x_0 + p_0 v_1 - \frac{b_1 - b_2 v_1}{m_2}\right) e^{-m_2(t_0 - t)} - \frac{b_1 - b_2 v_1}{m_2}\right]\right\}
\end{aligned} \tag{3.7}
$$

Finally, the optimal objective function value is found as:

$$J^* = M(t) + \frac{1}{4q} g(t). \tag{3.8}$$

4. NUMERICAL EXAMPLE AND MANAGERIAL IMPLICATIONS

Consider a ride-share platform that should optimize the price rate of change for optimum ride requests and ride price. In this numerical example, the base parameter values are as follows: $T = 20$, $\varepsilon = 0.01$, $\mu = 10$, $\beta = 0.3$, $\pi = 3.9$, $\phi = 5$, $R = 1$, $h = 0.1$, $x_0 = 2$, and $p_0 = 1$. The previous results found in Section 3 are implemented in Matlab, and the optimal ride requests $x(t)$, ride price $p(t)$, and price rate of change $u(t)$ are depicted in Figure 1. The maximum total profit is also calculated, and it is equal to $J^* = 609.44$.



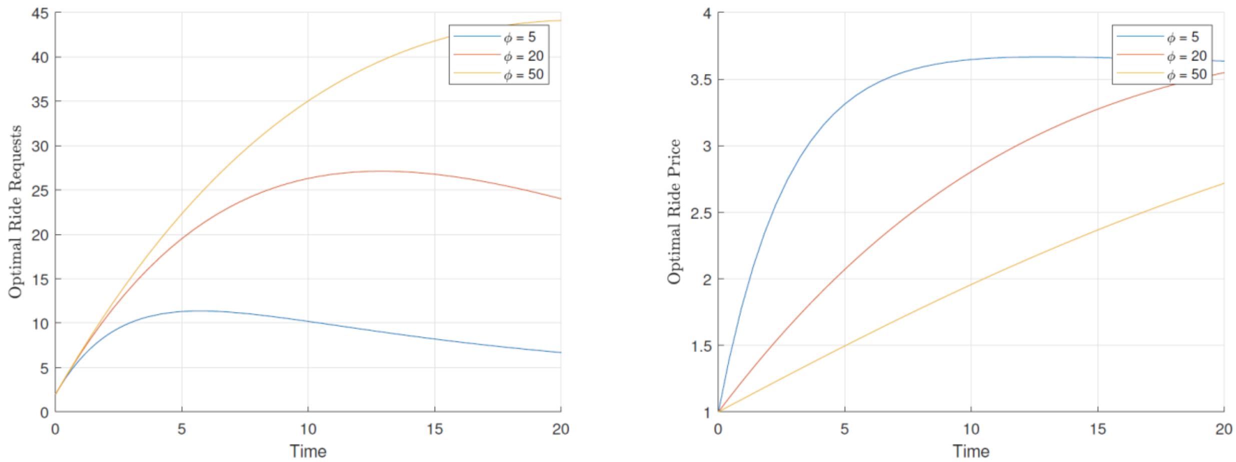
(a) (b)

Figure 1. (a) Optimal State Variables; (b) Control Variable (right)

In this part, the effect of the different model parameters on the state (number of ride requests and ride price), control variables (price rate of change) as well as the objective function value will be discussed. In each numerical experiment, we vary one parameter, and the rest of the parameters is set at their base values. Managerial implications for each numerical experiment are also be added to equip the decision maker (the platform operator) with the necessary tools to control state variables and achieve maximum profit.

4.1. Effect of ϕ, sensitivity of riders to the ride price rate:

Figure 2 presents the effect of the sensitivity of riders to the ride price rate ϕ on the optimal rider requests and ride price. According to Figure 2(a), the number of ride requests increases as the sensitivity to the ride price rate increases for $\phi = [5, 20, 50]$. This is due to the decrease in the ride price, which is considered comparatively high, as depicted in Figure 2(b).



(a) (b)

Figure 2. Effect of ϕ on the State Variables

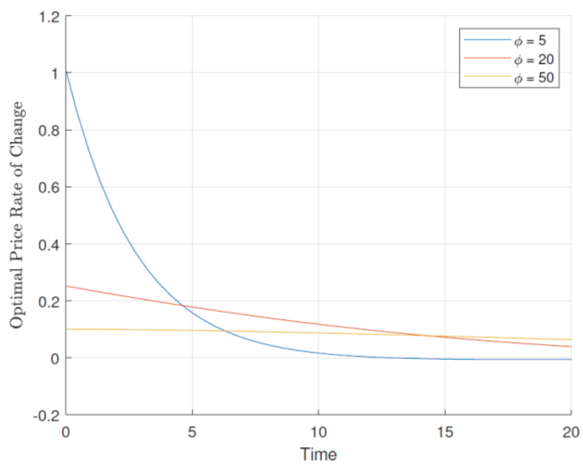


Figure 3. Effect of ϕ on the Control Variable

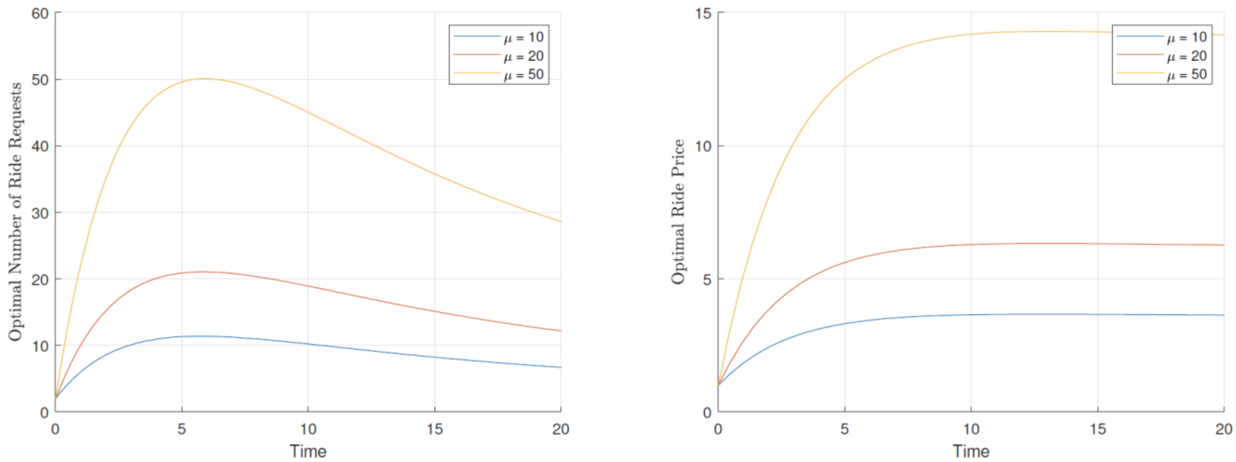
In fact, when customers are more sensitive to the ride price, the decision maker (the ride-sharing provider) should reduce the ride price to attract more customers. Actually, decreasing ride prices has a positive impact on the objective function that increases from 609 to 1238 for $\phi = 5$ and $\phi = 20$, respectively (Table 2). However, when sensitivity gets very high $(\text{i.e.}, \phi = 50)$, the ride-sharing provider strategy should be offering aggressive price discounts in order to keep almost the same profit levels. Following this strategy, the decrease income per rider will be compensated by the increase of ride requesters. This strategy will help the decision-maker secure a higher market share at the expense of income per rider. For example, the profit is equal to 1238 and 1307 for $\phi = 20$ and $\phi = 50$, respectively.

Table 2. Variations of J^* with ϕ

ϕ	5	20	30
Objective function value	609	1238	1307

4.2. Effect of μ, market growth rate for ride requesters:

The impact of the market growth rate μ on the optimal ride requests and the optimal price rate trajectories is investigated and is depicted in Figure 4.



(a) (b)

Figure 4. Effect of μ on the State Variables

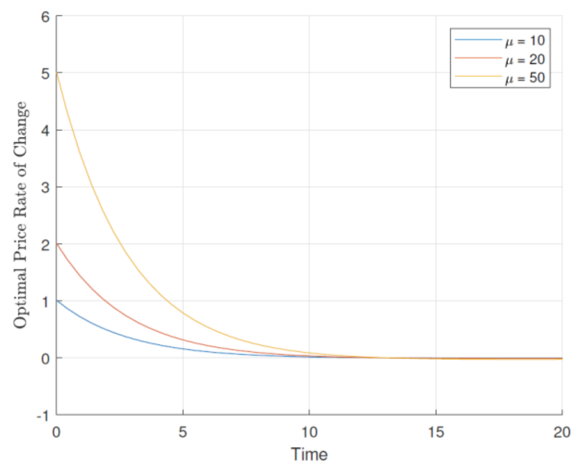


Figure 5. Effect of μ on the Control Variable

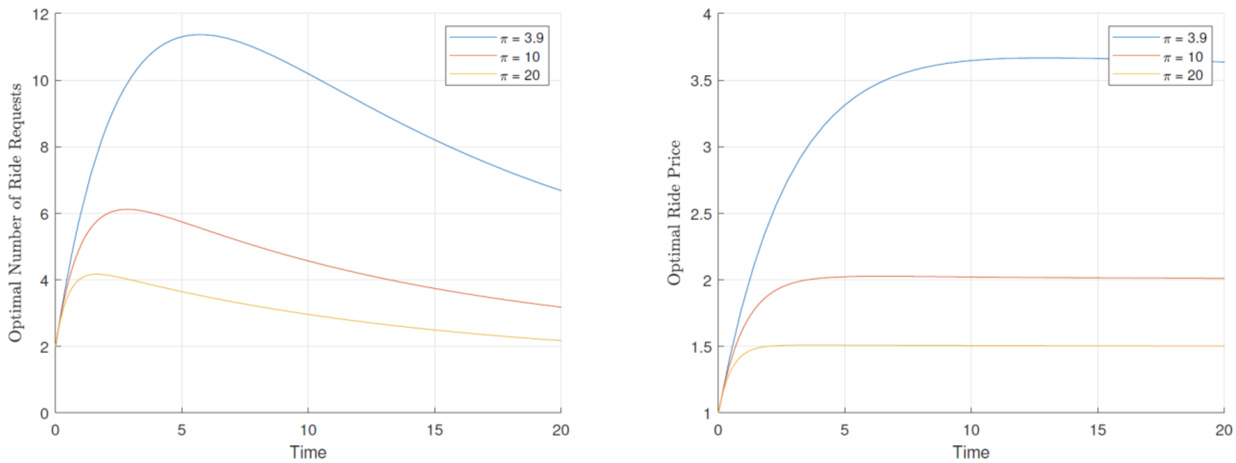
Clearly, the market growth rate has a positive effect on the number of ride requests. When the ride requests increase, the ride-share provider will gain leverage and capitalize on the growth of the customer base by increasing prices and generating more profit. In fact, market growth has always had a positive impact on the ride-share provider and its competitors alike, even though the market share is the same. For instance, the profit increased from 609 to 1908 for $\mu = 10$ and $\mu = 20$, respectively.

Table 3. Variations of J^* with μ

μ	10	20	50
Objective function value	609	1908	10125

4.3. Effect of π, utility factor for passengers for difference between the reference price and shown price:

The effect of the utility factor π on the objective function, the price rate and the number of ride requests depends largely on the reference price R. When R is set at a low level $R = 1$, it can be seen from Figure 6 (right) that the ride-sharing platform cannot highly raise the ride price rate. From a management point of view, this scenario happens in a very competitive market where many ride-sharing providers are competing for demand and focused on attracting a bigger customer base by reducing prices to get a higher market share and try to drive out competitors from the market.



(a) (b)

Figure 6. Effect of π on the State Variables for Low R

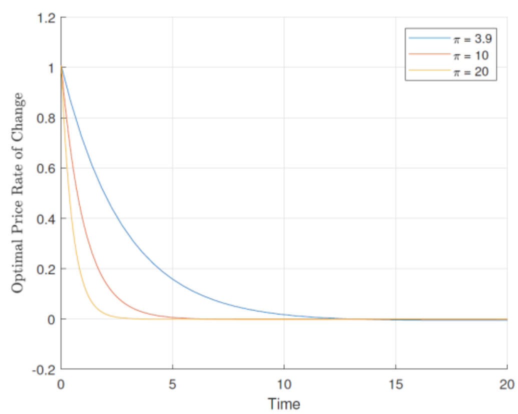


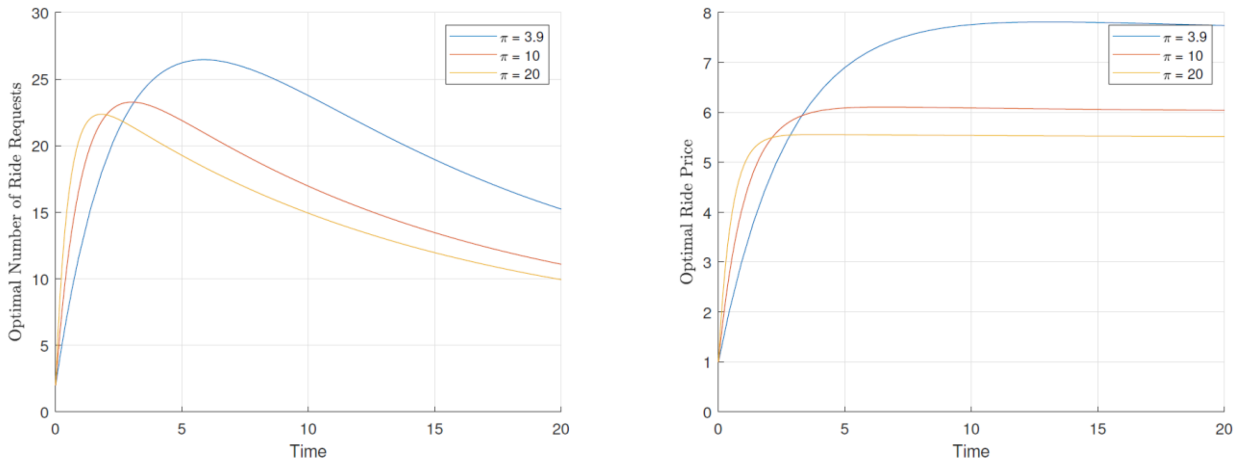
Figure 7. Effect of π on the Control Variable for Low R

This will result in the ride-sharing platform not being able to charge enough for their customers, which negatively affects the profit (Table 4).

Table 4. Variations of J^* with π when $R = 1$

π	3.9	10	20
Objective function value	609	178	89

However, when R gets higher $R = 5$, the ride-sharing platform can set prices at higher levels, as depicted in Figure 8 (right). Also, when R is set at high levels $R = 5$, the ride-sharing platform can attract more customers by setting its price lower than the reference price, as shown by the trajectories of the price rate and ride requests depicted in Figure 8. From a management perspective, this scenario happens in markets where transportation is expensive for reasons such as markets with high purchasing power or markets with poor public transportation infrastructure.



(a) (b)

Figure 8. Effect of π on the State Variables for High R

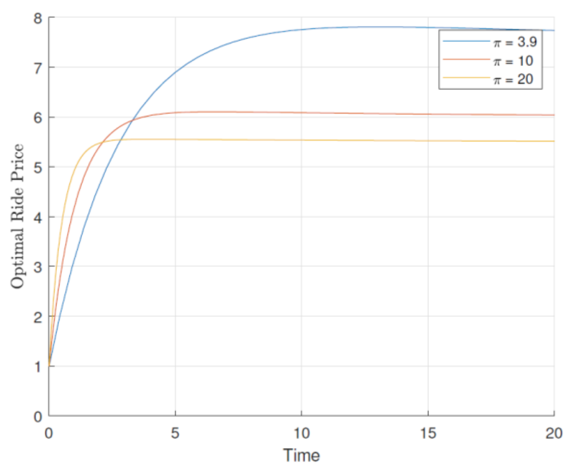


Figure 9. Effect of π on the Control Variable for High R

Therefore, this results in higher profit levels, as shown in Table 5.

Table 5. Variations of J^* with π when $R = 5$

π	3.9	10	20
Objective function value	2950	1948	1651

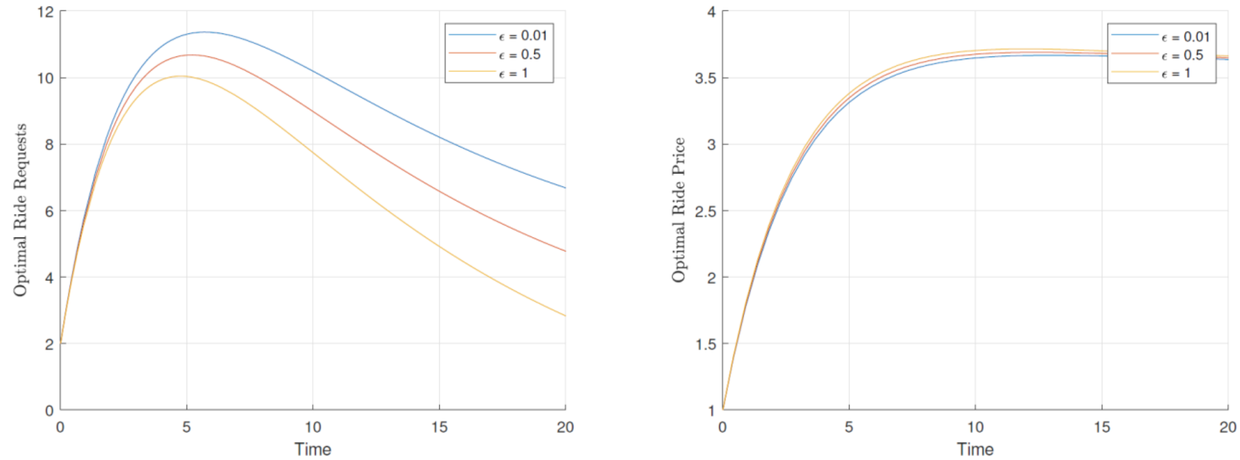
4.4 Effect of ε, cost of serving riders:

In real life, the cost of serving customers can be attributed to market regulations, the strength of unions and their negotiation power, the cost of developing and maintaining the platform, income taxes, …etc. As the cost of serving riders increases, the profit decreases, as shown in Table 6.

Table 6. Variations of J^* with ε

ε	0.01	0.5	1
Objective function value	609	467	322

This is due to the fact that an increase in the cost of serving riders will push the ride-sharing provider to increase prices in order to maintain its profit margin (Figure 11 (right)) and pass this cost increase to the customers. However, when the cost of serving is high (i.e., $\varepsilon = 1$), this may lead to the ride-sharing provider losing their customer base (Figure 10 (left)), which is reflected negatively on the profit, as can been seen from the profit level that drops from 609 to 322 for $\varepsilon = 0.01$ and $\varepsilon = 1$, respectively.



(a) (b)

Figure 10. Effect of ε on the State Variables

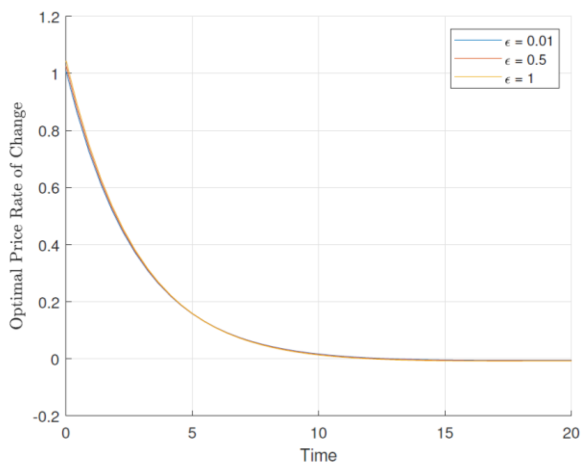
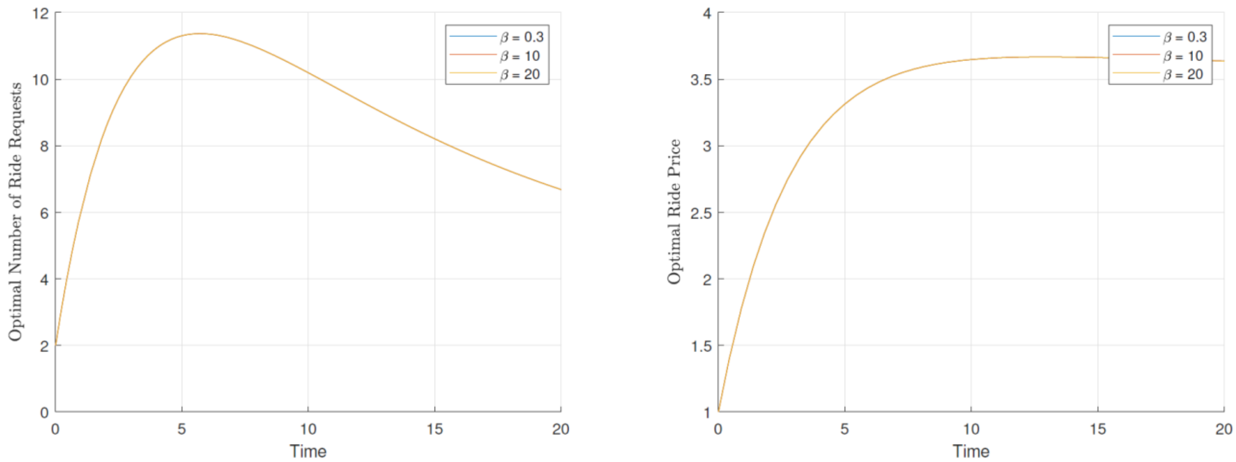


Figure 11. Effect of ε on the Control Variable

4.5. Effect of $\boldsymbol{\beta}$, cost of changing the price rate:

In Figure 12, the effect of the cost of changing the price rate on the optimal price rate and number of ride requests is investigated.



(a) (b)

Figure 12. Effect of β on the State Variables

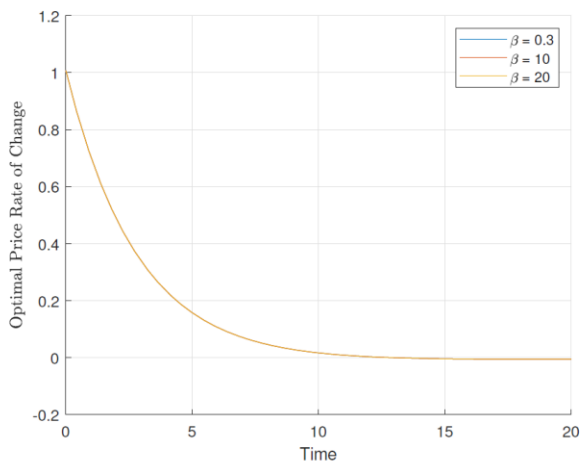


Figure 13. Effect of β on the Control Variable

It can be seen that the trajectories (of both price and ride requests) for the different values of β are almost identical and don't show any major differences when β changes from 3 to 20. This is also reflected on the objective function that slightly decreases as β increases, as depicted in Table 7. In fact, this decrease in the objective function happens because of the cost of changing the price rate, which in real-life scenarios might happen because of marketing expenses to develop the brand name or by improving the ride experience quality.

Table 7. Variations of J^* with β

β	3	10	20
Objective function value	609	596	582

5. CONCLUSION

Significant technological advances in the last few years have changed the ways businesses are conducted. Oftentimes, information is needed on the spot in order to make the best possible decision. We have considered in this paper a model for dynamic pricing in ride-share platforms. Using the model predictive control approach, the optimal rate of change in the ride price is derived in closed form. Managerial insights are obtained that provide useful information about the system parameters and how they affect the model solution.

The managerial implications results show that the number of ride requests is negatively affected by the ride price rate, which pushes platform operators to lower their prices to attract a bigger customer base and market share. On the other side, if the number of ride requests increases because of a higher utility factor or market growth, decision-makers can capitalize on that to increase ride rates. For example, when the utility factor changes from 3.9 to 10, the objective function decreases from 609 to 178 because the platform can charge its customers enough to drive their profits up. Moreover, when the market growth rate increases from 10 to 20, the objective function increases from 609 to 1908.

Also, the discussed results reveal that the cost of serving each ride (because of market regulations, strength of unions and their negotiation power, income taxes, …etc.) and the cost of changing price rates (marketing expenses, brand building, ride quality improvement, …etc.) is found to have a negative effect on the ride price and the number of subscribers. For instance, the objective function decreased from 609 to 467 when the cost of serving riders changed from 0.01 to 0.5.

As an expansion to this work, it is recommended to include more platform dynamics like the number of riders and service providers joining the platform and raising prices during certain times (i.e., rush hours). Also, this research can be extended to consider a situation where two platforms, e.g., Uber vs. Lyft, compete for customers. Such a model could be tackled from a game theory viewpoint and involve different techniques and tools other than optimal control.

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